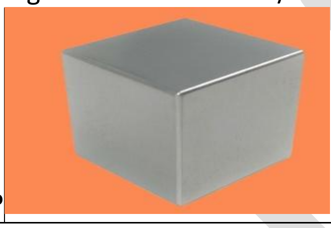
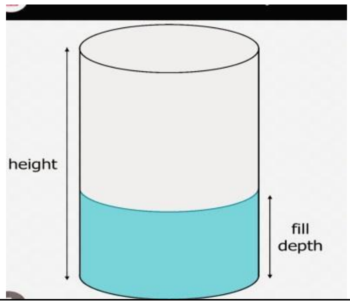
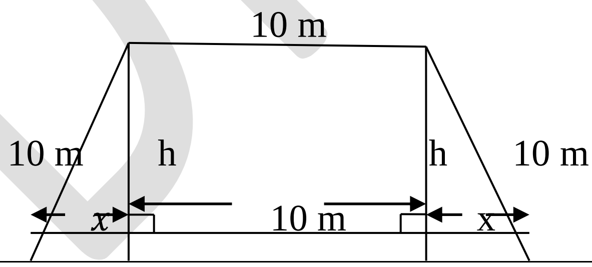
CHAPTER-6
APPLICATION OF DERIVATIVES
02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm,how fast is the enclosed area increasing ? 	2
2.	The volume of a cube is increasing at the rate of 9 cm^3/s.How fast is its surface area increasing when the length of an edge is 10 cm? 	2
3.	Find the values of x for which y = $[x(x-2)]^2$ is an increasing function.	2
4.	The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue(marginal revenue).If the total revenue(in rupees) received from the sale of x units of a product is given by $R(x)=3x^2+36x+5$,find the marginal revenue when x=5. 	2
5.	Prove that the function f given by f(x) = log sinx is strictly increasing on (0, $\frac{\pi}{2}$) and strictly decreasing on ($\frac{\pi}{2}$,).	2
6.	A car is moving along a straight road. Its position at a time t is given by $s(t) = 2t^3 - 3t^2 - 12t + 1$ meters. At what time when the car changes direction? (a)30 seconds (c)1 seconds (b)2 seconds (d) 22 seconds 	2
7.	A spherical snowball is melting in such a way that its radius decreasing at a rate of 0.1 cm/min. At	2

	what rate is the volume of the snowball is decreasing when the radius is 5 cm?	
8.	A cylindrical tank is being filled with water at a constant rate. The tank has a height of 6m and radius of 4m. If the water level rises at a rate of 2m per hour, how fast is the volume of water increasing when the water level is 3m high?  	2
9.	A stone is dropped into a calm lake, creating a circular ripple that expands at a rate of 1.5m/s. Find the rate at which the area of the ripple is increasing when the radius is 4m.	2
10.	The side length of a square is increasing at a rate of 2cm/s. At what rate is the area of the square increasing when the side length is 5cm?	2
11.	**Assertion** (A) : The function $y = \log(1+x) - \frac{2x}{2+x}$ is decreasing throughout its domain. **Reason (R)** :The domain of the function $y = \log(1+x) - \frac{2x}{2+x}$ is (- 1, ∞). A. Both A and R are true and R is the correct explanation oF A B. Both A and R are true but R is NOT the correct explanation of A C. A is true but R is false. D. A is false but R is true. E.Both A and R are false.	2
12.	The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are of 10 m each. The height of the gate is h meter. On the basis of above figure, answer the following questions:  	2

	  (i) Find the Area (A) of the gate expressed as a function of x . (ii) Find the value of x when Area (A) is maximum (iii) Find the Maximum value of Area (A)	
13.	Read the following passage and the answer the questions given below. The temperature of a person during an intestinal illness is given by f (x)= -- $0.1x^2 + mx + 98.6$, $0 \leq x \leq 12$, m being a constant, where f (x) is the temperature in ^{0} F at x days. **(i)** is the function differentiable in the interval (0,12) ? Justify your answer. **(ii)** If 6 is the critical point of the function, then find the value of the constant m.	2
14.	Sahaj wants to prepare a handmade gift for his father's birthday at Home.For making lower part of box,he takes a square piece of Cardboard of side 20cm.If x cm be the length of each side of the Square cardboard which is to be cut from corners from square pieceof side 20 cm then Sahaj is interested in maximising the volume of the box. So what should be the value of x to be cut off so that volume of box is maximum?	2
15.	A particle is moving along the curve represented by the polynomial f(x) =$(x-2)^2(x-1)$. Based on above information answer the following questions: Find the interval where f(x) is strictly increasing.	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let x be the radius and A be the area of circle $\frac{dx}{dt}$ = 5 cm/s A = πr^2 $\frac{dA}{dt}$ =$2\pi r \frac{dx}{dt}$ = 2π x 8 x 5 = $80\pi cm^2/s$	2
2.	Let V be the volume and S be the surface area of cube of side x cm. $\frac{dV}{dt}$ = 9 cm^3/s V = x^3 $\frac{dV}{dt}$ = $3x^2 . \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{3}{x^2}$ Again S = $6x^2$ $\frac{dS}{dt}$ =12x . $\frac{3}{x^2}$ = 36/x = 36/10 = 3.6 cm^2/s	2
3.	Y=$[x(x-2)]^2$ $\frac{dy}{dx}$ = 2[x(x-2)] x (2x-2) = 4x(x-1)(x-2) , critical points x=0/1/2 For increasing function $\frac{dy}{dx} > 0$ From sign rule y increasing for $x \epsilon (0,1) \cup (2,\infty)$	2
4.	R(x)=$3x^2$+36x+5 $\frac{dR}{dx}$ = 6x +36 $\frac{dR}{dx}$ at x= 5 is 66	2
5.	f'(x) = $\frac{1}{sinx}$. cosx = cotx When x ϵ (0, $\frac{\pi}{2}$) then f'(x) $>$ 0 and when x ϵ ($\frac{\pi}{2}, \pi$) then f'(x) $<$ 0 So, f(x) is strictly increasing on (0, $\frac{\pi}{2}$) and strictly decreasing on ($\frac{\pi}{2}$,).	2
6.	Given, position of a car at time t is given as $s(t) = 2t^3 - 3t^2 - 12t + 1$ A car changes direction when its velocity changes its sign. So the velocity is the derivative of the position at time t. i.e. $v(t)$=$s'(t)$ So in order to know when the velocity changes we have to set $v(t) = 0$. $6t^2 - 6t - 12$=0 Or $t^2 - t - 2$=0 Or $(t-2)(t+1) = 0$ So, $t = 2 \; or \; t = -1$ As time can't be negative so the car changes its direction at 2 seconds.	2
7.	There is a snowball of spherical shape. Let its radius be r . Then the volume of the snowball is same as the volume of the sphere with radius r. The volume of the sphere is $v = \frac{4}{3}\pi r^3$ Now $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ Since the radius is decreasing so $\frac{dr}{dt} = -0.1$ cm/min and radius is 5cm so $\frac{dv}{dt} = 4\pi(5)^2(-0.1) = -10\pi$ cm^3/min. So the volume of the snowball is decreasing at a rate of 10π cm^3/min.	2
8.	If r is the radius of the cylindrical tank and its height is h then volume of the cylindrical tank will be	2

	$v = \pi r^2 h$ Given that $\frac{dh}{dt}$= 2 m/hr , h = 3m and r = 4m Now $\frac{dv}{dt} = 2\pi rh\frac{dv}{dt} + \pi r^2 \frac{dh}{dt}$ $\frac{dv}{dt} = 2\pi(4)(3)(0) + \pi(4)^2(2)$ =32π m3/hr The volume of water is increasing at a rate of 32πcubic meters per hour.	
9.	Let *r* be the radius of the circular ripple and *A* be the area of the ripple. The area *A* of a circle is $A=\pi r^2$ Differentiate *A* with respect to time *t*: $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ Given $\frac{dr}{dt}$=1.5m/s and r=4m, substitute and solve for $\frac{dA}{dt}$: $\frac{dA}{dt}=2\pi(4)\cdot 1.5=12\pi m^2/s$ So, when the radius of the ripple is 4 m4m, the area of the ripple is increasing at a rate of12πm^2/s.	2
10.	Let *s* be the side length of the square and *A* be the area of the square. The area *A* of a square is $A=s^2$ Differentiate *A* with respect to time *t*: $\frac{dA}{dt}=2s\frac{ds}{dt}$ Given $\frac{ds}{dt}$=2cm/s and s=5cm, substitute and solve for $\frac{dA}{dt}$: $\frac{dA}{dt}=2(5)\cdot 2=20cm^2/s$ So, when the side length of the square is 5cm, the area of the square is increasing at a rate of 20cm2/s.	2
11.	d	2
12.	**(i)** $(10 + x)\sqrt{100 - x^2}$ **(ii)** $5\sqrt{3}$ m $(iii)\ \frac{75\sqrt{3}}{2}$ m^2	2
13.	(i) f (x)= -- $0.1x^2 + m x + 98.6$ being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12) (ii) $f'(x)) = -0.2x + m$ Since, 6 is the critical point, $f'(x) = 0 \Longrightarrow m = 1.2$	2
14.	for x= $\frac{10}{3}$,volume is maximum	2
15.	f(x) is strictly inc in $(-\infty,4/3)\cup[2,\infty)$	2