CHAPTER-8

BINOMIAL THEOREMS

03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Which is larger $(1.01)^{1000000}$ or 10,000 .	3
2.	Prove that $\sum_{r=0}^{n} 3^{r\,n}C_r = 4^n$.	3
3.	Using binomial theorem. Prove that $6^n - 5n$ always leaves remainder I when divided by 25 .	3
4.	Find m such that the coefficient of x^2 in expansion of $\left(mx + \frac{m}{x}\right)^6$ is 960	3
5.	If the expansion of $(1 - ax)^n = 1 - 21x + 189x^2 - 945x^3 + \cdots$, then find a and n	3
6.	Find the term which is at equal distance from beginning and from end in the expansion of $\left(\frac{x}{3} + \frac{1}{x}\right)^8$	3
7.	Prove by using binomial theorem that $6^n - 5n - 1$ is divisible by 25, for all natural number n.	3
8.	Expand $(a + b)^5$ by using Pascal's triangle.	3
9.	Find the coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$	3
10.	Show that $9^{n+1} - 8n - 9$ is divisible by 64 , whenever n is a positive integer	3
11.	The first three terms in the expansion of a binomial are 1 , 10 and 40..Find the expansion	3
12.	Which is larger $(1.01)^{1000000}$ *or* 10,000 ?	3
13.	Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.	3
14.	Find the coefficient of x 4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.	3
15.	Find n, in the expansion of $(1 + x)^n$, coefficients of 2nd, 3rd and 4th terms are in A.P. Then n is equal to	3
16.	Find the coefficient of x^5 in binomial expansion of $(1 + 2x)^6(1 - x)^7$	3
17.	What are the last three digits of 27^{26}?	3
18.	Show that 2^{3n}-7n-1 is divisible by 49, where n is a positive integer.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$(1.01)^{10,00000}$ or $10,000$ $(1.01)^{10,00000} = (1+0.01)^{10,00000}$ $= \quad {}^{10,00000}C_0 + \quad {}^{10,00000}C_1(0.01)$ + other +ve term. $= 1 + 10,00000 \times 0.01$+ other positive term $= 1 + 10,000$ =10,001 Hence $(1.01)^{10,00000} > 10,000$	3
2.	(2) $\sum_{n=0}^{n} 3^r \quad {}^nc_r = \sum_{r=0}^{n} \quad {}^nC_r 3^r$ $= \quad {}^nC_0 + \quad {}^nC_1 \cdot 3 + \quad {}^nC_2 \cdot 3^2 + \cdots + \quad {}^nC_n 3^n$ $= \quad (1+3)^n$ $= \quad 4^n$ (RHS)	3
3.	$6^n = (1+5)^n$ $= 1 + \quad {}^nC_1 5 + \quad {}^nC_2 5^2 + \quad {}^nC_3 5^3 + \cdots + 5^n$ $= 1 + 5n + 5^2(\quad {}^nC_2 + \quad {}^nC_3 5 + \cdots + 5^{n-2})$ $6^n - 5n \quad = 1 + 25(\quad {}^nC_2 + \quad {}^nC_3 5 + \cdots + 5^{n-2})$ $= 1 + 25k$ (Hence Proved)	3
4.	$\left(mx + \frac{m}{x}\right)^6 = m^6\left(x + \frac{1}{x}\right)^6$ Then, coefficient of x^2 is $m^6 6_{C_2}$ $m^6 6_{C_2} = 960$ $m^6 = \frac{960}{15} = 64$ $m = 2$	3
5.	$(1-ax)^n = 1 - nax + \frac{n(n-1)}{2}a^2x^2 - \frac{n(n-1)(n-2)}{6}a^3x^3$ …, Comparing $na = 21$ $\frac{n(n-1)}{2}a^2 = 189$ $n(n-1)a^2 = 378$ Thus, $(n-1)a = 18$ Also $\frac{n}{n-1} = \frac{7}{6}$ $n = 7$ and $a = 3$	3
6.	The term which is at equal distance from beginning and from end is middle term	3

	Thus, $T_5 = 8_{C_4}\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right)^4 = \frac{70}{81}$	
7.	$(1+x)^n = C(n,0)+C(n.1)x+C(n.2)x^2+......C(n,n)x^n$ So, $(1+x)^n - C(n,1)x - 1 = C(n.2)x^2+......C(n,n)x^n$ Putting x=5,we get $6^n - 5n - 1$ is divisible by 25.	3
8.	By using Pascal triangle, the coefficients are 1,5,10,10,5,1 $(a+b)^5 = a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + b^5$	3
9.	$(1+x)(1-x)^n = (1+x)\{1 - nx + \frac{n(n-1)x^2}{2} - \cdots .. + (-1)^n x^n\}$ So,coefficient of $x^n = (-1)^n C(n,n) + (-1)^{-1}C(n,n-1) = (-1)^n(1-n)$	3
10.	$(1+a)^m = \overset{m}{\underset{0}{c}} + \overset{m}{\underset{1}{c}}a + \overset{m}{\underset{2}{c}}a^2 + ... + \overset{m}{\underset{m}{c}}a^m$ putting a=8 and m=n+1 we get $(1+8)^{n+1} = \overset{n+1}{\underset{0}{c}} + \overset{n+1}{\underset{1}{c}}8 + \overset{n+1}{\underset{2}{c}}8^2 + ... + \overset{n+1}{\underset{n+1}{c}}8^{n+1}$ $9^{n+1} = 9 + 8n + 64\left[\overset{n+1}{\underset{2}{c}} + \overset{n+1}{\underset{3}{c}}8 + ... + \overset{n+1}{\underset{n+1}{c}}8^{n-1}\right]$ So $9^{n+1} - 8n - 9 = 64k$ .implies the expression is divisible by 64	3
11.	Let the expansion be $(a+b)^n$, $(1+2)^5$ $a^n = 1........(i)$ $na^{n-1}b = 10........(ii)$ $\frac{n(n-1)}{2}a^{n-2}b^2 = 40$ on simplifying a=1 , b = 2	3
12.	Splitting 1.01 and using binomial theorem to write the first few terms we have $(1.01)^{1000000} = (1+0.01)^{1000000} = 1 + 10000 + other\ \ positive\, terms$ Which is greater than 10000	3
13.	$(27a^6 - 54ax^3 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6)$	3
14.	990	3
15.	7	3
16.	**Using binomial theorem we will expand both the terms.** **We know that,** **$(x+y)^n$ = $nC0x^n$ + $nC1x^{n-1}y$ + $nC2x^{n-2}y^2$ + $nC3x^{n-3}y^3$ +……..+ $nCrx^{n-r}y^r$ +….. + $nC0x^0y^n$** **Applying the formula we get,** **$(1 + 2x)^6 (1 – x)^7$ = (1 + 6C1(2x) + $6C2(2x)^2$ + $6C3(2x)^3$ + $6C4(2x)^4$ + $6C5(2x)^5$ + $6C6(2x)^6$)** **(1 – 7C1x + $7C2(x)^2$ -$7C3(x)^3$ + $7C4(x)^4$ -$7C5(x)^5$ + $7C6(x)^6$ - $7C7(x)^7$)** **= (1 + 12x + $60\times^2$ + $160\times^3$ + $240\times^4$ + $192\times^5$ + $64\times^6$) (1 – 7x + $21\times^2$ – $35\times^3$ + $35\times^4$ – $21\times^5$ + $7\times^6$ – x^7)** **Clearly, it can determined that the coefficient of x^5 is**	3

	⇒ 1 * (-21) + 12 * 35 + 60(-35) + 160 * 21 + 240 * (-7) + 192 * 1 **⇒ 171** **Therefore, the coefficient of x^5 in $(1 + 2x)^6 (1 – x)^7$ is 171.**	
17.	**In order to determine the same, the value 27^{26} needs to be reduced into the form $(730 – 1)^n$. After using simple binomial expansion, the digits can be obtained.** **Now, we have, $27^2 = 729$** **Thus,** **$27^{26} = (729)^{13} = (730 – 1)^{13}$** **$= 13C0\ (730)^{13} – 13C1\ (730)^{12} + 13C2\ (730)^{11} – . . . . . –$** **$13C10\ (730)^3 + 13C11(730)^2 – 13C12\ (730) + 1$** **= 1000m + [(13 × 12)]/2] × (14)2 – (13) × (730) + 1** **Herein, we can say that 'm' is a positive integer** **Thus,** **= 1000m + 15288 – 9490 = 1000m + 5799** **Therefore, the last three digits of 27^{26} are 799.**	3
18.	**Given $2^{3n}-7n-1= 8^n-7n-1$** **$=(1+7)^n-7n-1$** **$=nC_0+nC_1\ 7+nC_2 7^2+……..+nC^n 7^n -7n-1$** **$=1+7n+nC_2 7^2+……..+nC^n 7^n -7n-1$** **$=nC_2 7^2+……..+nC^n 7^n$** **$=49(nC_2+…….+nC_n 7^{n-2})$** **Which is divisible by 49**	3