CHAPTER-10

VECTORS

02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = 4$, $\|\vec{c}\| = 5$ and each one of them being perpendicular to the sum of the other two find $\| \vec{a} + \vec{b} + \vec{c} \|$	**2**
2.	If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$	**2**
3.	Show that the points A$(-2\hat{i}+3\hat{j}+5\hat{k})$, B$(\hat{i}+2\hat{j}+3\hat{k})$, C$(7\hat{i}-\hat{k})$ are collinear.	**2**
4.	If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.	**2**
5.	If $\vec{p}$ and $\vec{q}$ are the unit vectors forming an angle of 300 , find the area of the parallelogram having $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$ as its diagonals.	**2**
6.	Find the direction ratios and direction cosines of the vector $\vec{a} = (5\hat{i} - 3\hat{j} + 4\hat{k})$.	2
7.	Write the value of p for $\vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b} = \hat{i}+p\hat{j}+3\hat{k}$ are parallel vectors.	2
8.	Find $\vec{a}.(\vec{b}\times\vec{c})$ if $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$.	2
9.	If $\vec{a} = x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b} = 3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors, then write the value of y^x+5z.	2
10.	Find a unit vector parallel to the sum of the vectors $(\hat{i}+\hat{j}+\hat{k})$ and $(2\hat{i}-3\hat{j}+5\hat{k})$.	2
11.	*If* $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{k}$, then find the value of $\hat{a} - \hat{b} + 2\hat{c}$.	2
12.	The sum of two unit vectors is a unit vector. Show that the value of their difference is $\sqrt{3}$.	2
13.	Find a vector in the direction of $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8.	2
14.	Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, OZ	2
15.	If $\|\vec{a}\| = 10, \|\vec{b}\| = 1$ and $\|\vec{a}.\vec{b}\| = 6$, then find $\|\vec{a} \times \vec{b}\|$	2
16.	Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.	2
17.	Prove that the points A,B and C with position vectors $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ respectively are collinear if and only if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$.	2
18.	Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$	2
19.	If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.	2
20.	Show that the points A,B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ *and* $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively from the vertices of a right angled triangle.	2
21.	For what value of a, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?	2
22.	Find unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.	2
23.	If $\vec{a} = 2$, $\vec{b} = \sqrt{3}$ and $\vec{a}.\vec{b} = \sqrt{3}$ find the angle between $\vec{a}$ and $\vec{b}$.	2
24.	If $\vec{p}$ is unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$,then find $\|\vec{x}\|$.	2
25.	Show that the points A$(-2\hat{i} + 3\hat{j} + 5\hat{k})$,B$(\hat{i} + 2\hat{j} + 3\hat{k})$, and C$(7\hat{i} - \hat{k})$ are collinear.	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\left\|\vec a+\vec b+\vec c\right\|^2=\left(\vec a+\vec b+\vec c\right).\left(\vec a+\vec b+\vec c\right)$ $=\vec a.\vec a+\vec a.\left(\vec b+\vec c\right)+\vec b.\vec b+\vec b.\left(\vec a+\vec c\right)+\vec c.\vec c+\left(\vec a+\vec b\right)$ $=\left\|\vec a\right\|^2+\left\|\vec b\right\|^2+\left\|\vec c\right\|^2$ $=9+16+25$ $=50$ $\left\|\vec a+\vec b+\vec c\right\|=\sqrt{50}$ $=5\sqrt{2}$	**2**
2.	[redacted] is given that $\vec a=2\hat i-3\hat j+\hat k, \vec b=-\hat i+\hat k, \vec c=2\hat j-\hat k$ $\therefore \vec a+\vec b=\left(2\hat i-3\hat j+\hat k\right)+\left(-\hat i+\hat k\right)=\hat i-3\hat j+2\hat k$ $\vec b+\vec c=\left(-\hat i+\hat k\right)+\left(2\hat j-\hat k\right)=-\hat i+2\hat j$ We know that the area of parallelogram is $\frac{1}{2}\left\|\vec{d_1}\times\vec{d_2}\right\|$, where $\vec{d_1}$ and $\vec{d_2}$ are the diagonal vectors. Now, $\left(\vec a+\vec b\right)\times\left(\vec b+\vec c\right)=\begin{vmatrix}\hat i & \hat j & \hat k\\ 1 & -3 & 2\\ -1 & 2 & 0\end{vmatrix}=-4\hat i-2\hat j-\hat k$ $\therefore$ Area of the parallelogram having diagonals $\left(\vec a+\vec b\right)$ and $\left(\vec b+\vec c\right)$ $=\frac{1}{2}\left\|\left(\vec a+\vec b\right)\times\left(\vec b+\vec c\right)\right\|$ $=\frac{1}{2}\left\|-4\hat i-2\hat j-\hat k\right\|$ $=\frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2}$ $=\frac{\sqrt{21}}{2}$ square units Thus, the required area of the parallelogram is $\frac{\sqrt{21}}{2}$ square units.	**2**

<table>
<tr><td>3.</td><td>We have
vector AB = (1 + 2)i + (2 - 3)j + (3 - 5)k = 3i - j - 2k
vector BC = (7 - 1)i + (0 - 2)j + (-1 - 3)k = 6i - 2j - 4k
vector CA = (7 + 2)i + (0 - 3)j + (-1 - 5)k = 9i - 3j - 6k
Now, |vector AB|^2 = 14, |vector BC|^2 = 56, |vector CA|^2 = 126
⇒ |vector AB| = √14, |vector BC| = 2√14, |vector CA| = 3√14
⇒ |vector CA| = | vector AB| + |vector BC|
Hence the points A, B and C are collinear.</td><td>2</td></tr>
<tr><td>4.</td><td>Ans: $\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$
$=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$
$\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0\left[\because \vec{a}+\lambda\vec{b}\perp\vec{c}\right.$
$\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right].\left(3\hat{i}+\hat{j}\right)=0$
$3(2-\lambda)+(2+2\lambda)=0$
$-\lambda=-8$
$\lambda=8$</td><td>2</td></tr>
<tr><td>5.</td><td>$\vec{a}=\hat{p}+2\,\hat{q}$
$\vec{b}=2\,\hat{p}+\hat{q}$
$\vec{a}\times\vec{b}=(\vec{p}+2\vec{q})\times(2\vec{p}+\vec{q})$
$=2\vec{p}\times\vec{p}+\vec{p}\times\vec{q}+4\vec{q}\times\vec{p}+2\vec{q}\times\vec{q}$
$=2(0)+\vec{p}\times\vec{q}-4\vec{p}\times\vec{q}+2(0)$
$=-3\vec{p}\times\vec{q}$
Area of the parallelogram $=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$
$=\frac{1}{2}\left|-3(\vec{p}\times\vec{q})\right|$
$=\frac{3}{2}\left|\vec{p}\right|\left|\vec{q}\right|\sin 30^o$
$=\frac{3}{2}(1)(1)\left(\frac{1}{2}\right)$ ($\because \vec{p}$ and $\vec{q}$ are unit vectors)
$=\frac{3}{4}$ sq. units</td><td>2</td></tr>
<tr><td>6.</td><td>Given that $\vec{a}$ = $(5\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$
For any vector $\vec{a}$ = $a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}$ the direction ratios are represented as (a_x,a_y,a_z)
The direction ratios are (5, -3, 4)
$|\vec{a}| = \sqrt{25+9+16} = \sqrt{50} = 5\sqrt{2}$
∴ The direction cosines are = $\frac{5}{5\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}} = \frac{1}{\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$</td><td>2</td></tr>
</table>

7.	$Given\ that\ \vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b} = \hat{i}+p\hat{j}+3\hat{k}$ Since these two vectors are parallel to each other, so the angle between them is $\theta = 0$. Therefore $\vec{a}\times\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin 0 = 0$ We know that $\vec{a}\times\vec{b} = (a_2b_3-b_2a_3)\hat{i} + (a_3b_1-a_1b_3)\hat{j} + (a_1b_2-a_2b_1)\hat{k}$ $\therefore \vec{a}\times\vec{b} = 0$ $\Rightarrow (a_2b_3-b_2a_3)\hat{i} + (a_3b_1-a_1b_3)\hat{j} + (a_1b_2-a_2b_1)\hat{k} = 0$ $\Rightarrow \hat{i}(6-9p) + \hat{j}(9-9) + \hat{k}(3p-2) = 0$ $\Rightarrow -3\hat{i}(3p-2) + \hat{k}(3p-2) = 0$ $\Rightarrow 3p-2 = 0 \Rightarrow$ Thus p = 2/3	2
8.	Given that $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b} = -\hat{\imath}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}+2\hat{k}$ To find $\vec{a}.(\vec{b}\times\vec{c})$ We know that $\vec{b}\times\vec{c} = \hat{i}(b_2c_3-c_2b_3) + \hat{j}(c_1b_3-b_1c_3) + \hat{k}(b_1c_2-c_1b_2)$ Here $a_1=2, a_2=1, a_3=3, b_1=-1, b_2=2, b_3=1, c_1=3, c_2=1, c_3=2$ $\therefore \vec{b}\times\vec{c} = \hat{i}(4-1)+\hat{j}(3+2)+\hat{k}(-1-6) = 3\hat{i}+5\hat{j}-7\hat{k}$ Therefore, $\vec{a}.(\vec{b}\times\vec{c}) = (2\hat{i}+\hat{j}+3\hat{k}).(3i+5\hat{j}-7\hat{k}) = ((2\times3)+(1\times5)+(3\times(-7))) = 6+5-21$ = -10	2
9.	Given that $\vec{a} = x\hat{i}+2\hat{j}-z\hat{k}$ and $\vec{b} = 3\hat{i}-y\hat{j}+\hat{k}$ are two equal vectors. $\therefore$ x = 3, y = -2 and z = -1 $\therefore y^x+5z = (-2)^3+5(-1) = -8-5 = -13$	2
10.	Let $\vec{a} = (\hat{i}+\hat{j}+\hat{k})$ and $\vec{b} = (2\hat{i}-3\hat{j}+5\hat{k})$ $\vec{a}+\vec{b} = (\hat{i}+\hat{j}+\hat{k})+(2\hat{i}-3\hat{j}+5\hat{k}) = 3\hat{i}-2\hat{j}+6\hat{k}$ The unit vector parallel to the sum of the given vectors = $\frac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert} = \frac{3\hat{i}-2\hat{j}+6\hat{k}}{\sqrt{9+4+36}} = \frac{3\hat{i}-2\hat{j}+6\hat{k}}{\sqrt{49}} =$ $\frac{3\hat{i}-2\hat{j}+6\hat{k}}{7} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$	2
11.	$\lvert\hat{a}-\hat{b}+2\hat{c}\rvert = \sqrt{4+25+1} = \sqrt{30}$ $d.r = -\frac{2}{\sqrt{30}}.-\frac{5}{\sqrt{30}},-\frac{1}{\sqrt{30}}$	2
12.	$\lvert\vec{a}\rvert = 1, \lvert\vec{b}\rvert = 1, \lvert\vec{a}+\vec{b}\rvert = 1$ $(\lvert\vec{a}+\vec{b}\rvert)^2 + (\lvert\vec{a}-\vec{b}\rvert)^2 = 2\left\{\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2\right\} = 4$ $(\lvert\vec{a}-\vec{b}\rvert)^2 = 3$ $\lvert\vec{a}-\vec{b}\rvert = \sqrt{3}$	2
13.	$\vec{a} = 5\hat{i}-\hat{j}+2\hat{k}, \quad \hat{a} = \frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{30}}$ $Req\ vector = 8\hat{a} = 8.\frac{5\hat{\imath}-\hat{j}+2\hat{k}}{\sqrt{30}}$	2
14.	$\lvert\vec{a}\rvert = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ d.c = $(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$ $cos\alpha = \frac{1}{\sqrt{3}}, cos\beta = \frac{1}{\sqrt{3}}, cos\gamma = \frac{1}{\sqrt{3}}$ $\alpha = \beta = \gamma$(where α, β, γ are the inclination of $\vec{a}\ with\ OX, OY, OZ$ resp.)	2
15.	$(\lvert\vec{a}.\vec{b}\rvert)^2 + (\lvert\vec{a}\times\vec{b}\rvert)^2 = \left\{\lvert\vec{a}\rvert^2.\lvert\vec{b}\rvert^2\right\}$ $(\lvert\vec{a}\times\vec{b}\rvert)^2 = 64$	2

	$\left\|\vec{a}\times\vec{b}\right\| = 8$	
16.	$-\frac{1}{\sqrt{6}}\hat{\imath}+\frac{2}{\sqrt{6}}\hat{\jmath}-\frac{1}{\sqrt{6}}\hat{k}$	2
17.	Proving $\overrightarrow{AB}\times\overrightarrow{BC}=0, (\vec{b}-\vec{c})\times(\vec{c}-\vec{b})=0$ and proceeding further to prove.	2
18.	Expanding and solving.	2
19.	$2\hat{\imath}-4\hat{\jmath}+4\hat{k}$	2
20.	$\left\|\overrightarrow{AB}\right\|=\sqrt{35}, \left\|\overrightarrow{BC}\right\|=\sqrt{41}\ and\ \left\|\overrightarrow{CA}\right\|=\sqrt{6}$ and apply Pythagoras theorem.	2
21.	Let $\vec{A}$ = $2\,\hat{\imath}-3\hat{\jmath}+4\hat{k}$ $\vec{B}$ = $a\hat{\imath}+6\hat{\jmath}-8\hat{k}$ $\vec{A}$ and $\vec{B}$ are collinear so , $\vec{A}= \lambda\vec{B}$ $\frac{2}{a}=\frac{-3}{6}=\frac{4}{-8}$ a= -4	2
22.	Unit vector perpendicular to $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}$ is $\hat{n}=\frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|}$ … (i) $\vec{a}\times\vec{b}=\begin{bmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{bmatrix}$ = $-\hat{\imath}+\hat{\jmath}$ $\left\|\vec{a}\times\vec{b}\right\|=\sqrt{1+1}=\sqrt{2}$ From equation (i) $\hat{n}=\frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|}=\frac{1}{2}(-\hat{\imath}+\hat{\jmath})$	2
23.	We know that angle between $\vec{a}$ and $\vec{b}$ is given by $\cos\theta=\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|}$ $\cos\theta=\frac{\sqrt{3}}{2\sqrt{3}}$ $\cos\theta=\frac{1}{2}$ $\theta=\frac{\pi}{3}$	2
24.	It is given that $\vec{p}$ is unit vector and $(\vec{x}-\vec{p})\cdot(\vec{x}+\vec{p})=80$ $\|\vec{x}\|^2-\|\vec{p}\|^2=80$ $\|\vec{x}\|^2=80+1=81$ $\|\vec{x}\|=9$	2
25.	Given points A$(-2\hat{\imath}+3\hat{\jmath}+5\hat{k})$ B$(\hat{\imath}+2\hat{\jmath}+3\hat{k})$ C$(7\hat{\imath}-\hat{k})$ $\overrightarrow{AB}$=P.V. of B – P.V. OF A $=(\hat{\imath}+2\hat{\jmath}+3\hat{k})-(-2\hat{\imath}+3\hat{\jmath}+5\hat{k})$ $=(3\hat{\imath}-\hat{\jmath}-2\hat{k})$ $\overrightarrow{BC}$=P.V. of C – P.V. of B $=(7\hat{\imath}-\hat{k})-(\hat{\imath}+2\hat{\jmath}+3\hat{k})$	2

	$=(6\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$ $\overrightarrow{BC}=2\,\overrightarrow{AB}$ $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$. B is common. Hence A,B,C are collinear.	