CHAPTER-5
CONTINUITY & DIFFERENTIABILITY
03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	If the $f(x)$ given by $f(x) = \{3ax + b\ ,\ \ if\ x > 1\ 11\qquad ,\ if\ x = 1\ 5ax - 2b\ ,\ if\ x < 1$ is continuous at $x = 1$, find the values of a and b .	3
2.	Let $f(x) = \{\frac{1 - coscos\,4x}{x^2}\ \ ,\ \ if\ x < 0\ a\qquad ,\ if\ x = 0\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}\ ,\ if\ x > 0$, Determine the value of a so that $f(x)$ is continuous at $x = 0$	3
3.	If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2)\frac{dy}{dx} + y = 0$.	3
4.	If $(x - a)^2 + (x - b)^2 = c^2$ for some c>0.Prove that $\dfrac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant,	3
5.	Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$	3
6.	$y = (\log x)^{\cos x} + \frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$	3
7.	Find the points of discontinuity , if any, of the function, f(x) = |x| + 3 , x≤ -3 -2x , -3 < x< 3 6x +2 , x> 3	3
8.	If siny = x sin (a + y) , prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	3
9.	Discuss the differentiability of f(x) = x|x| at x = 0	3
10.	For what value of k is the following function continuous at x= $-\frac{\pi}{6}$? $f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}} & , x \neq -\frac{\pi}{6} \\ k, & x = -\frac{\pi}{6} \end{cases}$	3
11.	Prove that the greatest integer function defined by f(x)=[x], 0<x<2 is not differentiable at x=1.	3
12.	If $x\sqrt{1+y}\ \ + \ y\sqrt{1+x}$=0 and x ≠ y prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.	3

13.	If the $f(x)$ given by $f(x) = \begin{cases} 3ax + b\,, & if\ x > 1 \\ 11\,, & if\ x = 1 \\ 5ax - 2b\,, & if\ x < 1 \end{cases}$ is continuous at $x = 1$, find the values of a and b .	3
14.	Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}\,, & if\ x < 0 \\ a\,, & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}\,, & if\ x > 0 \end{cases}$, Determine the value of a so that $f(x)$ is continuous at $x = 0$	3
15.	If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2)\frac{dy}{dx} + y = 0$.	3
16.	If siny = x sin(a+ y), then prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$	3
17.	 A potter made a vessel, where the shape of the pot is based on $f(x) = \|x - 3\| + \|x - 2\|$, where $f(x)$ represents the height of the pot. when x > 4 what will be the height in terms of x? And if the potter is trying to make a pot using the function f(x) = [x], will he get a pot or not? why?	3
18.	If the function f(x) given by f(x) = $\begin{cases} 3ax + b, & if\ x > 1 \\ 11, & if\ x = 1 \\ 5ax - 2b, & if\ x < 1 \end{cases}$ is continuous at x = 1, then find the area of the sign board having base 'b' and height 'a' 	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$ $5a - 2b = 3a + b = 11 \;\Rightarrow\; a = 3 \text{ and } b = 2$	3
2.	$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \frac{1 - coscos\,4x}{x^2} = \lim_{x\to0^-} \frac{2\sin^2 2x}{x^2} = 2 \times 4 \times \lim_{x\to0^-} (\frac{sinsin\,2x}{2x})^2 = 8$ $\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} \times \frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4} = \lim_{x\to0^+} \sqrt{16 + \sqrt{x}} + 4 = 4 + 4 = 8$ $\Rightarrow$ a = 8	3
3.	$\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{d}{dx}\left(\frac{1-x}{1+x}\right) \Rightarrow \frac{dy}{dx}$ $= -\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2} [Multiplying\ by\ (1-x^2)]$	3
4.	Correct proof should be there without step missing	3
5.	$\frac{dy}{dx} = \frac{logtany + ytanx}{logcosx - xsecycosecy}$	3
6.	$(logx)^{cosx} \left\{-sinxlog(logx) + \frac{cosx}{xlogx}\right\} + \frac{4x}{(1+x^2)^2}$	3
7.	Discontinuous at x = 3	3
8.	$x = \frac{\sin y}{\sin(a+y)}$ $\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$ $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	3
9.	LHD = RHD = 0 , f(x) is differentiable at x = 0	3
10.	$\lim_{x\to\pi/6} f(x) = \lim_{x\to\pi/6} \frac{\sqrt{3}sinx + cosx}{x+\pi/6}$ $\lim_{x\to\pi/6} f(x) = \lim_{x\to\pi/6} \frac{2\sin(x+\frac{\pi}{6})}{x+\pi/6} = 2$ $f(\pi/6) = k$ k=2	3
11.	RHD= $\lim_{h\to0} \frac{f(1-h)-f(1)}{h} = \lim_{h\to0} \frac{[1+h]-[1]}{h} = \lim_{h\to0} \frac{(1-1)}{h} = 0$ LHD = $\lim_{h\to0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to0} \frac{[1-h]-[1]}{-h} = \lim_{h\to0} \frac{(0-1)}{-h} = \infty$ Since LHD ≠ RHD Therefore f(x) is not differentiable at x=1	3

12.	$x\sqrt{1+y}+y\sqrt{1+x}=0 \Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$ Squaring to get: $x^2(1+y)=y^2(1+x)$ Simplifying to get: $(x-y)(x+y+xy)=0$ As, $x \neq y \therefore y=-\frac{x}{1+x}$ $\frac{dy}{dx}=\frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2}=\frac{-1}{(1+x)^2}$	3
13.	$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=f(1)$ $5a-2b=3a+b=11 \quad \Rightarrow \quad a=3 \text{ and } b=2$	3
14.	$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{1-\cos 4x}{x^2}=\lim_{x\to 0^-}\frac{2\sin^2 2x}{x^2}=2\times 4\times \lim_{x\to 0^-}\left(\frac{\sin 2x}{2x}\right)^2=8$ $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\times\frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4}=\lim_{x\to 0^+}\sqrt{16+\sqrt{x}}+4=4+4=8$ $\Rightarrow a=8$	3
15.	$\frac{dy}{dx}=\frac{1}{2}\sqrt{\frac{1+x}{1-x}}\times\frac{d}{dx}\left(\frac{1-x}{1+x}\right)\Rightarrow\frac{dy}{dx}$ $=-\sqrt{\frac{1+x}{1-x}}\times\frac{1}{(1+x)^2}[Multiplying\ by\ (1-x^2)]$	3
16.	Given, $siny=xsin(a+y)$ $x=\frac{\sin y}{\sin(a+y)}$ On differentiating both side w.r.t x , we get $\Rightarrow\frac{dx}{dy}=\frac{\frac{d}{dy}(siny)\sin(a+y)-\frac{d}{dx}(\sin(a+y))\sin y}{\sin^2\ (a+y)}$ $\Rightarrow\frac{dx}{dy}=\frac{(\cos y)\sin(a+y)-(\cos(a+y))\sin y}{\sin^2\ (a+y)}$ $\Rightarrow\frac{dx}{dy}=\frac{\sin(a+y-y)}{\sin^2\ (a+y)}$ $\Rightarrow\frac{dx}{dy}=\frac{\sin(a)}{\sin^2\ (a+y)}$ $\Rightarrow\frac{dy}{dx}=\frac{\sin^2(a+y)}{sina}$ **Hence proved.**	3
17.	$The\ given\ function\ can\ be\ written\ as\ f(x)=\begin{cases}5-2x & ,if\ x<2\\ 1 & ,if\ 2,\leq x<3\\ 2x-5 & ,if\ x\geq 3\end{cases}$ $when\ x>4,\ f(x)$ = 2x – 5 no the potter will not get a pot because f(x) = [x] is not a continuous function as it is a greatest integer function so it is discontinuous at every integral point	3

18.	The given function is f(x) = $\begin{cases} 3ax + b, \ if \ \ x > 1 \\ 11, \ if \ x = 1 \\ 5ax - 2b, \ if \ x < 1 \end{cases}$ given that f(x) is continuous at x = 1 therefore, L.H.L = R.H.L = f(x) Now,L.H.L = $\lim_{x \to 1-} f(x)$ = $\lim_{x \to 1-} f(5ax - 2b)$ = 5a – 2b R.H.L = $\lim_{x \to 1+} (3ax + b)$ = 3a + b also, given that f(1) = 11 now, L.H.L = R.H.L = F(1) = 11 3a + b = 11 5a – 2b = 11 on solving we get a = 3 amd b = 2 now area of sign board = 1/2bh = 1/2 *2*3 = 3 sq. unit	3