CHAPTER-11

THREE DIMENSIONAL GEOMETRY

03 MARKS TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Find the distance of a point (2,4,-1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$	3
2.	Find the shortest distance between the lines $\vec{r} = (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \gamma(\mathbf{i} - \mathbf{j} + \mathbf{k})$ and $\vec{r} = (2\mathbf{i} - \mathbf{j} - \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	3
3.	Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z axis respectively.	3
4.	Find the shortest distance between the following lines: $\vec{r} = (2\hat{i} + 4\hat{j} - 8\hat{k}) + \beta(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\vec{r} = (\hat{i} - 2\hat{j} - 4\hat{k}) + \alpha(4\hat{i} + 6\hat{j} + 12\hat{k})$	3
5.	Find the shortest distance between the following lines whose vector equation are given: $\vec{r} = (2\hat{i} + 4\hat{j} - 8\hat{k}) + \beta(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\vec{r} = (\hat{i} - 2\hat{j} - 4\hat{k}) + \alpha(\hat{i} + 2\hat{j} + 4\hat{k})$	3
6.	Find the angle between the pair of lines: $\vec{r} = (6\hat{i} + 4\hat{j} - 8\hat{k}) + \gamma(2\hat{i} + 4\hat{j} + 4\hat{k})$ $\vec{r} = (10\hat{i} - 4\hat{j}) + \delta(6\hat{\imath} + 4\hat{j} + 12\hat{k})$	3
7.	 Read the following text and answer the question on the basis of the same. A cycle race was organized in a town , where the maximum speed limit was set by the organizers . No participant are allowed to cross the specified speed limit, but two cycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ and $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 2k + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ Find the angle between two lines.	3
8.		3

	 An insect is crawling along the line $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{5z-10}{11}$ and another insect is crawling along the line $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ .Find the value of p so that the lines are perpendicular to each other.	
9.	 An insect is crawling along the line which passes through the point (-2,4,-5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ then find the cartesian equation of of the line.	3
10.	Find the distance of the point P $(-2, 4, -5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.	3
11.	Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1).	3
12.	Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.	3
13.	Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ from the point (1,2,3).	3
14.	Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P (1,3,3).	3
15.	Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{2-5}{3}$ are coplanar.	3

<u>**ANSWERS:**</u>

Q. NO	ANSWER	MARKS
1.	7 units	
2.	$\frac{3\sqrt{2}}{2}$	
3.	$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$	
4.	Since, the given lines are parallel as their direction ratios are proportional, so shortest distance between these lines is given by; $d = \left\lvert\frac{\vec{b}\ X\ (\vec{a_2}-\vec{a_1})}{\lvert\vec{b}\rvert}\right\rvert$, where $\vec{a_1} = (2\hat{\imath} + 4\hat{\jmath} - 8\hat{k})$ $\vec{a_2} = (\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$ $\vec{b} = (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ $d = \frac{\sqrt{2581}}{7}$	3
5.	Since, the given lines are not parallel as their direction ratios are not proportional, so shortest distance between these lines is given by; $d = \left\lvert\frac{(\vec{b_1}\ X\ \vec{b_2})\ .\ (\vec{a_2}-\vec{a_1})}{\lvert\vec{b_1}\ X\ \vec{b_2}\rvert}\right\rvert$, where $\vec{a_1} = (2\hat{\imath} + 4\hat{\jmath} - 8\hat{k})$ $\vec{a_2} = (\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$ $\vec{b_1} = (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ $\vec{b_2} = (\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$ $d = \frac{16}{\sqrt{5}}$	3
6.	The angle between the two given lines is given by: $\theta = \left\lvert\frac{\vec{b_1}.\vec{b_2}}{\lvert b_1\rvert\lvert b_2\rvert}\right\rvert$, where $\vec{b_1} = (2\hat{\imath} + 4\hat{\jmath} + 4\hat{k})$ $\vec{b_2} = (6\hat{\imath} + 4\hat{\jmath} + 12\hat{k})$ $\theta = \cos^{-1}\frac{19}{21}$	3
7.	$b_1 = (\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ and $b_2 = (2\hat{\imath} + \hat{\jmath} + \hat{k})$ $b_1 . b_2 = (\hat{\imath} + 2\hat{\jmath} - 2\hat{k}).(2\hat{\imath} + \hat{\jmath} + \hat{k})$ $= 2+2-2$ $= 2$ I b_1I = $\sqrt{1+4+4} = 3$ I b_2 I = $\sqrt{4+1+1} = \sqrt{6}$ the angle between two lines, cos $\theta = 2/3\sqrt{6}$ So, $\theta = \cos^{-1}(2/3\sqrt{6})$	3
8.	The given lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{5z-10}{11}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ OR, $\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-2}{11/5}$ and $\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$ Direction ratios of the lines are -3,2p/7, 11/5 and -3p/7 , 1 ,-5 As the lines are perpendicular So, -3 X -3p/7 + 2p/7 X 1 + 11/5 X (-5) = 0 9p/7 + 2p/7 -11 = 0 11p -77 = 0 11 p = 77 So, p = 7.	3
9.	The equation of given line is $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$	3

	Direction ratios of the line are 3,5 and 6 Now, the equation of the line passing through point (-2,4,5) and having direction ratios 3,5,6 is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$	
10.	Any general point on the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$………(1) is given by Q $(-3+3\lambda, 4+5\lambda, -8+6\lambda)$……….(2) If this point Q is to be foot of the perpendicular drawn to the line (1) from the point $P(-2,4,-5)$, then Direction ratios of line $\overrightarrow{PQ}$ are given by $(3\lambda-3+2, 5\lambda+4-4, 6\lambda-8+5) = (3\lambda-1, 5\lambda, 6\lambda-3)$ Now, as $\overrightarrow{PQ}$ is perpendicular to the line (1) hence, we have $3.(3\lambda-1)+5.(5\lambda)+6.(6\lambda-3)=0$ $\Rightarrow 70\lambda-21=0 \Rightarrow \lambda = \frac{21}{70} = \frac{3}{10}$ Hence, $\overrightarrow{PQ} = \left(-1+\frac{9}{10}\right)\hat{\imath} + \frac{15}{10}\hat{\jmath}\left(-3+\frac{18}{10}\right)\hat{k} = \frac{1}{10}\hat{\imath} + \frac{15}{10}\hat{\jmath} - \frac{12}{10}\hat{k}$ Therefore, $\lvert\overrightarrow{PQ}\rvert = \frac{1}{10}\sqrt{1+225+144} = \sqrt{\frac{37}{10}}$.	1 1 1
11.	Let Q be the foot of perpendicular drawn from the points A (1, 8, 4) to the line passing through B and C as shown in the Fig. 11.2. The equation of line BC by using the formula, $\vec{r} = \overrightarrow{a_1} + \lambda(\overrightarrow{a_2} - \overrightarrow{a_1})$ Here, $\overrightarrow{a_1} = -\hat{\jmath} + 3\hat{k}, \overrightarrow{a_2} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ So that equation of $\overrightarrow{BC} = -\hat{\jmath} + 3\hat{k} + \lambda(2\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$………(1) Any general point Q on line (1) is given by Q $(2\lambda, -1-2\lambda, 3-4\lambda)$…..(2) If this point Q is to be foot of the perpendicular drawn to the line (1) from the point $P(1,8,4)$, then Direction ratios of line $\overrightarrow{PQ}$ are given by $2\lambda-1, -1-2\lambda-8, 3-4\lambda-4) = (2\lambda-1, -2\lambda-9, -4\lambda-1)$ Now, as $\overrightarrow{PQ}$ is perpendicular to the line (1) hence, we have $2.(2\lambda-1)-2.(-2\lambda-9).-4.(-4\lambda-1)=0 \Longrightarrow 24\lambda+20=0 \Longrightarrow \lambda = \frac{-5}{6}$ The required point is obtained by putting value of λ in (2) which is Q $(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3})$	1 1 1
12.	Any general point on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$………(1) is given by Q $(\lambda, 1+2\lambda, 2+3\lambda)$……….(2) Let P (1, 6, 3) be the given point and let Q be the foot of perpendicular from point P to the line (1) Direction ratios of line $\overrightarrow{PQ}$ are given by $(\lambda-1, 1+2\lambda-6, 3\lambda+2-3) = (\lambda-1, 2\lambda-5, 3\lambda-1)$ Now, as $\overrightarrow{PQ}$ is perpendicular to the line (1) hence, we have $1.(\lambda-1)+2.(2\lambda-5)+3.(3\lambda-1)=0$ $\Rightarrow 14\lambda-14=0 \Rightarrow \lambda=1$ Hence, co – ordinates of point Q are : Q $(1,3,5)$ Now, if R (x,y,z) be image point of the point P (1, 6, 3) then, Q (1, 3, 5) will be mid – point of line – segment PR. So that, $\frac{x+1}{2}=1, \frac{y+6}{2}=3, \frac{z+3}{2}=5$ Hence, $x=1, y=0, z=7$.	1 1 1

	So that image point is : (1, 0, 7).	
13.	A $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$	3
14.	R(4,3,7) or R(-2,-1,3)	3
15.	-51 -141 +192=0	3