CHAPTER-7

INTEGRALS

02 MARKS TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Evaluate : $\int_0^1 x^2 e^x$dx	2
2.	Find : $\int \frac{tan^3 x}{cos^3 x}$ dx	2
3.	Find: $\int \frac{x-1}{(x-2)(x-3)}$dx	2
4.	Find: $\int_{-\frac{\pi}{4}}^{0} \frac{1+tanx}{1-tanx}$dx	2
5.	Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^x$dx	2
6.	Write the value of ∫ sec x(sec x + tan x) dx	2
7.	Evaluate: $\int \frac{x^3-x^2+x-1}{x-1}$ dx	2
8.	Evaluate: $\int \frac{dx}{5-8x-x^2}$	2
9.	Evaluate: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$	2
10.	Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5 x\ dx$	2
11.	If $f(x) = \int_0^x tsintdt, then\ find\ the\ value\ of\ f'(x)$	2
12.	Find $\int \frac{sin^6 x}{cos^8 x} dx$	2
13.	$Evaluate : \int_e^{e^2} \frac{dx}{xlogx}$	2
14.	Evaluate : $\int (sinx)dx$	2
15.	Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xcos^2 xdx$	2

16.	Find the value of $\int \quad sinx\sqrt{1-cos2x}\,dx$	2
17.	find the value of $\int \quad \frac{1}{1+e^{-x}}\,dx$	2
18.	Find the value of $\int_0^{2\pi} \quad |sinx|dx$	2
19.	Find the value of $\int \quad 5^{x+x}\left(\frac{x^2+2}{x^2+1}\right)\,dx$	2
20.	Find the value of $\int \quad \sqrt{1+sinx}\,dx$	2
21.	Evaluate $\int \frac{dx}{cosx+\sqrt{3}sinx}$	2
22.	Evaluate $\int \frac{(x+3)e^x}{(x+5)^3}\,dx$	2
23.	Evaluate $\int cosec^3x\,dx$	2
24.	Evaluate $\int_{-\pi}^{\pi} \frac{cos^2x}{1+a^x}\,dx, \quad a>0$	2
25.	Evaluate: $$\int tan^{-1}\left(\frac{cosx}{1-sinx}\right)dx, \quad x\in\left(-\frac{\pi}{2},\frac{3\pi}{2}\right)$$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\int_0^1 x^2e^x dx = [x^2e^x]_0^1 - \int_0^1 2xe^x dx$ $= [x^2e^x - 2xe^x + 2e^x]_0^1$ $= e - 2$	2
2.	Given I = $\int \frac{tan^3x}{cos^3x} dx$ Let cos x = t So, sinx dx = −dt I = $\int [\frac{-1}{t^6} + \frac{1}{t^4}] dt = \frac{t^{-5}}{5} + \frac{t^{-3}}{-3} + C = \frac{1}{5(cosx)^5} - \frac{1}{3(cosx)^3} + c = \frac{sec^5x}{5} - \frac{sec^3x}{3} + C$	2
3.	$\int \frac{x-1}{(x-2)(x-3)} dx$ Since , $\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$, on solving A = -1 and B = 2 $\Rightarrow \frac{x-1}{(x-2)(x-3)} = \frac{-1}{x-2} + \frac{2}{x-3}$ $\Rightarrow \int \frac{x-1}{(x-2)(x-3)} dx = -\int \frac{dx}{x-2} + 2\int \frac{dx}{x-3}$ = - log (x – 2) + 2 log (x – 3) + C = - log (x – 2) + log$(x-3)^2$ +C = log $\frac{(x-3)^2}{(x-2)}$+ C	2
4.	$\int_{-\frac{\pi}{4}}^{0} \frac{1+tanx}{1-tanx} dx = \int_{-\frac{\pi}{4}}^{0} tan\, tan\, (\frac{\pi}{4} + x)\, dx =$ $= [logsec(\frac{\pi}{4} + x)]_{-\frac{\pi}{4}}^{0}$ = log sec $\frac{\pi}{4}$ − log sec ($\frac{\pi}{4} - \frac{\pi}{4}$) = log ($\sqrt{2}$) – log (sec 0) = log ($\sqrt{2}$) – log 1 = log ($\sqrt{2}$) = $\frac{1}{2}$ log 2	2
5.	*Evaluate*: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^x dx$ Put 2x = t , ∴ dx = $\frac{1}{2}$ dt $\therefore \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^x dx = \int_2^4 [\frac{1}{t} - \frac{1}{t^2}] e^t dt = [\frac{1}{t} e^t]_2^4 = \frac{e^4}{4} - \frac{e^2}{2}$.	2
6.	I = ∫sec x(sec x + tan x)dx = ∫(sec^{2}x + sec x tan x)dx = ∫sec^2 x dx + ∫sec x tan x dx = tan x + sec x + C	2
7.	Let I = $\int \frac{x^3 - x^2 + x - 1}{x-1} dx = \int \frac{x^2(x-1)+1(x-1)}{(x-1)} dx$ $= \int \frac{(x^2+1)(x-1)}{(x-1)} dx = \int (x^2+1)\, dx = \frac{x^3}{3} + x + c$	2
8.	$\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{5 - 2.4x - x^2 + 4^2 - 4^2}$ $= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$ $= \frac{1}{2\sqrt{21}} \log\left(\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4}\right) + c$	2
9.	$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = [\tan^{-1} x]_1^{\sqrt{3}}$ $= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$	2
10.	Use the property, $\int_{-a}^{a} f(x)\, dx = 0$, if $f(-x) = -f(x)$; f(x) is a odd function $f(x) = \sin^5x \Longrightarrow f(-x) = \sin^5(-x) = -\sin^5x = -f(x)$ so, f(x) is a odd function $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5x\, dx = 0$	2
11.	Differentiating both sides w.r.t to x we get $f'(x) = [tsint]_0^x$ $= xsinx - 0 = xsinx$	1 1

12.	$\int \frac{sin^6x}{cos^8x}dx = \int tan^6xsec^2xdx$ $= \int t^6dt$, where tanx = t $\Rightarrow sec^2xdx = dt$ $= \frac{t^7}{7} + C = \frac{tan^7x}{7} + C$	1 1
13.	$Let\ I = \int_e^{e^2} \frac{dx}{xlogx}$, Put logx = t $\Rightarrow \frac{1}{x}\ dx = dt$ When x = e, t = loge = 1 and when x = e^2, t = 2loge = 2 $Let\ I = \int_e^{e^2} \frac{dx}{xlogx} = \int_1^2 \frac{dt}{t} = [logt]_1^2 = log2 - log1 = log2$	1 1
14.	$\int (sinx)dx = \int \left(cos(\frac{\pi}{2} - x)\right) dx$ $= \int (\frac{\pi}{2} - x)dx = \frac{\pi}{2}x - \frac{x^2}{2} + C$	1 1
15.	Let $f(x) = xcos^2x$ $\Rightarrow f(-x) = (-x)cos^2(-x) = -xcos^2x = -f(x)$ $\Rightarrow f(x)\ is\ an\ odd\ function$ $\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xcos^2xdx = 0$	1 1
16.	$\int sinx\sqrt{1 - cos2x}\, dx$ $= \int (sinx)(\sqrt{2}sinx)\ dx$ $= \frac{\sqrt{2}}{2}\int 2(sinx)^2dx$ $= \frac{1}{\sqrt{2}}\int (1 - cos2x)dx$ $= \frac{x}{\sqrt{2}} - \frac{sin2x}{2\sqrt{2}} + c$ (Answer)	2
17.	$\int \frac{1}{1 + e^{-x}}\ dx$ $= \int \frac{e^x}{e^x + 1}dx$ $= \int \frac{d(e^x + 1)}{e^x + 1}$ $=log\ log\ \lvert e^x + 1\rvert + c$(Answer)	2
18.	$\int_0^{2\pi} \lvert sinx\rvert dx$ $= \int_0^{\pi} sinx\ dx + \int_{\pi}^{2\pi} - sinx\ dx$ $= [-cosx]_0^{\pi} + [cosx]_{\pi}^{2\pi}$ $= -cos\pi + cos0 + cos2\pi - cos\pi$	2

	$= 4$(answer)	
19.	$\int 5^{x+x}\left(\frac{x^2+2}{x^2+1}\right)dx$ $= \int 5^{x+x}\left(1+\frac{1}{x^2+1}\right)dx$ $= \int 5^u\, du \quad \left[Taking,\ u = x + x, hence\ du = (1+\frac{1}{1+x^2})dx\right]$ $= \frac{5^u}{log5} + c$ $=\frac{5^{x+x}}{loglog\,5} + c$ (Answer)	2
20.	$\int \sqrt{1+sinx}\, dx$ $= \int \sqrt{(sin\frac{x}{2})^2 + (cos\frac{x}{2})^2 + 2cos\frac{x}{2}sin\frac{x}{2}}\, dx$ $= \int (sin\frac{x}{2} + cos\frac{x}{2})dx$ $= -2cos\frac{x}{2} + 2sin\frac{x}{2} + c$ (Answer)	2
21.	$\int \frac{dx}{cosx+\sqrt{3}sinx} = \frac{1}{2}\int \frac{dx}{\frac{1}{2}cosx+\frac{\sqrt{3}}{2}sinx} = \frac{1}{2}\int \frac{dx}{sin\frac{\pi}{6}cosx+cos\frac{\pi}{6}sinx} = \frac{1}{2}\int \frac{dx}{sin\left(x+\frac{\pi}{6}\right)}$ $= \frac{1}{2}\int cosec\left(x+\frac{\pi}{6}\right)dx = \frac{1}{2}logtan\left(\frac{x}{2}+\frac{\pi}{12}\right) + C$	2
22.	$\int \frac{(x+3)e^x}{(x+5)^3}dx = \int \frac{(x+5-2)e^x}{(x+5)^3}dx = \int e^x\left\{\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\right\}dx = e^x\frac{1}{(x+5)^2} + C$	2
23.	$I = \int cosec^3x\ dx = \int cosecx\ cosec^2x\ dx =$ $= cosecx\int cosec^2x\ dx - \int \frac{d}{dx}(cosecx)\int cosec^2x\ dx$ $= -\ cosecx\ cotx - \int cosecx\ cot^2x\ \ dx$ $= -\ cosecx\ cotx - \int cosecx\ (cosec^2x - 1)\ dx$ $= -\ cosecx\ cotx - \int cosec^3x\ \ dx + \int cosecx\ dx$ $= -\ cosecx\ cotx - I + \int cosecx\ dx$ $2I = -\ cosecx\ cotx + log\left\|tan\frac{x}{2}\right\|$ $\therefore I = -\frac{1}{2}cosecx\ cotx + \frac{1}{2}log\left\|tan\frac{x}{2}\right\| + C$	2
24.	$I = \int_{-\pi}^{\pi}\frac{cos^2x}{1+a^x}dx$ ----- (i) Also $I = \int_{-\pi}^{\pi}\frac{cos^2(-x)}{1+a^{-x}}dx = \int_{-\pi}^{\pi}\frac{a^xcos^2x}{1+a^x}dx$ ---- (ii) Adding $2I = \int_{-\pi}^{\pi}cos^2x\,dx = 2\int_0^{\pi}cos^2x\,dx = 2.2\int_0^{\frac{\pi}{2}}cos^2x\,dx$ $I = 2\int_0^{\frac{\pi}{2}}cos^2x\,dx = 2\int_0^{\frac{\pi}{2}}sin^2x\,dx0 = 2\int_0^{\frac{\pi}{2}}dx - 2\int_0^{\frac{\pi}{2}}cos^2x\,dx$ $I + I = 2\int_0^{\frac{\pi}{2}}dx = \pi \therefore\ I = \frac{\pi}{2}$	2
25.	$\int tan^{-1}\left(\frac{cosx}{1-sinx}\right)dx, x \in \left(-\frac{\pi}{4},\frac{\pi}{4}\right)$	2

	$= \int tan^{-1}\left(\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}\right)dx$ $=\int tan^{-1}\left(\frac{cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}-sin\frac{x}{2}}\right)dx = \int tan^{-1}\left(\frac{1+tan\frac{x}{2}}{1-tan\frac{x}{2}}\right)dx = \int tan^{-1}\left(tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)dx$ $= \int\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \frac{x^2}{4}+\frac{\pi x}{4}+C$	