CHAPTER-4
DETERMINANTS
02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	If W is an imaginary cube root of unity, find the value of $\begin{vmatrix} W^2 & W & 1 \\ 1 & W^2 & W \\ W & 1 & W^2 \end{vmatrix}$	2
2.	If $\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix}$ Write the cofactors of a_{21} , a_{22} a_{31} a_{33}	2
3.	Prove that $\begin{vmatrix} x+5 & x & x \\ x & x+5 & x \\ x & x & x+5 \end{vmatrix} = 75x^2 + 125x$	2
4.	**If A is an invertible matrix of order 2, then det is equal (A^{-1}) to**	2
5.	If $\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix}$ Write the cofactors of a_{21} , a_{22} a_{31} a_{33}	2
6.	If A is a symmetric matrix and B is skew-symmetric matrix such that $A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $\lvert 2A \rvert$.	2
7.	If A is any matrix such that $A^5 = I$, then find the value of $\lvert A^{-1} \rvert$.	2
8.	Evaluate the determinant $\begin{vmatrix} \log_4 9 & \log_3 8 \\ \log_4 3 & \log_3 512 \end{vmatrix}$.	2
9.	If A is a square matrix of order 3 and $\lvert A \rvert = 4$, then find $\lvert adj.(2A) \rvert$	2
10.	If A and B are invertible matrices of order 3 such that $\lvert A \rvert = 2$ and $\lvert (AB)^{-1} \rvert = \frac{-1}{6}$,then find $\lvert B \rvert$.	2
11.	If the Matrix A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, find $\lvert adjA \rvert$ without computing adj A.	2
12.	If the matrix $\begin{bmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{bmatrix}$ is singular, find x.	2
13.	If A = $\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$,show that $A^{-1} = A^2$	2
14.	Find the value(s) of k if the area of the triangle with vertices (-2,0), (0,4) and (0 ,k) is 4 square unit.	2
15.	If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $A^{-1} = kA$, then find the value of k.	2
16.	Barun visited three places Kolkata , Bhubaneswar and Bangalore with his younger Karan. He observed on map that the three places make a straight line. Karan wrote these places as points	2

	$(2x, x+3), (0, x)$ *and* $(x+3, x+6)$.  Find the coordinates of Kolkata and Bangalore	
17.	A square matrix A is invertible if A is non singular. If $A = \begin{bmatrix} 2 & p & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of p so that A^{-1} exists.	2
18.	The place of Peace and reconciliation, also known as the pyramid of peace and Accord is a 62- meter high pyramid in Mursultan, the capital of Kazakistan, that serves as anon-demonstrational national spiritual centre and an event house. It has 25 equal smaller equilateral triangles as shown in figure.  If the vertices of one triangle are (0, 0), $(3, \sqrt{3})$ and $(3, -\sqrt{3})$, then find the of the triangle. Also find the area of one face of the Pyramid	2
19.	It is well known that for a square matrix, $AA^{-1} = A^{-1}A = I$ and AI = IA=A. Now find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$,	2
20.	If A(3,4) ,B(7,2) ,C (x, y) are collinear, then write the equation of the line passing through A, B, C.	2
21.	Find the Value of $$\begin{vmatrix} \cos 15 & \sin 15 & 0 \\ \sin 15 & \cos 15 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$	2
22.	Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$	2
23.	Given A= $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find A^{-1}.	2
24.	Evaluate the product AB where	2

	$A = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{vmatrix}$	
25.	Suppose A is any 3× 3 non-singular matrix and $(A-3I)(A-5I)=O$, where $I = I_3$ *and* $O = O_3$. If $\alpha A+\beta A^{-1}$=4I, then what will be value of $\alpha+\beta$.	2
26.	If $If\ A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then what will be the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$.	2
27.	Given that $A = [a_{ij}]$ is a square matrix of order 3 and $\|A\| = -7$, then find the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$.	2
28.	Let A be a square matrix of order 3 such that A (adj A) = 2I, where I is the identity matrix. Write the value of $\|adjA\|$.	2
29.	Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then find the value of Det(B).	2
30.	Find the value of y if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2y & 4 \\ 6 & y \end{vmatrix}$	2
31.	Find the equation of the line joining (1, 2) and (3,6) using determinants.	2
32.	Verify A(adj. A) =(adj. A)A = (det A). I for A= $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$	2
33.	What is the inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$?	2
34.	If we find positive integral power of a symmetric matrix then we get which type of matrix- Symmetric or Skew symmetric?	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\begin{vmatrix} w^2 & w & 1 \\ 1 & w^2 & w \\ w & 1 & w^2 \end{vmatrix}$ $R_1 \longrightarrow R_1 + R_2 + R_3$ $\begin{vmatrix} w^2+w+1 & w & 1 \\ 1+w^2+w & w^2 & w \\ w+1+w^2 & 1 & w^2 \end{vmatrix}$ So we know that $w^2+w+1=0$ $\begin{vmatrix} 0 & w & 1 \\ 0 & w^2 & w \\ 0 & 1 & w^2 \end{vmatrix}=0$	2
2.	$\begin{vmatrix} w^2 & w & 1 \\ 1 & w^2 & w \\ w & 1 & w^2 \end{vmatrix}$ $R_1 \longrightarrow R_1 + R_2 + R_3$ $\begin{vmatrix} w^2+w+1 & w & 1 \\ 1+w^2+w & w^2 & w \\ w+1+w^2 & 1 & w^2 \end{vmatrix}$ So we know that $w^2+w+1=0$ $\begin{vmatrix} 0 & w & 1 \\ 0 & w^2 & w \\ 0 & 1 & w^2 \end{vmatrix}=0$	2
3.	<u>L.H.S</u> $\begin{vmatrix} x+5 & x & x \\ x & x+5 & x \\ x & x & x+5 \end{vmatrix}$ $C_1 \to C_1 + C_2 + C_3$ $\begin{vmatrix} 3x+5 & x & x \\ 3x+5 & x+5 & x \\ 3x+5 & x & x+5 \end{vmatrix}$ Taking common (3x+5) from C_1 (3x+5) $\begin{vmatrix} 1 & x & x \\ 1 & x+5 & x \\ 1 & x & x+5 \end{vmatrix}$ $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ =(3x+5)$\begin{vmatrix} 1 & x & x \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix}$ Expand R_3 (3x+5)5(5x) =25x(3x+5) =75x^2+125x = R.H.S	2

4.	A is invertible $AA^{-1} = I$ $\det(AA^{-1}) = \det(I)$ $\det A.(\det A^{-1}) = \det(\sqrt{2})$ $\left[\lvert AB\rvert = \lvert A\rvert\lvert B\rvert\right]$ $\det A^{-1} = \frac{1}{\det A}$ $\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 0 \;\; i.e. \lvert I\rvert = 1\right\}$	2
5.	$\begin{vmatrix}x+5 & x & x\\ x & x+5 & x\\ x & x & x+5\end{vmatrix}$ $C_1 \to C_1 + C_2 + C_3$ $\begin{vmatrix}3x+5 & x & x\\ 3x+5 & x+5 & x\\ 3x+5 & x & x+5\end{vmatrix}$ Taking common (3x+5) from C_1 $(3x+5)\begin{vmatrix}1 & x & x\\ 1 & x+5 & x\\ 1 & x & x+5\end{vmatrix}$ $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ $=(3x+5)\begin{vmatrix}1 & x & x\\ 0 & 5 & 0\\ 0 & 0 & 5\end{vmatrix}$ Expand R_3 $(3x+5)5(5x) = 25x(3x+5)$ $= 75x^2 + 125x$ = R.H.S	2
6.	$A^T = A, B^T = -B$ and $A - B = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} \Rightarrow (A-B)^T = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^T$ $\Rightarrow A^T - B^T = A + B = \begin{bmatrix}1 & 3\\2 & 4\end{bmatrix} \Rightarrow A + B + A - B = \begin{bmatrix}2 & 5\\5 & 8\end{bmatrix} \Rightarrow 2A = \begin{bmatrix}2 & 5\\5 & 8\end{bmatrix}$ $\therefore \lvert 2A\rvert = 16 - 25 = -9$	2
7.	$A^5 = I \Rightarrow A^{-1}.A^5 = A^{-1}.I \Rightarrow A^4 = A^{-1}$ $\Rightarrow \lvert A^{-1}\rvert = \lvert A^4\rvert = \lvert A\rvert^4 = 1^4 = 1$	2
8.	$\log_4 9 = \frac{\log 9}{\log 4}; \log_3 8 = \frac{\log 8}{\log 3}; \log_4 3 = \frac{\log 3}{\log 4}; \log_3 512 = \frac{\log 512}{\log 3}$ $\begin{vmatrix}\log_4 9 & \log_3 8\\ \log_4 3 & \log_3 512\end{vmatrix} = \log_4 9.\log_3 512 - \log_4 3.\log_3 8 = \frac{\log 9}{\log 4}.\frac{\log 512}{\log 3} - \frac{\log 3}{\log 4}.\frac{\log 8}{\log 3}$ $= \frac{2\log 3}{2\log 2}.\frac{9\log 2}{\log 3} - \frac{\log 3}{2\log 2}.\frac{3\log 2}{\log 3} == 9 - \frac{3}{2} = \frac{15}{2}$	2
9.	$\because \lvert adj.A\rvert = \lvert A\rvert^{n-1} \;\; \therefore \lvert adj.(2A)\rvert = \lvert 2A\rvert^{3-1} = \lvert 2A\rvert^2 = \left(2^3\lvert A\rvert\right)^2 = \left(2^3.4\right)^2 = 2^{10}$	2
10.	$\lvert (AB)^{-1}\rvert = \frac{-1}{6} \Rightarrow \frac{1}{\lvert AB\rvert} = \frac{-1}{6} \Rightarrow \frac{1}{\lvert A\rvert.\lvert B\rvert} = \frac{-1}{6} \Rightarrow \frac{1}{2.\lvert B\rvert} = \frac{-1}{6} \Rightarrow \frac{1}{\lvert B\rvert} = \frac{-1}{3}$	2

11.	Here Determinant A = $\begin{bmatrix}1 & -1 & 2\\ 3 & 1 & -2\\ 1 & 0 & 3\end{bmatrix}$ = 1(2-2) + 3(1+3) = 12 A is no singular Adj A = (12) * (12) = 144	2
12.	Use the determinant expansion and expand the matrix ,find the value of x using simple equation. x = -4/3	2
13.	By definition of matrix b is inverse of A if AB = I = BA .Here we have to show that A^2 is inverse of A there it is sufficient to shoe that A^2 A = I = AA^2 i.e. A^3 = I	2
14.	The absolute value of ½ $\begin{bmatrix}-2 & 0 & 1\\ 0 & 4 & 1\\ 0 & k & 1\end{bmatrix}$ = 4 The absolute value of ½ (-2)(4-k) = 4 Solving above ,we get k = 8 and 0	2
15.	Here $\lvert A\rvert = -19, adjA = \begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix}, A^{-1} = \frac{1}{\lvert A\rvert}adjA$ $\therefore A^{-1} = \frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix} \Rightarrow \frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix} = \begin{bmatrix}2k & 3k\\ 5k & -2k\end{bmatrix} \Rightarrow k = \frac{1}{19}$	2
16.	As points are on a straight line $\therefore \begin{vmatrix}2x & x+3 & 1\\ 0 & x & 1\\ x+3 & x+6 & 1\end{vmatrix} = 0 \Rightarrow -12x + 3(x+3) = 0 \Rightarrow x = 1$ Therefore co-ordinates of Kolkata are (2,4) and Bangalore are (4,8)	2
17.	A^{-1} exist if $\lvert A\rvert \neq 0 \Rightarrow \begin{vmatrix}2 & p & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{vmatrix} \neq 0 \Rightarrow p \neq \frac{-8}{5}$	2
18.	Area of a triangle = $\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 3 & \sqrt{3} & 1\\ 3 & -\sqrt{3} & 1\end{vmatrix} = 3\sqrt{3} sq.unit$ $\therefore$ area of one of the face of the Pyramid = $25 \times 3\sqrt{3} = 75\sqrt{3}$ sq. unit	2
19.	$\begin{vmatrix}-3 & 2\\ 5 & -3\end{vmatrix} = -1, \therefore \begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix} = \begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ $\therefore P = \begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}^{-1} = \begin{bmatrix}13 & 8\\ 1 & 1\end{bmatrix}$	2
20.	A(3,4) ,B(7,2) ,C (x, y) are collinear so ½ $\begin{vmatrix}3 & 4 & 1\\ 7 & 2 & 1\\ x & y & 1\end{vmatrix} = 0$ 4(y-4)+2(x-3)= 0 2x+4y =22 is the equation of line passing through A(3,4) ,B(7,2).	1 1
21.	$\cos^2 15° - \sin^2 15°$ =cos 30° =√3/2	1 ½ ½
22.	$\Delta = (x+y)(-3x+3y)$-(y+z)(-3z+3y)+(z+x)(-3z+3x)=3(y^2-x^2-y^2+z^2-z^2+x^2)==0	1 1
23.	$\lvert A\rvert$=10 ,	½

	Adj A= $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ $\Rightarrow \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	1 ½
24.	$A = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 2 + 4 + 0 = 6$ and $B = \begin{vmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{vmatrix}$=12+24+0=36 AB=216	½ 1 ½
25.	Given, $(A - 3I)\ (A - 5I) = O$ Or, $A^2 - 8A + 15I = O$ Post multiplying by A^{-1} on both sides , we have, $\frac{1}{2}A + \frac{15}{2}A^{-1} = 4I \quad \ldots \ldots (i)$ Comparing (i) with $\alpha A + \beta A^{-1} = 4I$, $\alpha = \frac{1}{2}\ \ and\ \beta = \frac{15}{2}$ $\alpha + \beta = 8.$	2
26.	We have, $A^2 = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$ $\lvert A\rvert = -1$ $\lvert A^{2016} - 2A^{2015} - A^{2014}\rvert = \lvert A^{2014}\rvert \lvert A^2 - 2A - I\rvert$ $= -25.$	2
27.	$\lvert A\rvert = -7$ $\sum_{i=1}^{3} a_{i2}A_{i2} = Determinant\ of\ the\ matrix\ A\ expanded\ along\ C_2$ $= \lvert A\rvert$ $= -7$	2
28.	Since, $A.(adj\ A) = \lvert A\rvert I$ So, $A.$(adj A) = 2I Or, $\lvert A\rvert = 2$ Now, $\lvert adj\ A\rvert = \lvert A\rvert^{n-1}$ Or, $\lvert adj\ A\rvert = 2^{3-1}$ = 4	2
29.	$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$ $A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$ $B = A + A^4$ $\lvert B\rvert = 2 + 2\cos 3\theta$ $= \frac{5-\sqrt{5}}{2} \in (1, 2)$	2

30.	.$+\sqrt{3}$, $-\sqrt{3}$	2
31.	$2x-y=0$	2
32.	Verification	2
33.	$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	2
34.	symmetric	2