CHAPTER-5
CONTINUITY & DIFFERENTIABILITY
01 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	*The function* $f(x) = \{\frac{x^2-9}{x-3},\ x \neq 3\ 5\ \ ,\ x = 3$ (a) *is continuous at* $x = 3$ (b) *has removable discontinuity at* $x = 3$ (c) *has discontinuity of first kind at* $x = 3$ (d) *has discontinuity of second kind at* $x = 3$	1
2.	*The function* $f(x) = \{x + 3, \quad if\ x \leq 2$ $x + 4, \quad if\ x > 2$ (a) *is continuous at* $x = 2$ (b) *has removable discontinuity at* $x = 2$ (c) *has discontinuity of first kind at* $x = 2$ d) *has discontinuity of second kind at* $x = 2$	1
3.	*The greatest integer function* $f(x) = [x]$, *at integer points,* (a) *is continuous* (b) *has removable of first kind* (c) *has removable discontinuity* (d) *has discontinuity of second kind.*	1
4.	*The function* $f(x) = \{(x-1)^n\ sin\ sin\left(\frac{1}{x-1}\right),\quad x \neq 1\quad 0\ ,\quad x = 1$ *is continuous at* $x = 1$ (a) *for all value of* n (b) *for* $n > 0$ (c) $n = 0\ only$ (d) for $n < 0$	1
5.	*The function* $f(x) = \{cos\ cos\ \frac{1}{x}\ ,\ x \neq 0\quad 0\ ,\ x = 0$ (a) *is continuous at* $x = 0$(b) *has discontinuity of second kind at* $x = 0$ (c) has removable discontinuity at $x = 0$ (d) *has discontinuity of first kind at* $x = 0$	1
6.	The number of points of discontinuity of $f(x) = [x]\ in\ [3, 7]$ is (a) 4 (b) 5 (c) 6 (d) 8	1
7.	Let f and g be two real functions continuous at $x = a$, then $f + g$ (a) is continuous at $x = a$ (b) may or may not be continuous at $x = a$ (c) is discontinuous at $x = a$ (d) is continuous at $f(a) + g(a)$.	1
8.	If and g are two real functions continuous at a and $f(a)$ respectively, then (a) $g$of is continuous at $f(a)$ (b) $g$of is continuous at a (c) $fog$is continuous at a (d) fog is continuous at $f(a)$	1
9.	$If\ f(x) = \{\frac{e^x-1}{1n(1+2x)},\ x \neq 0\ k\quad ,\ x = 0$ is continuous at $x = o$, then $k =$	

	(a) $\frac{2}{3}$ (b) 3 (c) 2 (d) $\frac{3}{2}$	1
10.	The function $f(x) = \{\frac{\sin\ ax^{\circ}}{x},\ x \neq 0\ k\quad,\ x = 0$ is continuous at $x = 0, if\ k =$ (a) $\frac{\pi}{180}$ (b) $\frac{a\,\pi}{180}$ (c) $\frac{\pi}{180\,a}$ (d) $\frac{180\,a}{\pi}$	1
11.	What value of k, the function $\begin{cases} kx^2 & , \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at x=2. (a)0 (b)1 (c) 3/4 (d) 3/2	1
12.	The relationship between "a" and "b" so that the function 'f' defined by: f(x)= $\begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at x=3. (a) $a - b = 2/3$ (b) $a = -b$ (c) $a - b = 3$ (d) none of these	1
13.	$x = a\cos^3\theta\ and\ y = a\sin^3\theta, then\ find\ the\ value\ of\ \frac{d^2y}{dx^2}\ at\ \theta = \frac{\pi}{6}$ (a)1 (b) 0 (c) 7 (d) none of these	1
14.	If $y = \left(1 + \frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$ (a) $(1 + \frac{1}{x})^x[\log(1+\frac{1}{x}) - \frac{1}{x+1}]$ (b) $(1 + \frac{1}{x})^x[\log(1+\frac{1}{x})]$ (c) 0 (d) 1	1
15.	The differential coefficient of $f(logx)$ with respect to x, where $f(x) = logx$ is (a) $\frac{x}{logx}$ (c) $\frac{logx}{x}$ (b) $(xlogx)^{-1}$ (d) none of these	1
16.	Choose correct option If $f(x) = t^5$ then $\frac{dy}{dx}$ is (a) $5t^4$ (c) $5t^5$ (b) $\frac{t^6}{6}$ (d) none of these	1
17.	Choose correct option If $y = x^6$ find $\frac{dy}{dt}$ (a) $6x^5$ (c) 0 (b) 1 (d) none of these	1
18.	In the given question , a statement of Assertion (A) is followed by a statement of reason (R) Choose the correct answer out of the fallowing choice (a) Both A and B are true and R is the correct explanation of A (b) Both A and B are true and R is not the correct explanation of A (c) A is true but R is false	1

	(d)A is false but R is true **Assertion (A)** $f(x)$ is continuous at $x = a$ if $\lim_{x\to a} f(x)$ exist and equal to $f(a)$ **Reason (R)** $f(x)$ is continuous at a point , then $\frac{1}{f(x)}$ is also continuous at the point	
19.	In the given question , a statement of Assertion (A) is followed by a statement of reason (R) Choose the correct answer out of the fallowing choice (a) Both A and B are true and R is the correct explanation of A (b) Both A and B are true and R is not the correct explanation of A (c) A is true but R is false (d)A is false but R is true **Assertion (A):** Modulus function is continuous function **REASON (R)**: Modulus function is differentiable function	1
20.	The function f(x) = cotx is discontinuous on the set a) { $x = n\pi$: $n \in Z$} b) { $x = 2n\pi$: $n \in Z$} c) { $x = (2n+1)\frac{\pi}{2}$: $n \in Z$} d) { $x = \frac{n\pi}{2}$: $n \in Z$}	1
21.	Let f(x) = |sin x|. Then a) f is everywhere differentiable b) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$ c) f is everywhere continuous but not differentiable at $x = (2n+1)\frac{\pi}{2}$, $n \in Z$ d) none of these	1
22.	If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to a) $\frac{4x^3}{1-x^4}$ b) $\frac{-4x}{1-x^4}$ c) $\frac{1}{4-x^4}$ d) $\frac{-4x^3}{1-x^4}$	1
23.	If f(x) = mx + 1 , if $x \le \frac{\pi}{2}$ sinx + n , if $x > \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$,then a) m = 1 , n = 0 b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$	1
24.	If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to a) $\frac{\cos x}{2y-1}$ b) $\frac{\cos x}{1-2y}$ c) $\frac{\sin x}{1-2y}$ d) $\frac{\sin x}{2y-1}$	1
25.	The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is a) 2 b) $\frac{-1}{2\sqrt{1-x^2}}$ c) $\frac{2}{x}$ d) $1 - x^2$	1
26.	The function f(x) = [x] , where [x] denotes the greatest integer function, is continuous at a) 4 b) – 2 c) 1 d) 1.5	1
27.	The number of points at which the function $f(x) = \frac{1}{x-[x]}$ is not continuous is a) 1 b) 2 c) 3 d) none of these	1
28.	The set of points where the function f given by f(x) = |x – 3|cosx is differentiable is a) R b) R – {3} c) (0,∞) d) none of these	1
29.	If y = a sinmx + b cosmx , then $\frac{d^2y}{dx^2}$ is equal to a) $-m^2y$ b) m^2y c) – my d) my	1
30.	If the function f defined as	1

$$f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \ne 3 \\ k, & x = 3 \end{cases}$$

	is continuous at x=3, find the value of k.	
31.	If the following functions $f(x)$ is continuous at x = 0, then write the value of k. $f(x) = \begin{cases} \dfrac{\sin \frac{3x}{2}}{x} & , \quad x \neq 0 \\ k & , \quad x = 0 \end{cases}$	1
32.	If $y = x\lvert x\rvert$, find dy/dx for x<0.	1
33.	The value of 'k' for which the function $f(x) = \begin{cases} \frac{1-cos4x}{8x^2}, \ if\ x \neq 0 \\ k, \ if\ x = 0 \end{cases}$ is continuous at x=0 is A) 0 B) -1 C) 1 D) 2	1
34.	If $y = \ sin^{-1} x$, the (1-x^2)y_2 is equal to A) B) C) x^2	1
35.	If a function defined by $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 3 & \text{if } x = \dfrac{\pi}{2} \end{cases}$ is continuous at x=$\frac{\pi}{2}$, then the value of k is: A) 2 B) 3 C) 6 D) -6	1
36.	The function f(x) = cot x is discontinuous on the set: **(A)** $\{x = n\pi;\ n \in Z\}$ **(B)** $\{x = 2n\pi;\ n \in Z\}$ **(C)** $\left\{x = (2n+1)\frac{\pi}{2};\ n \in Z\right\}$ **(D)** $\left\{x = \frac{n\pi}{2};\ n \in Z\right\}$	1
37.	If y=$log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals A) -1/x B) -1/x^2 C) 2/x^2 -2/x^2	1
38.	A) Both A and R are true, and R is the correct explanation of A.	1

	B) Both A and R are true, and R is not the correct explanation of A. C) A is true but R is false. D) A is false but R is true. Assertion(A): Let y=t^{10} + 1 and x=$t^8 + 1$, then $\frac{d^2y}{dx^2} = 20t^8$. Reason(R): $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dx}{dt}$	
39.	A) Both d R is the correct explanation of A. B) Both d R is not the correct explanation of A. C)A is true but R is false. D)A is false but R is true. Assertion(A): If f(x)=$x^n, n \neq 0$ is differentiable for all x, then x can be any element of the interval [1, ∞). Reason(R): If f(x)=$x^n, n \neq 0$ is differentiable for all x, then x can be any element of the interval (1, ∞).	1
40.	If function defined by $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & if\ \ x \neq \frac{\pi}{2} \\ 3 & ,if\ \ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is (A) 2 (B) 3 (C) 6 (D) −6	1
41.	The function $f(x) = [x]$, denotes the greatest integer function, is continuous at (A) 4 (B) −2 (C) 1 (D) 1.5	1
42.	The function $f(x) = \begin{cases} \frac{sinx}{x} + cosx, & if\ \ x \neq 0 \\ k & ,if\ \ x = 0 \end{cases}$ is continuous at $= \frac{\pi}{2}$, then the value of $k$is. (A) 3 (B) 2 (C) 1 (D) 1.5	1
43.	The function $f(x) = \lvert x \rvert + \lvert x - 1 \rvert$is (A) Continuous at $x = 0$ as well as at$x = 1$ (B) Continuous at $x = 1$ but not at $x = 0$ (C) Discontinuous at $x = 0$ as well as at $x = 1$ (D) Continuous at $x = 0$ but not at $x = 1$	1
44.	The function $f(x) = \begin{cases} \frac{e^{3x} - e^{-5x}}{x}, & if\ \ x \neq 0 \\ k & ,if\ \ x = 0 \end{cases}$is continuous at $x = 0$, then value of $k$is (A) 3 (B) 5 (C) 6 (D) 8	1
45.	The function$f(x) = tan\ x$ is discontinuous on the set (A)$\{n\pi : n \epsilon Z\}$ (B)$\{2n\pi : n \epsilon Z\}$ (C)$\left\{(2n+1)\frac{\pi}{2} : n \epsilon Z\right\}$ (D) $\left\{\frac{n\pi}{2} : n \epsilon Z\right\}$	1
46.	The set of points where the function f given by $f(x) = \lvert 2x - 1 \rvert\ sin\ x$ is differentiable in (A)R (B) $R - \{\frac{1}{2}\}$ (C) $(0, \infty)$ (D) none of these	1

47.	If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\frac{dy}{dx}$is: (A) $\frac{\cos\theta+\cos 2\theta}{\sin\theta-\sin 2\theta}$ (B) $\frac{\cos\theta-\cos 2\theta}{\sin 2\theta-\sin\theta}$ (C) $\frac{\cos\theta-\cos 2\theta}{\sin\theta-\sin 2\theta}$ (D) $\frac{\cos 2\theta-\cos\theta}{\sin 2\theta+\sin\theta}$	1
48.	If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ is equal to: (A) $-\frac{1}{x}$ (B) $-\frac{1}{x^2}$ (C) $\frac{2}{x^2}$ (D)$-\frac{2}{x^2}$	1
49.	If $\sin y = x\cos(a+y)$, then $\frac{dx}{dy}$is: (A) $\frac{\cos a}{cos^2(a+y)}$ (B) $\frac{-\cos a}{cos^2(a+y)}$ (C) $\frac{\cos a}{sin^2 y}$ (D) $\frac{-\cos a}{sin^2 y}$	1
50.	if f(x) = $\frac{acosx-cosbx}{x^2}$, x≠0 and f(0) = 4 is continuous at x=0, then the ordered pair (a,b) is (a) (±1,3) (b)(1,±3) (c)(-1,-3) (d)(-1,3)	1
51.	let A={9,10,11,12,13} and f:A→N be a function defined as f(x) = Highest prime factor of x. Then number of elements in the range of f(x) is (a) 5 (b)4 (c)3 (d)None of these	1
52.	which of the statements(s) is/are incorrect (a) if f+ g is continuous at x+ a, then f and g are continuous at x=a (b) if $\lim_{x\to a}(fg)$exists, then $\lim_{x\to a} f$ and $\lim_{x\to a} g$ both exists (c) Discontinuity at x = a implies that non existences of limit (d) All functions defined on a closed interval attain maximum or a minimum value in its interval	1
53.	The derivative of f(tanx) w.r.t g(secx) at x=$\frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$, is (a)$\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) 0	1
54.	If $y^2 = ax^2 + bx + c$, then $\frac{d}{dx}(y^3y_2)$ = (a) 1 (b)-1 (c)$\frac{4ac-b^2}{a^2}$ (d)0	1
55.	If $u = x^2 + y^2$ and x = s+3t, y = 2s – t, then $\frac{d^2u}{dx^2}$ is equal to (a)12 (b)32 (c)36 (d)10	1
56.	The function f(x) = [x], where [x] is greatest integer function, is continuous at (a)4 (b)-2 (c)1 (d)1.5	1
57.	The number of points at which the function f(x) = $\frac{1}{x-[x]}$ is not continous is (a)1 (b)2 (c)3 (d) none of these	1
58.	If u = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is (a) 2 (b)x (c) -1 (d)1	1

59.	If y = $\log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to (a) $\frac{4x^3}{1-x^4}$ (b) $\frac{-4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $\frac{-4x^3}{1-x^4}$	1

ANSWERS:

Q. NO	ANSWER	MARKS
1.	(Ans. (b) We find the find that $\frac{x^2-9}{x-3} = (x+3) = 6 \neq f(3)$ So, $f(x)$ has removable discontinuity at $x = 3$	1
2.	(Ans. (c): We find that $=$ $and\ f(x)\ \ f(x) = x + 4 = 6.$ $\therefore \quad f(x) \neq f(x)$ Hence, $f(x)$ has discontinuity of first kind at $x = 2$.	1
3.	Ans. (b): At any integer k, we find that $f(x) = f(k-h) = [k-h] = k-1$ and $f(x) = f(k+h) = [k+h] = k$ $\therefore \quad f(x) \neq f(x)$. So, $f(x)$ has discontinuity of first kind at $x = k$	1
4.	(Ans. (b) : We find that $f(x) = (x-1)^n \ \ sin\ sin\left(\frac{1}{x-1}\right)$ $= 0 \times (An\ oscillating\ number\ between - 1 and\ 1) = 0, if\ n > 0$ $= f(0), if\ n = 0.$ Hence, $f(x)$ is continuous at $x = 1$ for all $n > 0$.	1
5.	Ans. (b) : We find that $f(x)$ $=cos\ cos\ \frac{1}{x}$ = An oscillating number between -1 and 1. So, $f(x)$ does not exist. Hence, $f(x)$ has discontinuity of first kind at $x = 0$.	1
6.	(Ans. (a): The function $f(x) = [x]$ is discontinuous at $x = 4, 5, 6$ in $[3, 7]$. It is right continuous at $x = 3$ and left discontinuous $x = 7$. Hence, there are four points of discontinuity.	1
7.	Ans. (a): The sum of continuous functions is continuous function. Therefore, $f + g$ if continuous at $x = a$.	1
8.	Ans. (b): The composition of continuous functions is a continuous function. Therefore, gof is continuous at $x = a$.	1
9.	Ans. (d): If $f(x)$ is continuous at $x = 0$, then $f(x) = f(0)\ \frac{e^{3x}-1}{1n(1+2x)} = k \implies \frac{3^{3x}-1}{3x} \times \frac{2x}{1n(1+2x)} \times \frac{3}{2} = k \implies \frac{3}{2} = k$	1
10.	(Ans. (b): If $f(x)$ is continuous at $x = 0$, then	1

	$\frac{\sin \sin ax^{\circ}}{x} = k \Rightarrow \frac{\sin\frac{\pi\, ax}{180}}{\frac{\pi\, ax}{180}} \times \frac{\pi a}{180} = \frac{\pi a}{180}$	
11.	C	1
12.	A	1
13.	A	1
14.	D	1
15.	A	1
16.	B	1
17.	D	1
18.	D	1
19.	C	1
20.	D	1
21.	a	1
22.	b	1
23.	b	1
24.	c	1
25.	a	1
26.	a	1
27.	d	1
28.	d	1
29.	b	1
30.	a	1
31.	$\lim_{x\to3}\frac{x2-9}{x-3}$ = 6 ,therefore k= 6	1
32.	$\lim_{x\to0}\frac{\sin 3x/2}{x}$=$\lim_{x\to0}\frac{\frac{3}{2}\sin 3x/2}{3x/2}$ Or k=3/2	1
33.	for x<0, $y = x\|x\|$ =-x^2 $\therefore \ \frac{dy}{dx} = -2x$	1
34.	C	1
35.	A	1
36.	C	1
37.	A	1
38.	D	1
39.	A	1
40.	C	1
41.	(C) 6 $k\lim_{x\to\frac{\pi}{2}}\frac{\sin(\frac{\pi}{2}-x)}{2(\frac{\pi}{2}-x)} = 3 \Rightarrow \frac{k}{2}\times 1 = 3 \Rightarrow k = 6$	1
42.	(D) 1.5 Greatest integer function is continuous except at integer points.	1
43.	(B) 2	1

	$$\lim_{x\to 0}\frac{\sin x}{x}+\lim_{x\to 0} cosx = k \Rightarrow\ 1+1=k \Rightarrow k=2$$	
44.	(A) Continuous at $x=0$ as well as at $x=1$	1
45.	(D) 8 $$\lim_{x\to 0}\frac{e^{8x-5x}\ -\ e^{-5x}}{x}=k\ \Rightarrow 8\times\lim_{x\to 0}e^{-5x}\times\lim_{x\to 0}\frac{e^{8x}\ -\ 1}{8x}=k\ \Rightarrow k=8$$	1
46.	(C)$\left\{(2n+1)\frac{\pi}{2} : n \epsilon Z\right\}$	1
47.	(B) $R-\{\frac{1}{2}\}$	1
48.	(B) $\frac{\cos\theta-\cos 2\theta}{\sin 2\theta-\sin\theta}$ $$\frac{dx}{d\theta}=-2sin\theta+2sin2\theta\ and\ \frac{dy}{d\theta}=2cos\theta-2cos2\theta$$	1
49.	(D)$-\frac{2}{x^2}$ $$y=2\log_e x-\log_e e^2\quad\Rightarrow\ y=2\log_e x-2$$	1
50.	(A) $\frac{\cos a}{cos^2(a+y)}$ $$\frac{dx}{dy}=\frac{\cos(a+y)\cos y+siny\sin(a+y)}{cos^2(a+y)}$$ $$\frac{dx}{dy}=\frac{\cos[(a+y)-y])}{cos^2(a+y)}\Rightarrow\ \frac{dx}{dy}=\frac{\cos a}{cos^2(a+y)}$$	1
51.	b	1
52.	b	1
53.	a,b,c,d	1
54.	b	1
55.	d	1
56.	d	1
57.	d	1
58.	d	1
59.	d	1
60.	b	1