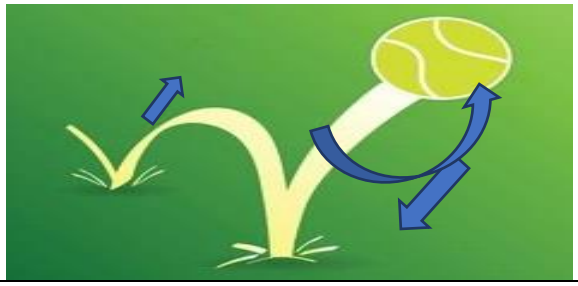
CHAPTER-9

SEQUENCES & SERIES

03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the sum of $1^2+(1^2+2^2)+(1^2+2^2+3^2)+(1^2+2^2+3^2+)+\ldots\ldots\ldots\ldots$	3
2.	The number of bacteria in a certain culture double every hour. If there were 30 bacteria in the culture originally, how may bacteria will be present at the end of 2nd hour, 4th hour and nth hour?	3
3.	The gate receipts at the show of 'comedy nights' amounted Rs.9500 on the first night and showed a drop of Rs.250 every succeeding night. If the operational expenses of the show are Rs.2000 a day, then find on which night, the show ceases to be profitable?	3
4.	If f is a function satisfying f(x+y) = f(x).f(y) for all x, y ∈ N such that f(1) = 3 and $\sum_{x=1}^{n} f(x) = 120$,find the value of n.	3
5.	A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it cost 50 paise to mail one letter, Find the amount spent on the postage when 8th set of letter is mailed.	3
6.	The sum of an infinite G.P. is 2 and the sum of G.P. made from the cubes of this infinite G.P. is 24. Find the G. P.	3
7.	The first term of a geometric progression is 1. The sum of the third and fifth term is 90. Find the common ratio of the geometric progression	3
8.	Ramesh being a plant lover decides to open a nursery and he bought few plants with pots. He wants to place pots in such a way that number of pots in first row is 2, in second row is 4 and in third row is 8 and so on. Answer the following questions based on the above information. (i) Find the number of pots in the 8th row. (ii) Find the total number of pots in 10 rows. (iii) If Ramesh wants to place 510 pots in all, how many rows will be formed?	3
9.	At the end of each year the value of a certain machine has depreciated by 20% of its value at the beginning of that year. If its initial value was Rs. 1250, find the value at the end of 5 years	3
10.	**Find the r^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.** **[Hint: $a_n = S_n - S_{n-1}$]**	3
11.	A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job? 	3

12.	 A tennis ball bounces 1/3 its height every time it is dropped.If the ball is dropped from a height of 12 m, find the total distance traveledby the ball when it hits the ground 7 times.	3
13.	A pattern of numbers is made in G.P. such that the fourth, seventh and the last term of it are 10, 80 and 2560 respectively. Find the first term and the number of term in the pattern.	3
14.	 A scientific observation has been carried out in which the number of bacteria in a certain culture triples every hour. If there were 45 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, and 5^{th} hour?	3
15.	Using the inequality relation between A.M. and G.M. find out the maximum value of $5^x + 5^{1-x}$.	3
16.	Deepak and his sister are playing number game during the summer vacation. They found that there are two number whose sum is 6 times their geometric mean. They obtained that the numbers are in the ratio $(3 + 2\sqrt{2}\ ) : (\ 3 - 2\sqrt{2}\ )$ Is that true? If it is true ,then show that.	3
17.	A GP consist of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places. Find the common ratio .	3
18.	What will be the sum of the series $7 + 77 + 777 + \cdots$ ... ... Of n^{th} term.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	nth term Tn = 12 + (12 + 22) + (12 + 22 + 32) + ……. + $n^2 = (n(n + 1)(2n + 1))/6 = (2n3 + 3n2 + n)/6$ $\therefore Tn = (2n^3 + 3n^2 + n)/6$ Sum to n terms, Sn = ΣTn $= 1/6[2\Sigma n^3 + 3\Sigma n^2 + \Sigma n]$ $\therefore Sn = (n(n + 1)2(n + 2))/12$	3
2.	Given, the number of bacteria was 30 and its doubles every hour. So, the series is in from of G.P. where First term of the series is 30 and common ratio is 2. Now, the third term of the G.P. represents the number of bacteria in 2 nd hour. a 3 =a r 2 =(30) (2) 2 =120 The number of bacteria at the end of 2 nd hour is 120. Now, the fifth term of the G.P. represents the number of bacteria in 4 th hour. a 5 =a r 4 =(30) (2) 4 =480 The number of bacteria at the end of 4 th hour is 480. Now, the n th term of the G.P. represent the number of bacteria in n th hour. a n+1 =a r n =(30) (2) n Thus, the number of bacteria at the end of 2 nd hour, 4 th hour and n th hour is 120, 480 and (30) (2) n respectively.	3
3.	The show ceases to be profitable on the night when the receipts are just Rs. 2000. Thus, by considering that it will happen at nth night. $\therefore T n = a + (n - 1) d$ $\therefore 2000 = 9500 + (n - 1) (- 250)$ $\Rightarrow (n - 1) = 38 - 8$ $\Rightarrow n = 31$ Hence, on 31st night, the show ceases to be profitable.	3
4.	n = 4	3
5.	$2.\frac{4^8 - 1}{4 - 1} = Rs.43690$	3
6.	$3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \ldots\ldots\ldots$	3
7.	The first term a = 1. Let common ratio is r. $a_3 + a_5 = ar^2 + ar^4 = 90$ $\Rightarrow r^2 + r^4 = 90$ (as a =1) $\Rightarrow r^4 + r^2 - 90 = 0$ which gives $r^2 = -10$ (not impossible) or $r^2 = 9$ $\Rightarrow r = \pm 3$	3
8.	i) 256 ii) 2046 iii) 8	3
9.	Since the value depreciates every year by 20%, so value remains 80%. Here, a = 1250, r = 80/100 = 4/5 and the series is a G.P. So, the value of machine at the end of 5 years will be $a_6 = ar^5 = 1250\ (4/5)^5$ = Rs. 409.60	3

10.	Sum of first n terms be S_n given as $S_n = 2n + 3n^2$ We have to find the r^{th} term that is a_r Using the given hint n^{th} term is given as $a_n = S_n - S_{n-1}$ $\Rightarrow a_r = S_r - S_{r-1}$ Using $S_n = 2n + 3n^2$ $\Rightarrow a_r = 2r + 3r^2 - (2(r-1) + 3(r-1)^2)$ $\Rightarrow a_r = 2r + 3r^2 - (2r - 2 + 3(r^2 - 2r + 1))$ $\Rightarrow a_r = 2r + 3r^2 - (2r - 2 + 3r^2 - 6r + 3) \Rightarrow a_r = 6r - 1$ Hence the r^{th} term is $6r - 1$	3
11.	Given first day he made 5 frames then two frames more than the previous that is 7 then 9 and so on Hence the sequence of making frames each day is 5, 7, 9… The sequence is AP with first term as a = 5 and common difference d = 2 Total number of frames to be made is 192 Let is requires n days hence $S_n = 192$ The sum of first n terms of AP is given by $S_n = n/2\ (2a + (n-1)\ d)$ Where a is the first term and d is common difference $S_n = (n/2)\ (2(5) + (n-1)\ 2)$ $192 = (n/2)\ (10 + 2n - 2)$ $\Rightarrow 384 = 10n + 2n^2 - 2n$ On computing and simplifying we get $\Rightarrow 2n^2 + 8n - 384 = 0$ $\Rightarrow n^2 + 4n - 192 = 0$ $\Rightarrow n^2 + 16n - 12n - 192 = 0$ $\Rightarrow n\ (n + 16) - 12\ (n + 16) = 0$ $\Rightarrow (n - 12)\ (n + 16) = 0$ $\Rightarrow n = 12$ and $n = -16$ But n represents number of days which cannot be negative hence n = 12 Hence number of days required to finish the job is 12 days.	3
12.	Total distance traveled when it hits the ground for the 1^{st} time = 12 Total distance traveled when it hits the ground for the 2^{nd} time = $12 + 2.\frac{12}{3}$ = (12 + 2x4) m distance traveled when it hits the ground for the 3^{rd} time = $(12 + 2x4 + 2x\frac{4}{3})$ So the Total distance traveled when it hits the ground for the 7^{th} time is $= 12 + 2x4 + 2x\frac{4}{3}$ + … up to 7^{th} terms $= 12 + 2\ (4 + \frac{4}{3}$ + … up to 6^{th} terms)	3

	$=12 + 2\left[\frac{4\{1-\left(\frac{1}{3}\right)^6\}}{1-\frac{1}{3}}\right]$ $= 12 + 12\left(\frac{3^6-1}{3^6}\right)$ $= \frac{24\times3^6-12}{3^6}$ m	
13.	General term $a_n = ar^{n-1}$ Now, $ar^3 = 10$ ..... (1). $ar^6 = 80$ ..... (2) $ar^{n-1} = 2560$ ..... (3) Dividing (2) by (1) $r^3 = 8$ $\Rightarrow r = 2$ Putting this value in (1), we get $a \times 8 = 10$ $\Rightarrow a = \frac{10}{8}$ Again substituting a and r in (3) $\frac{10}{8} \times 2^{n-1} = 2560$ $\Rightarrow 2^{n-1} = 256$ $\Rightarrow 2^{n-1} = 2^8$ $\Rightarrow n-1 = 8$ $\Rightarrow n = 9$	3
14.	Number of bacteria present in the culture, form a G.P. whose first term is 45 and the common ratio is 3 End of 2nd hour will be the 3rd term Bacteria present after 2nd hour = $ar^2 = 45 \times (3)^2 = 405$ ($T_n = ar^{n-1}$) End of 5th hour will be 6th term Bacteria present after 5th hour = $ar^5 = 45 \times (3)^5 = 10935$	3
15.	Since A.M. ≥ G.M. Therefore, $\frac{9^x + 9^{1-x}}{2} \geq \sqrt{9^x.9^{1-x}}$ $\Rightarrow 9^x + 9^{1-x} \geq 2.\sqrt{9^{x+1-x}}$ $\Rightarrow 9^x + 9^{1-x} \geq 2.3$ $\Rightarrow 9^x + 9^{1-x} \geq 6$	3
16.	Yes Let the two numbers are a and b	

	$\Rightarrow a+b=6\sqrt{ab}$ $\Rightarrow (a+b)^2=36ab$ $(a-b)^2=(a+b)^2-4ab$ $\Rightarrow (a-b)^2=36ab-4ab=32ab=$ $\Rightarrow (a-b)=4\sqrt{2ab}$ Solving these we have $a=(3+2\sqrt{2})\sqrt{ab}; b=(3-2\sqrt{2})\sqrt{ab}$ $a:b=(3+2\sqrt{2}):(3-2\sqrt{2})$	
17.	Let a be the first term and r be the common ratio, Let there be 2n terms in GP $a_1+a_2+.....a_{2n}=5(a_1+a_3+.....a_{2n-1})$ $\Rightarrow a+ar+.....+ar^{2n-1}=5(a+ar^2+ar^4+.....+ar^{2n-2})$ $\Rightarrow a\left(\frac{r^{2n}-1}{r-1}\right)=5\left(\frac{r^{2n}-1}{r^2-1}\right)$ $\Rightarrow r+1=5\Rightarrow r=4$	
18.	$7+77+777+.....$ $=7(1+11+111+.....)$ $=\frac{7}{9}(9+99+999+.....)$ $=\frac{7}{9}(10-1+100-1+1000-1+.....)$ $=\frac{7}{9}(10+100+1000+.....+(-1-1-1-1.....-1))$ $=\frac{7}{9}(10+10^2+10^3+.....10^n-n)$ $=\frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right]$	