CHAPTER-5

COMPLEX NUMBERS

03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	For any 2 complex numbers Z_1 and Z_2, prove that $Re(Z_1 Z_2)= Re\ Z_1 Re\ Z_2 - Im\ Z_1 Im\ Z_2$	3
2.	If $(\frac{1+i}{1-i})^m = 1$, then find the least positive integral value of m	3
3.	If $(x+iy)^3$= u+iv , then show that $\frac{u}{x}+\frac{v}{y}$ = $4(x^2+y^2)$	3
4.	Is $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ for all real numbers a & b ? Justify	3
5.	Evaluate : $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$	3
6.	Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$	3
7.	If $\overline{(x-\iota y)(3+5\iota)}=-6-24i$, then find the values of x and y.	3
8.	If $\lvert a+ib\rvert=1$, then show that $\frac{1+b+ai}{1+b-ai}=b+ai$	3
9.	If $z=\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)^{107}$, then show that $Im\ z=0$	3
10.	If $iz^3+z^2-z+i=0$, then show that $\lvert z\rvert=1$.	3
11.	Find the value of x^3+7x^2-x+16, when x=1+2i. x^3+7x^2-x+16	3
12.	Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.	3
13.	If $(x+iy)^{1/3}$ = a+ib, x , y ,ab ƐR ,show that $\frac{x}{a}+\frac{y}{b}$= $4(a^2-b^2)$	3
14.	If $(x+iy)^{1/3}$ = a+ib, x , y ,ab ƐR ,show that $\frac{x}{a}-\frac{y}{b}$= $-2(a^2+b^2)$	3
15.	Express the following complex number in the standard form. Also find their conjugate:$\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ Here Rez_1=x_1, Rez_2=x_2 Imz_1=y_1, Imz_2=y_2 $z_1 z_2$=$(x_1 + iy_1)(x_2 + iy_2)$ $x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$ $=x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$ $=x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2$ $=x_1x_2 - y_1y_2 + i(x_1y_2 + y_1x_2)$ L.H.S. Re(z_1z_2)=$x_1x_2 - y_1y_2$ R.H.S. Re Z_1Re Z_2 - Im Z_1Im Z_2=$x_1x_2 - y_1y_2$ Hence L.H.S.=R.H.S. (proved)	3
2.	$(\frac{1+i}{1-i})^m$=1 $\Longrightarrow (\frac{1+i}{1-i} \times \frac{1+i}{1-i})^m$=1 $\Longrightarrow (\frac{(1+i)^2}{(1-i)(1+i)})^m$=1 $\Longrightarrow (\frac{1+(i)^2+2i}{1^2-i^2})^m$=1 $\Longrightarrow (\frac{1-1+2i}{1+1})^m$=1 $\Longrightarrow \left(\frac{2i}{2}\right)^m$=1 $\Longrightarrow (i)^m$=1 Therefore m=4k,where k is some integer` The least positive integer is 1 Thus, the least positive integral value of m is 4(=4×1)	3
3.	$(x + iy)^3 = u + iv$ $\Longrightarrow (x)^3 + (iy)^3 + 3.x.iy(x + iy) = u + iv$ $\Longrightarrow x^3 + i^3y^3 + i3x^2y + 3xy^2i^2 = u + iv$ $\Longrightarrow x^3 - iy^3 + i3x^2y - 3xy^2 = u + iv$ $\Longrightarrow x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv$ On equating real and imaginary parts ,we obtain u=$x^3 - 3xy^2$,v=$3x^2y - y^3$ thus,$\frac{u}{x} + \frac{v}{y} = \frac{x^3-3xy^2}{x} + \frac{3x^2y-y^3}{y}$ $\Longrightarrow \frac{u}{x} + \frac{v}{y} = \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$ $\Longrightarrow \frac{u}{x} + \frac{v}{y} = x^2 - 3y^2 + 3x^2 - y^2$	3

	$\Rightarrow \frac{u}{x}+\frac{v}{y}=4x^2-4y^2$ $\Rightarrow \frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$ Hence proved.	
4.	$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ for all real numbers a & b except when $a<0$, $b<0$. Justification :- $-1 = i^2 = \sqrt{-1}\times\sqrt{-1}=\sqrt{(-1)(-1)}$ (assuming $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ for all real numbers a & b) = $\sqrt{1}=1$ Thus, we get $-1 = 1$, a contradiction . Hence, $\sqrt{a}\times\sqrt{b}\neq\sqrt{ab}$, when $a<0$, $b<0$. Otherwise $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ for all other cases, i.e. for - $a>0$, $b>0$; $a>0$, $b<0$; $a<0$, $b>0$; $a=0, b\neq 0$; $a\neq 0, b=0$	3
5.	$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3 = \left[i^2+\frac{1}{i^{25}}\right]^3 = \left[-1+\frac{1}{i}\right]^3 = [-1-i]^3 = -[1+i]^3$ $= -[1+3i-3-i]$ $= -(-2+2i)$ $= (2-2i)$	3
6.	$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{12+5i}{4+3i} = \frac{12+5i}{4+3i}\times\frac{4-3i}{4-3i} = \frac{63-16i}{25} = \frac{63}{25}-\frac{16}{25}i$ Therefore, conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{63}{25}+\frac{16}{25}i$	3
7.	Let $z=(x-iy)(3+5i)$ $z=3x+5xi-3yi-5yi^2=3x+5xi-3yi+5y=(3x+5y)+i(5x-3y)$ $\therefore \bar{z}=(3x+5y)-i(5x-3y)$ It is given that, $\therefore (3x+5y)-i(5x-3y)=-6-24i$, we obtain $3x+5y=-6$ ... (i) $5x-3y=24$ ... (ii) Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain $9x+15y=-18$ $25x-15y=120$ $34x = 102$ $\therefore x=\frac{102}{34}=3$ Putting the value of *x* in equation (i), we obtain $3(3)+5y=-6$ $\Rightarrow 5y=-6-9=-15$ $\Rightarrow y=-3$ Thus, the values of *x* and *y* are 3 and –3 respectively	3
8.	$\frac{1+b+ai}{1+b-ai}=\frac{(1+b+ai)^2}{(1+b)^2-(ai)^2}$ $=\frac{1+b^2-a^2+2b+2ia+2iab}{1+a^2+b^2+2b}=$ Since $\|a+ib\|=1 \Rightarrow a^2+b^2=1$	3

	LHS=$\frac{b^2+b+ai+bia}{1+b}=\frac{(1+b)(b+ai)}{1+b}=b+ai$	
9.	$z=\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-i\,\frac{1}{2}\right)^{107}$ $\bar{z}=\overline{\left(\frac{\sqrt{3}}{2}+\iota\,\frac{1}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\iota\,\frac{1}{2}\right)^{107}}$ $=\overline{\left(\frac{\sqrt{3}}{2}+\frac{\iota}{2}\right)}^{107}+\overline{\left(\frac{\sqrt{3}}{2}-\frac{\iota}{2}\right)}^{107}$ $=\left(\frac{\sqrt{3}}{2}-i\,\frac{1}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)^{107}$ $=z$ $=> z=\bar{z} => z\ is\ purely\ real => Im\ z=0$	3
10.	Given $iz^3+z^2-z+i=0$ Or $(z-i)(z^2+i)=0$ Or z=I or $z^2=-i$ Or$\|z\|^2=1$ Or$\|z\|=1$ proved	3
11.	We have x=1+2i or x-1=2i ⇒ $(x-1)^2=4i^2$ ⇒ $x^2-2x+5=0$………1 Since $x^3+7x^2-x+16=x(x^2-2x+5)+9(x^2-2x+5)+(12x-29)$ Therefore from 1 we get $x^3+7x^2-x+16=-17+24i$	3
12.	Since given that $\frac{3+2isin\theta}{1-2isin\theta}$ is purely real. Therefore $\frac{8sin\theta}{1+4sin^2\theta}=0$ Or sinθ=0 Or θ=nπ , n$\in Z$	3
13.	Solution: $(x+iy)^{1/3}=a+ib$ $(x+iy)=(a+ib)^3=a^3-3ab^2+i(3a^2b-b^3)$ $X=a^3-3ab^2$ $X/a=a^2-3b^2$---------(1) $Y=3a^2b-b^3$ $Y/b=3a^2-b^2$---------(2) (1)+(2) $=\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$	3
14.	$(x+iy)^{1/3}=a+ib$ $(x+iy)=(a+ib)^3=a^3-3ab^2+i(3a^2b-b^3)$ $X=a^3-3ab^2$ $X/a=a^2-3b^2$---------(1) $Y=3a^2b-b^3$ $Y/b=3a^2-b^2$---------(2)	3

	(1)-(2) $= \frac{x}{a} - \frac{y}{b} = -2a^2 - 2b^2$ $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$	
15.	Solution: $z = \frac{\sqrt{5+12i} + \sqrt{5-12i}}{\sqrt{5+12i} - \sqrt{5-12i}}$ On rationalising $((\sqrt{5+12i}) + (\sqrt{5-12i}))^2/(5+12i) - (-5-12i)$ $=10+2\sqrt{25+144}/24i$ = 3/2i Z = - 3i/2 $\bar{z}$ = 3i/2	3