CHAPTER-5

CONTINUITY & DIFFERENTIABILITY

02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the value of k for which $f(x) = \{\frac{1 - coscos\,4x}{8x^2},\ if\ \ x \neq 0\ k\ \ \ \ ,if\ \ x = 0$ is continuous at $x = 0$.	2
2.	Find the value of k for which $f(x) = \{\frac{x^2 + 3x - 10}{x-2},\ if\ \ x \neq 2\ k\ \ \ \ ,if\ \ x = 2$ is continuous at $x = 2$.	2
3.	Discuss the differentiability of $f(x) = x\|x\|$at $x = 0$.	2
4.	Differentiate $\left(\frac{1+ \cos x}{\sin x}\right)$ with respect to x .	2
5.	If $x = a(2\theta - sin\ sin\ 2\theta)$ and $= a(1 - cos\ cos\ 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.	2
6.	If $\log y = \tan^{-1} x$, *then prove that* $(1 + x^2)y_2 + (2x - 1)y_1 = 0$	2
7.	Find the value of k for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{2x-1}, & 0 \leq x < 1 \end{cases}$	2
8.	If siny = x sin(a+y),prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	2
9.	$x^2 + 2xy + y^3 = 42$ then find $\frac{dy}{dx}$	2
10.	If $y.\sqrt{x^2+1} = \log[\sqrt{x^2+1} - x], show that \left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0.$	2
11.	Determine the value of k so that the function f(x) = $\frac{\sin 2x}{5x}$, if $x \neq 0$ K , if x = 0 is continuous at x =0	2
12.	If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$	2

13.	Find $\frac{dy}{dx}$ for $\tan^{-1}(x^2 + y^2) = a$	2
14.	Differentiate $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ w.r.t. x	2
15.	Show that the derivative of the function f given by $f(x) = 2x^3 - 9x^2 + 12x + 9$, at x = 1 and x = 2 are equal.	2
16.	Find the value of p for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , x \neq 0 \\ p & , x = 0 \end{cases}$	2
17.	F the function $f(x) = \begin{cases} \frac{\sin x - \cos x}{4x - \pi} & , x \neq \frac{\pi}{4} \\ k & , x = \frac{\pi}{4} \end{cases}$ is continuous at x= x/4.	2
18.	Find the value of $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$, x=cos $\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$.	2
19.	If x=acos $\theta;\ y = b\sin\theta$, then find $\frac{d^2y}{dx^2}$.	2
20.	Find the derivative of $x^{\log x}$ w.r.t. logx.	2
21.	Find the value of $k$for which $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ \ x \neq 0 \\ k & , if\ \ x = 0 \end{cases}$ is continuous at $x = 0$.	2
22.	Find the value of k for which $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}, & if\ \ x \neq 2 \\ k & , if\ \ x = 2 \end{cases}$ is continuous at $x = 2$.	2
23.	Discuss the differentiability of $f(x) = x\lvert x\rvert$at $x = 0$.	2
24.	Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x .	2
25.	If $x = a(2\theta - \sin 2\theta)$ and $= a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.	2
26.	Using the fact that sin(A+B) = sin A cos B + cos A sin B and the differentiation, obtain the sum formula for cosines.	2
27.	If y = $\cos^{-l}\left(\frac{1-x^2}{1+x^2}\right)$ and cos2x = $\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$, find $\frac{dy}{dx}$	2
28.	if the function f(x) satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 1} f(x)$	2
29.	limit of a function exist at x = a if L.H.L of f(x) at x = a and R.H.L of f(x) at x = a are equal to f(a), using the above concept find $\lim_{x\to 5} f(x)$, where f(x) = IxI - 5	2
30.	Let $a_1, a_2, \ldots a_n$ be fixed real numbers and define a function $f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$. what is $\lim_{x\to a1} f(x)$? for some a $\neq a_1, a_2, \ldots a_n$, compute $\lim_{x\to a} f(x)$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{1 - cos\, cos\, 4x}{8x^2} = k \Rightarrow \lim_{x\to 0}\frac{2\, sin^2\, 2x}{8x^2} = k \Rightarrow \lim_{x\to 0}\left(\frac{sin\, sin\, 2x}{2x}\right)^2 = k \Rightarrow k = 1$	2
2.	$\frac{(x-2)(x+5)}{x-2} = k \Rightarrow k = 7$	2
3.	$f(x) = \{x^2 \;,\; x \geq 0 \; -x^2 \;,\; x < 0$ LHD $= \frac{f(x)-f(0)}{x-0} = \frac{-x^2-0}{x-0} = 0$ RHD $= \frac{f(x)-f(0)}{x-0} = \frac{x^2-0}{x-0} = 0$	2
4.	$\left(\frac{2\, cos^2 \frac{x}{2}}{2\, sin\, \frac{x}{2} \frac{x}{2}}\right) = \left(cot \frac{x}{2}\right) = \left[tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right] = \frac{\pi}{2} - \frac{x}{2}$ $f'(x) = -\frac{1}{2}$	2
5.	$\frac{dx}{d\theta} = a(2 - 2cos\, 2\theta) = 4asin^2\theta \,\&\, \frac{dy}{d\theta} = 2asin\, 2\theta = 4a\, sin\theta\, cos\theta \Rightarrow \frac{dy}{dx}\Big]_{\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$	2
6.	Correct proof should be there without step missing	2
7.	$K = \frac{-1}{\sqrt{2}}$	2
8.	Correct proof should be there without step missing	2
9.	$2x + 2y(1) + 2x.\frac{dy}{dx} + 3y^2.\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-2(x+y)}{2x+3y^2}$	2
10.	Correct proof should be there without step missing	2
11.	K = $\frac{2}{5}$	2
12.	xlogy = y – x dy/dx = y(1 + logy)/(y – x) $\frac{dy}{dx} = \frac{(1 + logy)^2}{logy}$	2
13.	dy/dx = - x/y	2
14.	-1	2
15.	f ’(x)= $6x^2$ - 18x + 12 , f ’(1) = f ‘(2) = 0	2
16.	$\lim_{x\to 0} f(x)$= f(0) $\lim_{x\to 0}\frac{4*2sin^2 2x}{4x^2}$= p	2

	$8\lim_{x\to0}(\frac{sin2x}{2x})^2$=p or p= 8	
17.	$\lim_{x\to\pi/4}\frac{sinx-cosx}{4x-\pi}$= f($\pi/4$) $\lim_{x\to\pi/4}\frac{\sqrt{2}\sin(x-\frac{\pi}{4})}{4(x-\frac{\pi}{4})}$ = k $\frac{\sqrt{2}}{4}=k$ K= 1/2$\sqrt{2}$	2
18.	$\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$ $\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ $\frac{dy}{dx}\Big\vert_{\theta=\frac{\pi}{3}} = \sqrt{3}$	2
19.	$\frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = b\cos\theta \Rightarrow \frac{dy}{dx} = -\frac{b}{a}\cot\theta$ $\frac{d^2y}{dx^2} = \frac{b}{a}\text{cosec}^2\theta\left(\frac{-1}{a\sin\theta}\right) = -\frac{b}{a^2}\text{cosec}^3\theta$	2
20.	Let $u = x^{\log x}$ and $v = \log x$ Now, $\log u = (\log x)^2$ $\Rightarrow \frac{1}{u}\frac{du}{dx} = 2\log x.\frac{1}{x}$ $\Rightarrow \frac{du}{dx} = \frac{2\log x}{x}.x^{\log x}$ Again, $v = \log x$ $\Rightarrow \frac{dv}{dx} = \frac{1}{x}$ $\therefore \frac{du}{dv} = 2.x^{\log x}\log x$	2
21.	$\lim_{x\to0}\frac{1-\cos 4x}{8x^2} = k \Rightarrow \lim_{x\to0}\frac{2\,sin^2\,2x}{8x^2} = k \Rightarrow \lim_{x\to0}(\frac{\sin 2x}{2x})^2 = k \Rightarrow k = 1$	2
22.	$\lim_{x\to2}\frac{(x-2)(x+5)}{x-2} = k \Rightarrow k = 7$	2
23.	$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ LHD = $\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to0^-}\frac{-x^2-0}{x-0} = 0$ RHD = $\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to0^+}\frac{x^2-0}{x-0} = 0$	2
24.	f(x) = $\tan^{-1}\left(\frac{2\,cos^2\frac{x}{2}}{2\,sin\,\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(cot\frac{x}{2}\right) = \tan^{-1}\left[tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right] = \frac{\pi}{2}-\frac{x}{2}$	2

	$f'(x) = -\frac{1}{2}$	
25.	$\frac{dx}{d\theta} = a(2-2cos\ 2\theta) = 4asin^2\theta$&$\frac{dy}{d\theta} = 2asin\ 2\theta = 4a\ sin\theta\ cos\theta \quad \Rightarrow \left.\frac{dy}{dx}\right]_{\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$	2
26.	We have, Sin(A+B) = sin A cos B + cos A cos B differentiating both side w.r.t x ⇒d/dx(sin(A+B) = d/dx(sinAcosB + cosAsinB) ⇒cos(A + B)[dA/dx +dB/dx] = -sinAsinBdB/dx +cosBcosAdA/dx + ⇒cosAcosBdB/dx –sinBsinAdA/dx ⇒cos(A + B)[dA/dx +dB/dx] = dB/dx[cosAcosB – sinAsinB] + ⇒dA/dx[cosAcosB – sinAsinB] ⇒cos(A + B)[dA/dx +dB/dx] = [cosAcosB – sinAsinB][dA/dx + dB/dx] ⇒cos(A + B) = cosAcosB – sinAsinB	2
27.	$y = cos^{-1}(\frac{1-x^2}{1+x^2})$ Let x = tanp $p = tan^{-1}x$ $\Rightarrow y = cos^{-1}(\frac{1-tan^2p}{1+tan^2p})$ $\Rightarrow y = cos^{-1}(cos2p)$ = 2p $\Rightarrow y = 2\ tan^{-1}x$ differentiating w.r.t x both side $dy/dx = \frac{2}{1+x^2}$	2
28.	Given that $\lim_{x\to1}\frac{f(x)-2}{x^2-1} = \pi$ $\Rightarrow \frac{\lim_{x\to1} f(x)-2}{\lim_{x\to1} x^2-1} = \pi$ $\Rightarrow \lim_{x\to1}(f(x)-2) = \pi \lim_{x\to1}(x^2-1)$ $\Rightarrow \lim_{x\to1}(f(x)-2) = \pi(1^2-1)$ $\Rightarrow \lim_{x\to1}(f(x)-2) = 0$ $\Rightarrow \lim_{x\to1} f(x) - 2 = 0$ $\Rightarrow \lim_{x\to1} f(x) = 2$	2
29.	The given function is: f(x) = IxI – 5 $\Rightarrow \lim_{x\to5^-} f(x) = \lim_{x\to5^-}[\text{IxI} - 5]$ $= \lim_{x\to5}(x-5)$ = (5 – 5) = 0 $\Rightarrow \lim_{x\to5^+} f(x) = \lim_{x\to5^+}[\text{IxI} - 5]$ $= \lim_{x\to5}(x-5)$ = (5 – 5) = 0 $\Rightarrow \lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x)$ Hence $\lim_{x\to5} f(x) = 0$	2

30.	Given function is: $f(x) = (x-a_1)(x-a_2)....(x-a_n)$ $\Rightarrow \lim_{x\to a1} f(x) = \lim_{x\to a1} [(x-a_1)(x-a_2)....(x-a_n)]$ $= [\lim_{x\to a1}(x-a_1)]\ [\lim_{x\to a1}(x-a_2)].......\ [\lim_{x\to a1}(x-a_n)]$ $= (a_1-a_1)(a_1-a_2)....(a_1-a_n)$ $\lim_{x\to a1} f(x) = 0$ now $\lim_{x\to a} f(x) = \lim_{x\to a}[(x-a_1)(x-a_2)....(x-a_n)]$ $= [\lim_{x\to a}(x-a_1)]\ [\lim_{x\to a}(x-a_2)].......\ [\lim_{x\to a}(x-a_n)]$ $= (a-a_1)(a-a_2)....(a-a_n)$	2