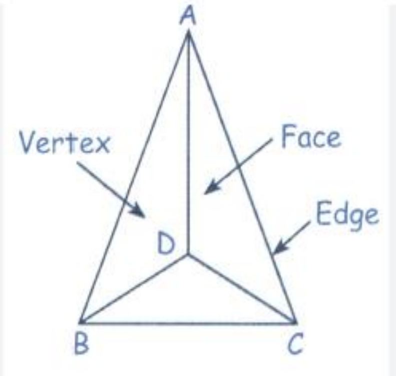
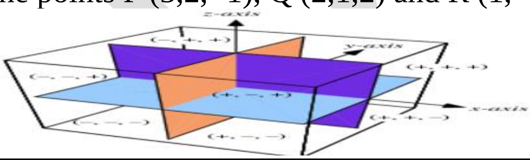
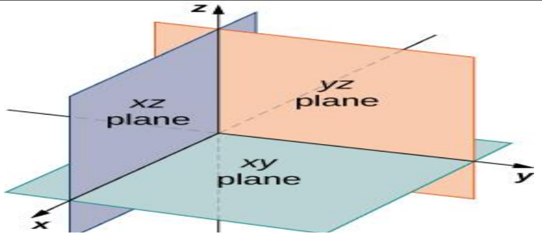
CHAPTER-12

INTRODUCTION TO 3D

03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (– 4, 0, 0) is equal to 10.	3
2.	A man visited India gate and moving on path, he observed that his distance from India gate is always 500 metres. Taking india gate as origin. Answer the following: (i) Find the equation of path of the man. (ii) If he saw a balloon seller is at point (600,800) then the man has to move towards India gate or away from it. (iii) If man is considered to be at point (400,300), then find the distance between balloon seller and man.	3
3.	  A building is to be constructed in the form of a triangular pyramid as shown in the figure. Let its angular points are A(0,1,2),B(3,0,1),C(4,3,6) and D(2,3,2) and G be the point of intersection of the median of ΔBCD. (i) Find the coordinates of point G. (ii) Find the length of AG.	3
4.	Two Racing cars started their linear journey from a fixed point. Find the condition for which the lines x cosA + y sinA = p_1 and x cosB + y sinB = p_2 will be perpendicular to each other. 	3
5.	A Marriage programme is arranged on a triangular plot costing Rs.5,000 per sq.m. Show that the area of triangle with vertices at (p - 4, p + 5), (p + 3, p - 2) and (p, p) is independent of p.Also find the arrangement cost .	3

		
6.	Mahendra went on a linear drive which passes through the point of intersection ofroads 3x + 4y - 1 = 0 and 2x - 5y + 7 = 0 and which is perpendicular to a crossroad 4x - 2y + 7 = 0. Find the equation of the straight line. 	3
7.	Find the point P on Z – axis such that PA = PB where A (1,5,7) B (5,1,-4)	3
8.	If the extremities (end points) of a diagonal of a square are (1,-2,3) and (2,-3,5) then find the length of the side of square	3
9.	If the distance between the points (a,0,2) and (1,1,0) is $\sqrt{14}$ units, then find the value of a.	3
10.	Show that the points A(1,2,1), B(1,-2,2),C(5,-2,1) and (5,2,2) are the vertices of a square.	3
11.	Three consecutive vertices of a parallelogram PQRS are P (1,1,2), Q(1,2,-4) and R(3,-1,2). Find the coordinates of the fourth vertex.	3
12.	Verify that the points A(3,-2,4), B(1,0,-2) and C(-1,2,-8) are collinear.	3
13.	Find the ratio in which the line segment joining $A(2,4,5)$ and $B(3,5,4)$ divided by the yz $plane$.	3
14.	The straight line joining the points (3,4,3) and (2,1,5) intersects the plane $2x + 2y - 2z = 1$ at P; find the coordinates of P.	3
15.	The mid points of the sides of a triangle are $(1,5,-1)$, $(0,4,-2)$ and $(2,3,4)$. Find its vertices. Also find the centroid of the triangle.	3
16.	Find the points in (i) ZX-plane (ii) YZ-plane and (iii) XY-plane which are equidistant from the points P (3,2, -1), Q (2,1,2) and R (1, -1,0). 	3
17.	Find the coordinates of the points which trisect the line segment joining the points P (4,2, -6) and Q (10, -16,6). P . . Q (4,2, -6) A B (10,-16,6)	3
18.	Find the ratio in which YZ-plane divides the line segment joining points (-2,4,7) and (3, -5,8). Also, find the coordinates of the point of intersection.	3

		
19.	Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point $P(4, 2, -6)$ and $Q(10, -16, 6)$	3
20.	If the points $P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear, find the valuesof P and q	3
21.	**Show that coordinator of the centroid of triangle with vertices A($x_1 y_1 z_1$), B($x_2 y_2 z_2$),and C($x_3 y_3 z_3$) is** $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$	3

ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$$
$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}=10-\sqrt{(x+4)^2+y^2+z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2+y^2+z^2=100-20\sqrt{(x+4)^2+y^2+z^2}+(x+4)^2+y^2+z^2$$
$$\Rightarrow x^2-8x+16+y^2+z^2=100-20\sqrt{x^2+8x+16+y^2+z^2}+x^2+8x+16+y^2+z^2$$
$$\Rightarrow 20\sqrt{x^2+8x+16+y^2+z^2}=100+16x$$
$$\Rightarrow 5\sqrt{x^2+8x+16+y^2+z^2}=(25+4x)$$

On squaring both sides again, we obtain

$$25(x^2+8x+16+y^2+z^2)=625+16x^2+200x$$
$$\Rightarrow 25x^2+200x+400+25y^2+25z^2=625+16x^2+200x$$
$$\Rightarrow 9x^2+25y^2+25z^2-225=0$$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

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<tr><td>2.</td><td>

(i) Equation of path as locus of man will be circle as his distance from India gate is constant which is equal to 500m.
Now, taking India gate as center (0,0)
Radius 500m
Equation of circle is $x^2 + y^2 = 250000$.

(ii) Distance of balloon seller from India gate
$= \sqrt{(600-0)^2+(800-0)^2}$
$= \sqrt{360000+640000}$ =1000 m
Since,ballon seller is standing at is distance 1000m which is more than 500m. so, man has to move away from India gate.

(iii) Distance between man (400,300) and balloon seller
$= \sqrt{(600-400)^2+(800-300)^2}$
$= \sqrt{40000+250000} = 100\sqrt{29}$ m.

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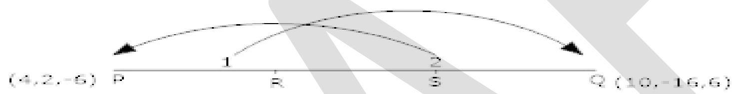
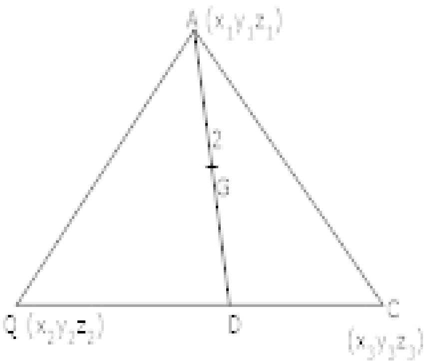
3.	(i) Clearly, G be the centroid of ΔBCD $= \left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3,2,3)$ (ii) AG=$\sqrt{(3-0)^2+(2-1)^2+(3-2)^2}$ $= \sqrt{3^2+1^2+1^2} = \sqrt{9+1+1} = \sqrt{11}$	3
4.	M_1=-CotA , m_2=-CotB , lines perpendicular=>m_1m_2=-1 , CotA.CotB=-1CosA.CosB+SinA.SinB=0 , Cos(A-B)=0 => A-B=90°	1 +1 +1
5.	Area of triangle =(1/2)[(p-4)(-2)+(p+3)(-5)+p(7)] = (1/2)l-7l = 7/2 is independent of p . Cost of programme Rs.5000x(7/2)=Rs.17,500	1 +1 +1
6.	Eq. of line through intersection of lines 3x+4y-1=0 & 2x-5y+7=0 is (3x+4y-1)+k(2x-5y+7)=0 ---(eq) It is perpendicular to line 4x-2y+7=0 $m_1.m_2$=-1 =>{-(3+2k)/(4-5k)}.2=-1 k=-2/9 . substituting in (eq.) we have (3x+4y-1)+(-2/9)(2x-5y+7)=0 ⇨ x+2y-1=0	1 +1 +1
7.	Given that A= (1,5,7) and B = (5,1,-4) Consider the point be P having co ordinates as it is on z – axis PA = PB on, $(PA)^2 = (PB)^2$ ⇒ $(1-0)^2 + (5-0)^2 + (7-a)^2 = (5 - 0)^2 + (1-0)^2 + (-4-a)^2$ ⟹ $1+25+49+a^2-14a = 25+1+16+a^2+8a$ ⇒ 14a + 8a = 49 – 16 ⇒ 22a = 33 ⇒ a = $\frac{3}{2}$ Therefore, point 'p' is (0,0, $\frac{3}{2}$)	3
8.	The points are B(1,-2,3) and D(2,-3,5) BD = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ $= \sqrt{(2-1)^2+(-3+2)^2+(5-3)^2}$ = $\sqrt{1+1+4}$ =$\sqrt{6}$ We know that the length of diagonal is equal to $\sqrt{2}$ times of side length $\therefore \sqrt{6} = \sqrt{2}$ x side length of square Length of side of square = $\sqrt{\frac{6}{2}}$ = $\sqrt{3}$	3
9.	Let the given points be A(a,0,2) and B(1,1,0) Given that the distance between A and B is 5. AB = 5 ⇒ $\sqrt{(1-a)^2+(1-0)^2+(0-2)^2} = \sqrt{14}$ ⇒ $(1-a)^2 + 1 + 4 = 14$	3

	$\Rightarrow 1+a^2-2a+5 = 14$ $\Rightarrow a^2 -2a - 8 = 0$ $\Rightarrow (a + 4)(a - 2) = 0$ $\Rightarrow a = -4, 2$	
10.	AB=BC=CD=DA=$\sqrt{17}$ Diagonal AC= Diagonal BD=$4\sqrt{2}$ Hence, ABCD is a square.	3
11.	We Know that the diagonals of a parallelogram bisect each other. Mid point of PR=Mid-point QS hence Find the coordinates of the fourth vertex S is (1,-2,8)	3
12.	Using Distance formula AB=$2\sqrt{11}$; BC=$2\sqrt{11}$;CA=$4\sqrt{11}$ We observe that AB+BC=CA Hence, A,B and C are collinear.	3
13.	Since the line segment joining $A(2,4,5) and\ B(3,5,4)$ is divided by the Y-Z plane C, so $\mathrm{x} = 0$. Let the ratio be $\mathrm{m:n}$ Coordinate of the point $C = (\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$ $$\frac{mx_2 + nx_1}{m+n} = 0$$ $$\frac{m3 + n2}{m+n} = 0$$ $$3m + 2n = 0$$ $$\frac{\mathrm{m}}{\mathrm{n}} = -\frac{2}{3}$$ $m:n = 2:3$ (divides externally)	3
14.	Let us suppose the point P divides the line segment joining the points $A(3,4,3) and\ B(2,1,5)$ in the ratio $m:n$ Then the coordinate of P is $(\frac{2m+3n}{m+n}, \frac{m+4n}{m+n}, \frac{5m+3n}{m+n})$ Since $2x + 2y = 2z = 1$ $2\frac{2m+3n}{m+n} + 2\ \frac{m+4n}{m+n} - 2\ \frac{5m+3n}{m+n}$=1 $\frac{2m+3n+m+4n-5m-3n}{m+n} = \frac{1}{2} \frac{m}{n} = \frac{7}{5}$ $$m:n = 7:5$$	3
15.	Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ be the vertices of the triangle and let $D(1,5,-1), E(0,4,-2) and\ F(2,3,4)$ be mid points of the sides BC,CA and AB respectively. Then, $\frac{\mathrm{x_2+x_3}}{2} = 1, \frac{\mathrm{y_2+y_3}}{2} = 5,\ \frac{\mathrm{z_2+z_3}}{2} = 1; \frac{\mathrm{x_3+x_1}}{2} = 0, \frac{\mathrm{y_3+y_1}}{2} = 4, \frac{\mathrm{z_3+z_1}}{2} = -2; \frac{\mathrm{x_1+x_2}}{2}$=2, $\frac{\mathrm{y_1+y_2}}{2} = 3, \frac{\mathrm{z_1+z_2}}{2} = 4.$ Thus, $\mathrm{x_2 + x_3 = 2, x_3 + x_1 = 0,\ x_1 + x_2 = 4;}$ $\mathrm{y_2 + y_3 = 10, y_3 + y_1 = 8,\ y_1 + y_2 = 6;}$	3

	$z_2 + z_3 = -2, z_3 + z_1 = -4,\ z_1 + z_2 = 8.$ Adding first three equations, we get $2(x_1 + x_2 + x_3) = 6$ *or* $x_1 + x_2 + x_3 = 3$ Thus $x_1 = 1,\ x_2 = 3,\ x_3 = -1.$ Adding next three equations, we get $y_1 = 2, \quad y_2 = 4, \quad y_3 = 6.$ Adding last three equations, we get $z_1 = 3, \quad z_2 = 5, \quad z_3 = -7.$ Hence the vertices of the given triangle are $A(1,2,3), B(3,4,5)$ *and* $C(-1,6,-7)$	
16.	(i) $(\frac{31}{10}, 0, \frac{1}{5})$ (ii) $(0, \frac{31}{16}, \frac{-3}{16})$ (iii) $(\frac{3}{2}, 1, 0)$	
17.	(6, -4,-2) and (8,-10,2)	
18.	2:3 and $(0, \frac{2}{5}, \frac{37}{5})$	
19.	Let R and S be the points of trisection of the segment PO. Then ∴  $\therefore PR = RS = SQ$ $\Rightarrow 2PR = RQ$ $\Rightarrow \frac{PQ}{RQ} = \frac{1}{2}$ R divides PQ in the ratio 1:2 Co-ordinates of point $R\left[\frac{1(10)+2\times 4}{1+2}, \frac{1(-16)+2\times 2}{1+2}, \frac{1\times 6+2(-6)}{1+2}\right]$ $R(6,-4,-2)$ $PS = 2SQ$ $\Rightarrow \frac{PS}{SQ} = \frac{2}{1}$ S divider PQ in the ratio 2:1 co-ordinates of point S $\left[\frac{2(10)+1(4)}{1+2}, \frac{2(-16)+1(2)}{1+2}, \frac{2(6)+1(-6)}{1+2}\right]$ $\therefore S(8,-10,2)$	3
20.	Given points P (1,0,-6) Q (-3,P,q) and R (-5,9,6) are collinear . Let point Q divides PR in the ratio k:1 Therefore co-ordinates of point P ($\frac{1-5K}{K+1}$, $\frac{0+9K}{K+1}$,$\frac{-6+6K}{K+1}$)and Q(-3,P,q) $\frac{1-5K}{K+1}$ = -3 1-5k= -3k-3	3

	-2k=-4 K=2 Therefore the value of P and q are 6 and -2 .	
21.	Let D be the mid point of AC. Then co ordinates of D are  $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$. Let G be the centroid of ΔABC. Then G, divides AD in the ratio 2:1. So coordinates of D are $\left(\frac{1.x_1+2\frac{(x_2+x_3)}{2}}{1+2},\frac{1.y_1+2\left(\frac{y_2+y_3}{2}\right)}{1+2},\frac{1.z_1+2\left(\frac{z_2+z_3}{2}\right)}{1+2}\right)$ $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$	3