CHAPTER-7

INTEGRALS

03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Find: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$	3
2.	Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$	3
3.	Evaluate :$\int \frac{sin^6x+cos^6x}{sin^2xcos^2x}\, dx$	3
4.	Find the value of ∫ sin x·log cos x dx.	3
5.	Evaluate: $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$	3
6.	Evaluate: $\int \frac{(x-3)}{(x-1)^3}e^x dx$	3
7.	Evaluate : $\int_0^{\pi} \frac{xtanx}{secxcosecx}dx$	3
8.	Evaluate : $\int_{-1}^{2} f(x)dx, where\ f(x) = \|x+1\| + \|x\| + \|x-1\|$	3
9.	Evaluate : $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16\,sin2x}dx$	3
10.	$Find\ the\ value\ of \int \frac{x^7}{x+1}dx$	3
11.	Find the value of $\int tan\ tan\ x\ tan\ tan\ 2x\ tan\ tan\ 3x\ dx$	3
12.	find the value $\int \frac{1}{\sqrt{(1-e^{2x})}}dx$	3
13.	Evaluate $\int \frac{6e^{2x}+7e^x}{\sqrt{(e^x-5)(e^x-4)}}dx$	3
14.	Find the value of $\int_{-\pi}^{\pi} \frac{2x(1+sinx)}{1+cos^2x}dx$	3
15.	Evaluate $\int_0^1 \frac{log(1+x)}{1+x^2}\, dx$	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$ Let $\sqrt{x}$ = t $\Rightarrow \frac{1}{2\sqrt{x}}$ dx = dt $\Rightarrow \frac{1}{\sqrt{x}}$ dx = 2 dt $= 2\int \frac{1}{(t+1)(t+2)}dt = 2\int \frac{(t+2)-(t+1)}{(t+1)(t+2)}dt = 2\left(\int \frac{1}{t+1}dt - \int \frac{1}{t+2}dt\right)$ $= 2[\log\|t+1\| - \log\|t+2\|] + C$ $= 2\log\left\|\frac{t+1}{t+2}\right\| + c$ $= 2\log\left\|\frac{\sqrt{x}+1}{\sqrt{x}+2}\right\| + C$	3
2.	Let I = $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$ Put e^x = t $\Rightarrow e^x$ dx = dt $\therefore I = \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{-(t^2+4t-5)}} = \int \frac{dt}{\sqrt{-(t^2+2.t.2+2^2-9)}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} = sin^{-1}\left(\frac{t+2}{3}\right) + C = sin^{-1}\left(\frac{e^x+2}{3}\right) + C$	3
3.	$\int \frac{sin^6x + cos^6x}{sin^2x cos^2x}dx$ Let I = $\int \frac{sin^6x+cos^6x}{sin^2xcos^2x}dx = \int \frac{(sin^2x)^3+(cos^2x)^3}{sin^2xcos^2x}dx$ $= \int \frac{(sin^2x + cos^2x)(sin^4x - sin^2xcos^2x + cos^4x)}{sin^2xcos^2x}dx$ $= \int \frac{sin^4x - sin^2xcos^2x+cos^4x}{sin^2xcos^2x}dx$ $= \int tan^2x\,dx - \int dx + \int cot^2x\,dx$ $= \int sec^2x - 1)\,dx - x + \int cosec^2x - 1)\,dx$ =tanx – cotx -3x + C	3
4.	*Put* cos x = t ⇒ – sin x dx = dt ∴ – ∫ log t dt ⇒ – ∫ (log t) · 1 dt ⇒ [log t ∫ 1 dt – ∫ {$\frac{d}{dx}$ (log t) ∫ 1 dt} dt] ⇒ [(log t)·t – ∫ $\frac{1}{t}$ · t dt] ⇒ – [t · log t – ∫ 1 dt] ⇒ – [t log t – t] + C ⇒ – t·log t + t + C ⇒ – cos x log cos x + cos x + C	3
5.	$\Rightarrow \frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$ ⇒x^2 + x + 1 = A(x^2 + 1) + (Bx + C) (x + 2) ⇒ x^2 + x + 1 = x^2 (A + B) + x(2B + C) + (A + 2C) On comparing the coefficients of x^2, x and constant terms both sides, we get A + B = 1 ……. (ii) 2B + C = 1 …….. (iii) and A + 2C = 1 ……. (iv) On substituting the value ofBfrom q. (ii) in Eq. (iii), we get 2(1 – A) + C = 1 ⇒ 2 – 2A + C = 1 ⇒ 2A – C = 1 ……. (v) From above equations we get $\Rightarrow A = \frac{3}{5}, B = \frac{2}{5}$ and $C = \frac{1}{5}$ $\Rightarrow \frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$	3

	$\Rightarrow \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x+1}{x^2+1}dx$ $\Rightarrow \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x}{x^2+1}dx + \frac{1}{5}\int \frac{1}{x^2+1}dx$ $\Rightarrow \frac{3}{5}\log(x+2) + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}(x) + c$	
6.	$\int \frac{(x-3)}{(x-1)^3}e^x dx = \int \frac{(x-1-2)}{(x-1)^3}e^x dx$ $\Rightarrow \int \left[\frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3}\right]e^x dx$ $\Rightarrow \int \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right]e^x dx$ we know that $\Rightarrow \int e^x\,[f(x) + f'(x)]dx = e^x.f(x) + c$ where $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = -\frac{2}{(x-1)^3}$ hence $\int \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right]e^x dx = \frac{e^x}{(x-1)^2} + c$	3
7.	$Let\ I = \int_0^{\pi} \frac{xtanx}{secxcosecx}dx = \int_0^{\pi} xsin^2xdx$ $\Rightarrow I = \int_0^{\pi} (\pi - x)sin^2(\pi - x)dx$ $\Rightarrow I = \int_0^{\pi} (\pi - x)sin^2xdx \Rightarrow 2I = \pi\int_0^{\pi} sin^2xdx$ $\Rightarrow 2I = \frac{\pi}{2}\int_0^{\pi} (1 - cos2x)\,dx = \frac{\pi}{2}\left[x - \frac{sin2x}{2}\right]_0^{\pi} = \frac{\pi^2}{2}$ $\Rightarrow I = \frac{\pi^2}{4}$	1 1 1
8.	We can redefine f as $f(x) = \begin{cases} 2 - x, & if\ -1 \le x < 0 \\ x + 2, & if\ 0 \le x < 1 \\ 3x, & if\ 1 \le x < 2 \end{cases}$ $\Rightarrow \int_{-1}^{2} f(x)dx = \int_{-1}^{0} (2-x)dx + \int_0^1 (x+2)dx + \int_1^2 3xdx$ $= \left[2x - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} + 2x\right]_0^1 + \left[\frac{3x^2}{2}\right]_1^2$ $= \frac{5}{2} + \frac{5}{2} + \frac{9}{2} = \frac{19}{2}$	1 1 1
9.	$Let\ I = \int_0^{\frac{\pi}{4}} \frac{sinx + cosx}{9 + 16\,sin2x}dx = \int_0^{\frac{\pi}{4}} \frac{sinx + cosx}{9 + 16\,[1 - (sinx - cosx)^2]}dx$ Put $sinx - cosx = t \Rightarrow (sinx + cosx)dx = dt$ When x = 0 , t = -1, when x = $\frac{\pi}{4}$, t = 0 $\Rightarrow I\int_{-1}^{0} \frac{1}{9 + 16(1 - t^2)}dt = \int_{-1}^{0} \frac{1}{25 + 16t^2}dt = \frac{1}{16}\int_{-1}^{0} \frac{1}{\left(\frac{5}{4}\right)^2 + t^2}dt$ It is of the form $\int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}log\left\|\frac{a+x}{a-x}\right\| + C$ After evaluating, we get I = $\frac{1}{20}log3$	1 1 1

10.	We know that, $x^7+1=(x+1)(x^6-x^5+x^4-x^3+x^2-x+1)$ $\int \frac{x^7}{x+1}\,dx$ $=\int \frac{x^7+1-1}{x+1}\,dx$ $=\int \frac{x^7+1}{x+1}\,dx-\int \frac{dx}{x+1}$ $=\int (x+1)\frac{x^6-x^5+x^4-x^3+x^2-x+1)}{(x+1)}\,dx - log\, log\,\lvert x+1\rvert$ $=\frac{x^7}{7}-\frac{x^6}{6}+\frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+x - log\, log\,\lvert x+1\rvert + c$ (Answer)	3
11.	We know that, $tan\, tan\, 3x = tan\, tan\,(2x+x) = \frac{tan2x+tanx}{1-tan2xtanx}$ So, we get $tan\, tan\, x\;\; tan\, tan\, 2x\;\; tan\, tan\, 3x = tan\, tan\, 3x - tan\, tan\, 2x - tan\, tan\, x$ $\int tan\, tan\, x\, tan\, tan\, 2x\, tan\, tan\, 3x\, dx$ $=\int (tan\, tan\, 3x - tan\, tan\, 2x - tan\, tan\, x)dx$ $=\int tan\, tan\, 3x\, dx - \int tan\, tan\, 2x\, dx - \int tan\, tan\, x\, dx$ $=\frac{1}{3} iog\,\lvert cos\, cos\, 3x\rvert - \frac{1}{2} log\, log\,\lvert cos\, cos\, 2x\rvert - log\, log\,\lvert cos\, cos\, x\rvert + c$ (Answer)	3
12.	$\int \frac{1}{\sqrt{(1-e^{2x})}}\,dx$ $=\int \frac{e^{-x}}{\sqrt{e^{-2x}-1}}\,dx \ldots (i)$ $taking, e^{-x}=u$ $\therefore -e^{-x}dx = du$ $(i) becomes \int \frac{-du}{\sqrt{u^2-1}}$ $=-\,log\, log\,\lvert u+\sqrt{u^2-1}\,\rvert + c$ $=-\,log\, log\,\lvert e^{-x} + \sqrt{e^{-2x}-1}\rvert + c$ (Answer)	3
13.	$\int \frac{(6e^x+7)e^x}{\sqrt{(e^x-5)(e^x-4)}}\,dx$ Let $e^x=t,\;\; then\; e^x dx = dt$ $\therefore I = \int \frac{(6t+7)}{\sqrt{(t-5)(t-4)}}\,dt$ Using the expression $6t+7 = A\frac{d}{dt}(t^2-9t+20)+B$ Solving we get $A=3\;\; and\;\; B=34$ $\therefore I = \int \frac{(6t+7)}{\sqrt{(t-5)(t-4)}}\,dt = 3\int \frac{(2t-9)}{\sqrt{t^2-9t+20}}\,dt + 34\int \frac{1}{\sqrt{t^2-9t+20}}\,dt$ $=6\sqrt{t^2-9t+20} + 34 log \left\lvert \left(x-\frac{9}{2}\right) + \sqrt{t^2-9t+20} \right\rvert + C\; where\; t=e^x$	3
14.	$I=\int_{-\pi}^{\pi} \frac{2x(1+sinx)}{1+cos^2x}\,dx = \int_{-\pi}^{\pi} \frac{2x}{1+cos^2x}\,dx + 2\int_{-\pi}^{\pi} \frac{xsinx}{1+cos^2x}\,dx$ $=0+2\int_{-\pi}^{\pi} \frac{xsinx}{1+cos^2x}\,dx\;\; (f(x)=\frac{2x}{1+cos^2x}\; is\; an\; odd\; fn.)$	3

	$= 4\int_0^{\pi} \frac{xsinx}{1+cos^2x} dx \ (g(x) = \frac{xsinx}{1+cos^2x} \ is\ an\ even\ fn.)$ Also $I = 4\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+cos^2(\pi-x)} dx$ Adding we get, $2I = 4\pi \int_0^{\pi} \frac{sinx}{1+cos^2x} dx$ $$I = 2\pi \int_0^{\pi} \frac{sinx}{1+cos^2x} dx$$ Putting $t = cosx, \quad dt = -sinxdx$, Also as $x = 0, t = 1$ & $x = \pi, t = -1$ The integral reduces to $I = 2\pi \int_{-1}^{1} \frac{dt}{1+t^2} = \pi^2$	
15.	$\int_0^1 \frac{log(1+x)}{1+x^2}\ dx$ Putting $x = tan\theta,$ then the integral reduces to $$I = \int_0^{\frac{\pi}{2}} \log(1 + tan\theta)\, d\theta$$ Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ $$I = \frac{\pi}{8} log2$$	3