CHAPTER-13

LIMITS & DERIVATIVES

03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{x}$	3
2.	If the function f(x) satisfies $$\lim_{x\to 1}\frac{f(x)-2}{x^2-1}=\pi, evaluate \ \lim_{x\to 1} f(x)$$	3
3.	Differentiate $\frac{secx+tanx}{secx-tanx}$ with respect to x.	3
4.	If $y=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........$, then show that $\frac{dy}{dx}=y$.	3
5.	Find the derivative of $(1-x)(2-x)(3-x)......(n-x)$ at $x=1$.	3
6.	Let $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & where\ x\neq\frac{\pi}{2}\\ 3, & where\ x=\frac{\pi}{2}\end{cases}$ and if $\lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$, then find the value of k.	3
7.	Let f(x) = $\begin{cases}1, for\ x<0\\ 1+\ sinx, for\ x\geq 0\end{cases}$ using first principle check f'(0) exits or not.	3
8.	Find $\lim_{x\to 0}\frac{xcosx-log(1+x)}{x^2}$	3
9.	Find the derivative of $\frac{x^2}{x^2+2x+2023}$	3
10.	Evaluate: $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$	3
11.	Find the derivative of $\frac{x^5-\cos x}{\sin x}$	3
12.	Findthe derivative of $\frac{x+\cos x}{\tan x}$	3
13.	Suppose $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$ and if $\lim_{x\to 1} f(x)=f(1)$. What are the possible values of a	3

	and b?	
14.	**Find the derivative of expression** $1+x+x^2+x^3+\ldots+x^{50}$ **for x=1.**	3
15.	If $y = \left[\sin\frac{x}{2} + \cos\frac{x}{2}\right]^2$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$-\frac{1}{2}$	3
2.	2	3
3.	$\frac{2cosx}{(1-sinx)^2}$	3
4.	$y=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........ \Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........\right)$ $=0+1+\frac{2x}{2!}+\frac{3x^2}{3!}+\frac{4x^3}{4!}+.........=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........=y$ $\therefore \frac{dy}{dx}=y$	3
5.	$\frac{d}{dx}\left[(1-x)(2-x)(3-x)......(n-x)\right]$ $=(-1)(2-x)(3-x)......(n-x)+$ $(-1)(1-x)(3-x)......(n-x)+......+(-1)(1-x)(2-x)(3-x)......[(n-1)-x)]$ $\frac{d}{dx}\left[(1-x)(2-x)(3-x)......(n-x)\right]_{x=1}$ $=(-1)(2-1)(3-1)......(n-1)+0+....+0$ $=(-1).(n-1)!$	3
6.	$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}} \frac{k\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}=\frac{k}{2}\times 1=\frac{k}{2}$ But given $\lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2}=3 \Rightarrow k=6$	3
7.	$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h}$= 1 but $\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h}$= 0 Hence f'(0) does not exist.	3
8.	We know that cos x = $1-\frac{x^2}{2!}+\frac{x^4}{4!}-........$ And log (1+ x) = $x-\frac{x^2}{2}+\frac{x^3}{3}+..........$ Use the above series we get the answer $\frac{1}{2}$	3

9.	This is into the form $\frac{u(x)}{v(x)}$, use derivative property $\frac{u(x)}{v(x)}$ we get the required answer	3
10.	$\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x} = \lim_{x\to 0}\frac{\sin x\,(\frac{1}{\cos x}-1)}{\sin^3 x}$ $= \lim_{x\to 0}\frac{1-\cos x}{\cos x\sin^2 x}$ $= \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\cos x\,(4\sin^2\frac{x}{2}\cos^2\frac{x}{2})}$ $= \frac{1}{2}$	3
11.	Let $f(x) = \frac{x^5-\cos x}{\sin x}$ Differentiating w.r.t x we get $f'(x) = \frac{(5x^4+\sin x)\sin x-(x^5-\cos x)\cos x}{\sin^2 x}$ $= \frac{-x^5\cos x+5x^4\sin x+1}{\sin^2 x}$	3
12.	Let $f(x) = \frac{x+\cos x}{\tan x}$ Differentiating w.r.t x we get $f'(x) = \frac{(1-\sin x)\tan x-(x+\cos x)\sec^2 x}{\tan^2 x}$	3
13.	We have $\lim_{x\to 1} f(x) = f(1) \Leftrightarrow \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$ $\Leftrightarrow \lim_{x\to 1^-} f(x) = f(1) \quad and \ \lim_{x\to 1^+} f(x) = f(1)$ $\Leftrightarrow \lim_{h\to 0} f(1-h) = 4 \quad and \ \lim_{h\to 0} f(1+h) = 4$ $\Leftrightarrow \lim_{h\to 0}\{a+b(1-h)\} = 4 \quad and \ \lim_{h\to 0}\{b-a(1+h)\} = 4$ ⟺a + b = 4 and b – a = 4 ⟺a = 0 and b = 4.	3
14.	By the power rule of differentiation we get, $f'(x)=1+2x+3x^2+4x^3+\ldots+50x^{49}$ By substituting x=1 in the above derivative we get, $f'(1) = 1+2+3+\ldots+50$ $=\frac{50(50+1)}{2}$	3

	$=25\times51$ $=1275$	
15.	$\frac{\sqrt{3}}{2}$.	3