AN EDUCATIONAL INSTITUTE

**XI– MATHEMATICS TERM 1- Practice Paper 02**

# Answer key

SECTION: A

1. (c) $\phi$
2. (c) 0
3. (c) {1/2, -2}
4. (c) Domain = R, Range = $(-\infty, 2)$
5. (d) $\pi/4$
6. (b) 81
7. (a) 10
8. (c) i
9. (b) (-1, 3)
10. (b) 10
11. (c) $-1760 \times 9 \times y^3$
12. (b) 15
13. (c) 216
14. (b) $2x \cos x - x^2 \sin x$
15. (d) $2 < x < 1$ and $x < 3$
16. (c) -1
17. (a) 604800
18. (d) -1
19. (b) Both Assertion and reason are correct but reason is not correct explanation for Assertion
20. (d) Assertion is false but reason is true.

## SECTION: B

### (VSA Questions of 2 marks each)

21. Using Binomial theorem,

$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 + {}^6C_4(2x)^2(3)^4 - 6\,C_5(2x)(3)^5 + {}^6C_6(3)^6$

$= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) + 15(4x^2)(81) - 6(2x)(243) + 729$

$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

22. $\frac{\tan x+\sec x-1}{\tan x-\sec x+1}$

$= \frac{\tan x+\sec x-(\sec^2 x-\tan^2 x)}{\tan x-\sec x+1}\left[\begin{array}{l} 1+\tan^2 x=\sec^2 x \\ \Rightarrow 1=\sec^2 x-\tan^2 x \end{array}\right]$

$= \frac{(\tan x+\sec x)-(\sec x-\tan x)(\sec x+\tan x)}{\tan x-\sec x+1}$ $[a^2-b^2=(a-b)(a+b)]$

$= \frac{(\tan x+\sec x)[1-(\sec x-\tan x)]}{\tan x-\sec x+1}$ [Taking common]

$= \frac{(\tan x+\sec x)(\tan x-\sec x+1)}{(\tan x-\sec x+1)}$

$= \tan x + \sec x = \frac{\sin x}{\cos x} + \frac{1}{\cos x} = \frac{1+\sin x}{\cos x}$

OR

L.H.S $= \frac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x}$

$\cos A - \cos B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$= \frac{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}}$

$= \frac{2\cos\frac{12x}{2}\cos\frac{2x}{2}}{2\cos\frac{12x}{2}\sin\frac{2x}{2}} = \frac{2\cos 6x\cos x}{2\cos 6x\sin x}$

$= \frac{\cos x}{\sin x} = \cot x$

23. Given that, $(x-1)(x^2-5x+7) < (x-1)$

$\therefore (x-1)(x^2-5x+6) < 0$

$\Rightarrow (x-1)(x-2)(x-3) < 0$

$\Rightarrow x \in (-\infty, 1) \cup (2,3)$

24. $= \frac{5+12i+5-12i+2\sqrt{25+144}}{5+12i-5+12i}$

$= \frac{3}{2i} = \frac{3i}{-2} = 0 - \frac{3}{2}i$

Therefore, the conjugate of $z = 0 + (3/2)i$

25. 6 or 9/10

## SECTION C
## (Short Answer Questions of 3 Marks each)

26. Domain: square root to be defined, $x - [x] \geq 0$

but $x - [x] \neq 0$

so, $x - [x] > 0$

$\Rightarrow$ x > [x]

this is true for all real value of $x$.

Range: we know, $\{x\} = x - [x]$ where $\{.\} is fractional part of x.$

so, $f(x) = \sqrt{\{x\}}$

we know, $0 \leq \{x\} < 1$

$\Rightarrow 0 \leq \sqrt{\{x\}} < 1$

Hence range of the function, $y \in [0,1)$

27. L.H.S:

$\frac{1}{2} \times \sin\ 10° \times \sin\ 50° \times \sin\ 70°\ \left(\because \sin\ 30° = \frac{1}{2}\right)$

$= \frac{1}{4} \times (2\sin\ 10° \times \sin\ 50°) \times \sin\ 70°$

$= \frac{1}{4} \times \left(\cos\ (50° - 10°) - \cos\ (50° + 10°)\right) \times \sin\ 70°$

$(\because 2\sin\ A\sin\ B = \cos\ (A - B) - \cos\ (A + B))$

$= \frac{1}{4} \times \left(\cos\ 40° - \frac{1}{2}\right) \times \sin\ 70°\ \left(\because \cos\ 60° = \frac{1}{2}\right)$

$= \frac{1}{8}.2\cos 40° - \sin 70° - \frac{1}{8}\sin 70°$

$= \frac{1}{8}\left(\sin\ (40° + 70°) + \sin\ (70° - 40°)\right) - \frac{1}{8}\sin\ 70°$

$(\because 2\cos\ A\sin\ B = \sin\ (A - B) - \sin\ (A + B))$

$= \frac{1}{8}\left(\sin\ 110° + \frac{1}{2}\right) - \frac{1}{8}\sin\ (180° - 110°)$

$= \frac{1}{8}\sin 110^\circ + \frac{1}{16} - \frac{1}{8}\sin 110^\circ$

$= \frac{1}{16} =$ R.H.S

$\cos\ 20^\circ \cos\ 40^\circ \cos\ 60^\circ \cos\ 80^\circ$

$= \sin\ 70 \sin\ 50 \sin\ 30 \sin\ 10 = 1/16$

28. f(x)= $\frac{1}{x^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \left[ \frac{\frac{1}{(x+h)^2} - \frac{1}{(x)^2}}{h} \right]$$

$$\lim_{h\to 0} \left[ \frac{\frac{(x)^2 - (x+h)^2}{(x+h)^2 (x)^2}}{h} \right]$$

$$\lim_{h\to 0} \frac{1}{h} \left[ \frac{(x)^2 - (x+h)^2}{(x+h)^2 (x)^2} \right]$$

$$= \frac{2}{x^3}$$

29. We have, $R_1 = \{(x, y) \mid y = 2x + 7$, where $x \in R$ and $-5 \le x \le 5\}$

Domain of $R_1 = \{-5 \le x \le 5, x \in R\} = [-5,5]$

$x \in [-5,5]$

$=> 2x \in [-10,10]$

$=> 2x + 7 \in [-3,17]$

Range is $[-3,17]$

30. 0

31. $\left(x^2 + \frac{3}{x}\right)^5$

$= {}^5C_0 \cdot (x^2)^5 \cdot \left(\frac{3}{x}\right)^0 + {}^5C_1 \cdot (x^2)^{5-1} \cdot \left(\frac{3}{x}\right)^1 + {}^5C_2 \cdot (x^2)^{5-2} \cdot \left(\frac{3}{x}\right)^2 + {}^5C_3 \cdot (x^2)^{5-3} \cdot \left(\frac{3}{x}\right)^3 + {}^5C_4 \cdot (x^2)^{5-4} \cdot \left(\frac{3}{x}\right)^4 + {}^5C_5 \cdot (x^2)^0 \cdot \left(\frac{3}{x}\right)^5$

$= x^{10} + 5 \cdot x^8 \cdot \frac{3}{x} + 10 \cdot x^6 \cdot \frac{9}{x^2} + 10 \cdot x^4 \cdot \frac{27}{x^3} + 5 \cdot x^2 \cdot \frac{81}{x^4} + \frac{243}{x^5}$

$= x^{10} + 15x^7 + 90x^4 + 270x + \frac{405}{x^2} + \frac{243}{x^5}$

**SECTION D**

**(Long answer type questions (LA) of 5 marks each)**

32. $\sec^2 x = 1 + \tan^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

$\sec x = \pm\frac{5}{4}$ or $\cos x = -\frac{4}{5} \because \pi < x < \frac{3\pi}{2}$

We have $2\sin^2 \frac{x}{2} = 1 - \cos x = 1 - \left(-\frac{4}{5}\right) = \frac{9}{5}$

$\Rightarrow \sin^2 \frac{x}{2} = \frac{9}{10}$

$\Rightarrow \sin \frac{x}{2} = \frac{3}{\sqrt{10}} \quad \because \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$\Rightarrow \sin \frac{x}{2} = \frac{3}{\sqrt{10}} \quad \because \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Again $2\cos^2 \frac{x}{2} = 1 + \cos x = 1 + \left(-\frac{4}{5}\right) = \frac{1}{5}$

$\cos^2 \frac{x}{2} = \frac{1}{10}$

or $\cos \frac{x}{2} = -\frac{1}{\sqrt{10}} \quad \because \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$\tan \frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}} = -3$

OR

We have, LHS $\cos^2 X + \cos^2 \left(X + \frac{\pi}{3}\right) + \cos^2 \left(X - \frac{\pi}{3}\right) + \cos^2 = \frac{3}{2}$

$= \cos^2 x + \left[\cos\left(x + \frac{\pi}{3}\right)\right]^2 + \left[\cos\left(x - \frac{\pi}{3}\right)\right]^2$

$= \cos^2 x + \left(\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3}\right)^2 + \left(\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3}\right)^2$

$= \cos^2 x + \left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right)^2 + \cos^2 x + \left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)^2$

$= \text{cis}^2 x + \left[\frac{1}{4}\cos^2 x + \frac{3}{4}\sin^2 x - \frac{\sqrt{3}}{2}\sin x\cos x + \frac{1}{4}\cos^2 x + \frac{3}{4}\sin^2 x + \frac{\sqrt{3}}{2}\sin x + \cos x\right]$

$= \cos^2 x + 2\left(\frac{\cos^2 x}{4} + \frac{3\sin^2 x}{4}\right) \ [\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)]$

$= \frac{4\cos^2 x + 2\cos^2 x + 6\sin^2 x}{4}$

$= \frac{6(\sin^2 x + \cos^2 x)}{4} = \frac{3}{2} = \text{ RHS } [\because \sin^2 \theta + \cos^2 \theta = 1]$

33.

Step 1: Finding mean

| | | | |
|---|---|---|---|
| 30 − 40 | 3 | $\frac{30+40}{2} = 35$ | 35 × 3 = 105 |
| 40 − 50 | 7 | $\frac{40+50}{2} = 45$ | 45 × 7 = 315 |
| 50 − 60 | 12 | $\frac{50+60}{2} = 55$ | 55 × 12 = 660 |
| 60 − 70 | 15 | $\frac{60+70}{2} = 65$ | 65 × 15 = 975 |
| 70 − 80 | 8 | $\frac{70+80}{2} = 75$ | 75 × 8 = 600 |
| 80 − 90 | 3 | $\frac{80+90}{2} = 85$ | 85 × 3 = 255 |
| 90 − 100 | 2 | $\frac{90+100}{2} = 95$ | 95 × 2 = 190 |
| | $\sum f_i = 50$ | | $\sum f_i x_i = 3100$ |

Mean $\overline{x} = \frac{\sum x_i f_i}{\sum f_i}$

$\Rightarrow \overline{x} = \frac{3100}{50}$

$\Rightarrow \overline{x} = 62$

| Frequency | Mid - point | $(x_i - \overline{x})^2$ | $f_i(x_i - \overline{x})^2$ |
|---|---|---|---|
| 3 | 35 | $(35-62)^2 = (27)^2 = 729$ | 3 × 729 = 2187 |
| 7 | 45 | $(45-62)^2 = (17)^2 = 289$ | 7 × 289 = 2023 |
| 12 | 55 | $(55-62)^2 = (7)^2 = 49$ | 12 × 49 = 588 |
| 15 | 65 | $(65-62)^2 = 3^2 = 9$ | 15 × 9 = 135 |
| 8 | 75 | $(75-62)^2 = (13)^2 = 169$ | 8 × 169 = 1352 |
| 3 | 85 | $(85-62)^2 = (23)^2 = 529$ | 3 × 529 = 1587 |
| 2 | 95 | $(95-62)^2 = (33)^2 = 1089$ | 2 × 1089 = 2178 |
| $\sum f_i = 50$ | | $\sum f_i(x_i - \overline{x})^2 = 10050$ | |

Variance $(\sigma^2) = \frac{1}{N}\sum f_i(x_i - \overline{x})^2$

$= \frac{1}{50} \times 10050$ $[\because N = \sum f_i = 50]$

$= 201$

Standard deviation $(\sigma) = \sqrt{201} = 14.17$

hence, the mean is 62, variance is 201 and the standard deviation is 14.17

34. (a)

Definition (Modulus): The modulus value of a real number $x$ can be denoted by $|x|$, it denotes the non-negative value of $x$ without depending on the sign. It is also called Absolute Value.

Now, we can define modulus function as $f(x) = |x|$

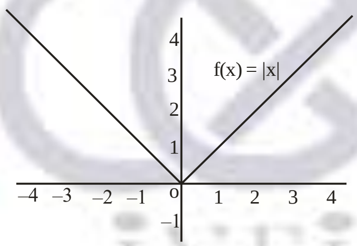
It can be represented as shown below

$f(x) = \{x$, if $x \geq 0$

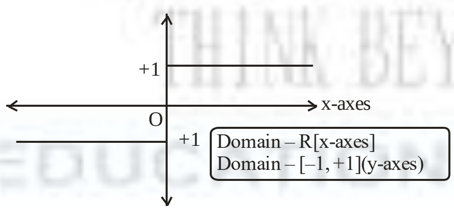
$-x$, if $x < 0\}$

Domain: $R$

Range: $[0, \infty)$




(b) $\text{Sgm}\,(f(x)) = \begin{cases} +1 & \text{for } x > 0 \\ -1 & \text{for } x < 0 \\ 0 & \text{for } x = 0 \end{cases}$




35. $(a+b)^4 - (a-b)^4 = ((a+b)^2 + (a-b)^2)((a+b)^2 - (a-b)^2)$

$\left(\text{Since } x^2 - y^2 = (x+y)(x-y)\right)$

$= ((a+b)^2 + (a-b)^2)\{((a+b) + (a-b))((a+b) - (a-b))\}$

$= ((a+b)^2 + (a-b)^2)((2a)(2b))$

$= 4ab(a^2 + b^2 + 2ab + a^2 + b^2 - 2ab)$

$= 8ab(a^2 + b^2)$

So, $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8(\sqrt{3})(\sqrt{2})\left((\sqrt{3})^2 + (\sqrt{2})^2\right)$

$= 8\sqrt{6}(3+2) = 40\sqrt{6}$

**SECTION E**

**(Case Studies/Passage based questions of 4 Marks each)**

36. (i) $R = \{(x, y): x = y^2, x \in \{9,4,25\}, y \in \{5,3,2,1,-1,-2,-3,-5\}\}$

(ii) $R = \{(x, y): x$ is a factor of $y, x \in \{1,2,4,8\}, y \in \{8\}\}$

(iii) Domain = {1,2,3} and Range = {2,3,4,5,6,7,8,9,10}

OR

Domain = {1,2,3,4,5,6} and Range = {1,2,3,4,5,6,7,8,9,10} )

37. (i) Permutations

(ii) mxnxpx.….

(iii) 720

Or

999

38. (i) $15x^3$

(ii) 54

(iii) $X = 3/32$