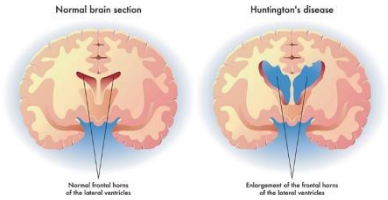
CHAPTER-15

STATISTICS

03 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	 The annual incidence rates of Huntington’s Disease per 100,000 individuals were recorded over a span of five years. The data is as follows: 4, 7, 8, 9, 10. Calculate the mean deviation about the mean for these rare disease rates.	3
2.	A group of athletes participated in a practice session for a particular exercise routine over the course of several days. The recorded practice times (in minutes) for each athlete were as follows: 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.  Calculate the mean deviation about the mean for the athletes' practice times.	3
3.	A group of cars went through a series of servicing sessions at a garage. The recorded mileage before each servicing session (in thousands of kilometers) for each car were as follows: 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.  Calculate the mean deviation about the median for the recorded mileages of the cars before their servicing sessions.	3
4.	The mean and standard deviation of 20 observation is found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in case of the wrong item is omitted.	3
5.	Find the mean and standard deviation of first n terms of an A.P. whose first term is a and the common difference is d.	3
6.	Calculate the mean deviation about median for the daily wages of 12 labours getting wages (in Rs.) 48, 45, 60, 50, 46, 48, 50, 45, 70, 65,47,50	3
7.	The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.	3
8.	Mean and standard deviation of 100 items are 50 and 4, respectively. Find the sum of all the item and the sum of the squares of the items.	3
9.	The mean and standard deviation of a group of 100 observations were found	3

to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

10. If a is a positive integer and the frequency distribution has a variance of 160 . Determine the value of a . — 3

x	a	$2a$	$3a$	$4a$	$5a$	$6a$
f	2	1	1	1	1	1

11. An analysis of monthly wages paid to workers in two firms A and B , belonging to the same industry, gives the following results : — 3

	Firm A	Firm B
No.of wages earners	586	648
Mean of monthly wages	Rs.5253	Rs.5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays out larger amount as monthly wages ?

(ii) which firm A or B is shows greater variability in individual wages ?

12. The mean and variance of 7 observations are 8 and 16 respectively.
If 5 of the observations are 2 , 4 , 10 , 12 , 14 . Find the remaining two observations. — 3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	To calculate the mean deviation about the mean, follow these steps: Calculate the mean: (4 + 7 + 8 + 9 + 10) / 5 = 7.6 Calculate the deviations from the mean for each value: Deviations: -3.6, -0.6, 0.4, 1.4, 2. Calculate the absolute values of the deviations: 3.6, 0.6, 0.4, 1.4, 2.4 Calculate the mean of the absolute deviations: (3.6 + 0.6 + 0.4 + 1.4 + 2.4) / 5 = 1.64 So, the mean deviation about the mean is approximately 1.64	3
2.	Let's calculate the mean deviation about the mean for the athletes' practice times: Calculate the mean practice time: (38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44) / 10 = 50. Calculate the deviations from the mean for each practice time: Deviations: -12, 20, -2, -10, -8, 5, 13, -4, 4, -6. Calculate the absolute values of the deviations: 12, 20, 2, 10, 8, 5, 13, 4, 4, 6. Calculate the mean of the absolute deviations: (12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6) / 10 = 8.4. The mean deviation about the mean for the athletes' practice times is indeed 8.4.	3
3.	To calculate the mean deviation about the median for the recorded mileages, follow these steps: 1. Arrange the mileages in ascending order: 36, 42, 45, 46, 46, 49, 51, 53, 60, 72. 2. Find the median: In this case, the median is the average of the fifth sixth values, which is (46 + 49) / 2 = 47.5. 3. Calculate the deviations from the median for each mileage: Deviations: -11.5, -5.5, -2.5, -1.5, -1.5, 1.5, 3.5, 5.5, 12.5, 24.5. 4. Calculate the absolute values of the deviations: 11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5. 5. Calculate the mean of the absolute deviations: (11.5 + 5.5 + 2.5 + 1.5 + 1.5 + 1.5 + 3.5 + 5.5 + 12.5 + 24.5) / 10 = 10.5. So, the mean deviation about the median for the recorded mileages of the cars is indeed 10.5.	3
4.	Here $\mathrm{n} = 20, \bar{x} = 10$ and $\sigma = 2$	3

	$\therefore \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$ $\Rightarrow \Sigma x_i = 20 \times 10 = 200$ Therefore Incorrect $\Sigma x_i = 200$ Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \sigma^2$ $\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$ If wrong item is omitted. When wrong item 8 is omitted from the data then we have 19 observations. Therefore Correct $\Sigma x_i =$ Incorrect $\Sigma x_i - 8$ Correct $\Sigma x_i = 200 - 8 = 192$ Therefore Correct mean $= \frac{192}{19} = 10.1$ Also correct $\Sigma x_i^2 =$ Incorrect $\Sigma x_i^2 - (8)^2$ $\Rightarrow$ Correct $\Sigma x_i^2 = 2080 - 64 = 2016$ Hence Correct variance $= \frac{1}{19}$(correct Σx_i^2) $-$ (correct mean $)^2$ $= \frac{1}{19} \times 2016 - \left(\frac{192}{19}\right)^2$ $= \frac{2016}{19} - \frac{36864}{361} = \frac{38304 - 36864}{361} = \frac{1440}{361}$ Correct S.D. $= \sqrt{\frac{1440}{361}} = \sqrt{3.99} = 1.997$	
5.	The terms of the A.P. are: $a, a+d, a+2d, a+3d, \ldots, a+(r-1)d, \ldots, a+(n-1)d$. Suppose $\bar{X}$ be the mean of these terms. $\bar{X} = \frac{1}{n}\{a + (a+d) + (a+2d) + \cdots + (a+(n-1)d) = \frac{1}{n}\left[\frac{n}{2}\{2a + (n-1)d\}\right]$ $= a + (n-1)\frac{d}{2}$ Suppose σ be the standard deviation of n terms of the A.P.	3

<table>
<tr>
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<td>$\sigma^2 = \frac{1}{n}\sum_{r=1}^{n} [\{a+(r-1)d\} - \bar{X}]^2 \left[\text{Using: } \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{X})^2\right]$
$\Rightarrow \sigma^2 = \frac{1}{n}\sum_{r=1}^{n}\left[\{a+(r-1)d\} - \left\{a+(n-1)\frac{d}{2}\right\}\right]^2$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left[\sum_{r=1}^{n}(2r-2-n+1)^2\right]$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left[\sum_{r=1}^{n}(2r-(n+1)\}^2\right]$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left[\sum_{r=1}^{n}\{4r^2-4(n+1)r+(n+1)^2\}\right]$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left[\left\{4\left(\sum_{r=1}^{n} r^2\right)-4(n+1)\left(\sum_{r=1}^{n} r\right)+\sum_{r=1}^{n}(n+1)^2\right]\right.$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left\{\frac{4n(n+1)(2n+1)}{6}-\frac{4(n+1)n(n+1)}{2}+n(n+1)^2\right\}$
$\Rightarrow \sigma^2 = \frac{d^2}{4n}\left\{\frac{2n(n+1)(2n+1)}{3}-n(n+1)^2\right\}$
$\Rightarrow \sigma^2 = \frac{d^2}{12n}n(n+1)\{2(2n+1)-3(n+1)\} = \frac{(n^2-1)d^2}{12}$
$\Rightarrow \sigma = d\sqrt{\frac{n^2-1}{12}}$</td>
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<td>6.</td>
<td>Arranging the wages in ascending order, we get
45,45,46,47,48,48,50,50,50,60,65,70
Here, $n = 12$, which is even.
$\therefore$ M = Median = Mean of $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)$ th observations $= \frac{6th+7th}{2}$
$\therefore M = \frac{48+50}{2} = 49$
Here, $n = 12, \sum\lvert x_i - M\rvert = 66$
$\therefore$ Mean deviation about median $= \frac{\sum\lvert x_i-M\rvert}{n} = \frac{66}{12} = 5.5$
Hence, the mean deviation about median is 5.5</td>
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<td>7.</td>
<td>We have,
$n_1=60$, $\bar{x}_1=650$, $s_1=8$
$n_2=80$, $\bar{x}_2=660$, $s_2=7$
$\sigma=\sqrt{\frac{n_1 s_1^2+n_2 s_2^2}{n_1+n_2}+\frac{n_1 n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}}$
$\sigma=\sqrt{\frac{60(64)+80(49)}{60+80}+\frac{60\times 80(650-660)^2}{(60+80)^2}}$
$\sigma=\sqrt{(3916/49)}$</td>
<td>3</td>
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$\sigma=\sqrt{79.9}$

$\sigma=8.9$

Hence, the overall standard deviation is 8.9

8. Here $\bar{x}=50$, n = 100 and $\sigma=4$ — 3

$\because \frac{1}{n}\sum x_i = \bar{x}$

$\Rightarrow \sum x_i = 50\times100$

$\Rightarrow \sum x_i = 5000$

$\because \sigma^2=\frac{1}{n}\sum x_i^2 - (\frac{1}{n}\sum x_i)^2$

$\Rightarrow 16=\frac{1}{100}\sum x_i^2 - (50)^2$

$\Rightarrow \sum x_i^2 = 251600$

Hence, sum of all items is 5000 and sum of squares of all items is 251600.

9. Here n = 100, $\bar{x}-20$ and $\sigma=3$ — 3

$\therefore \bar{x}=\frac{1}{n}\sum x_i$

$\Rightarrow \sum x_i = n \text{ x } \bar{x} = 100\times20=2000$

$\therefore$ Incorrect $\sum x_i = 2000$

Now $\frac{1}{n}\sum x_i^2 - (\bar{x})^2=9$

$\Rightarrow \sum x_i^2 = 40900$

When wrong items 21, 21 and 18 are omitted from the data then we have 97 observations.

Correct $\sum x_i$ = Incorrect $\sum x_i$ - 21 - 21 -18 = 1940

$\therefore$ Correct mean = 1940/97 = 20

Also

Correct $\sum x_i^2$ = Incorrect $\sum x_i^2 - (21)^2 - (21)^2 - (18)^2$

= 40900 - 441 -441 324 = 39694

$\therefore$ Correct variance = 197(correct$\sum x_i^2$ - (correct mean)^2

= $197\times39694-(20)^2$

= 409.22 - 400 = 9.22

Correct S.D. = $\sqrt{9.22}$ = 3.036

10. — 3

x	f	fx	fx^2
a	2	$2a$	$2a^2$
$2a$	1	$2a$	$4a^2$
$3a$	1	$3a$	$9a^2$
$4a$	1	$4a$	$16a^2$
$5a$	1	$5a$	$25a^2$
$6a$	1	$6a$	$36a^2$
	$\Sigma f = 7$	$\Sigma fx = 22a$	$\Sigma fx^2 = 92a^2$

Variance = $\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2$

$160 = \frac{92a^2}{7} - \left(\frac{22a}{7}\right)^2 \Rightarrow a = 7$

11. (i) Firm A : — 3

	Mean of monthly wages = $\frac{Total\ monthly\ wage}{Number\ of\ workers}$ $5253 = \frac{Total\ monthly\ wage}{586}$ Total monthly wages = 5253 × 586 = Rs.3078258 Firm B : Mean of monthly wages = $\frac{Total\ monthly\ wage}{Number\ of\ workers}$ $5253 = \frac{Total\ monthly\ wage}{648}$ Total monthly wages = 5253 × 648 = Rs.3403944 Clearly, firm B pays out larger amount as monthly wages. (ii) Since A and B have the same mean. Therefore, the firm with greater variance will have more variability. Thus, firm B has greater variability in individual wages.	
12.	Let x and y be remaining two observations. (2+4+10+12+14+x+y)/7 = 8 ⇒ x + y = 14 $\rightarrow (i)$ And $\frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + x^2 + y^2) - Mean^2 = 16$ $x^2 + y^2$=100 Now, $(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$ $x - y = \pm 2 \rightarrow (ii)$ Hence the remaining two observations are 6 and 8.	3