CHAPTER-10

VECTORS

03 MARKS TYPE QUESTIONS

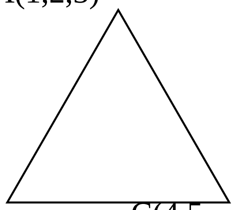
Q. NO	QUESTION	MARK
1.	The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.	**3**
2.	If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$,then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$	**3**
3.	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $\|\vec{a}\|=3$, $\|\vec{b}\|=5$, $\|\vec{c}\|=7$ find the angle between $\vec{a}$ and $\vec{b}$	**3**
4.	If $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-4\hat{k}$, then express $\vec{b}$ in the form $\vec{b}=\vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.	3
5.	If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.	3
6.	Using vector show that the points A(-2,3,5), B(7,0,-1), C(-3,-2,-5) and D(3,4,7) are such that AB and CD intersect at P(1,2,3).	3
7.	$\vec{a}=\hat{\imath}+2\hat{\jmath}-\hat{k}$, $\vec{b}=3\hat{\imath}+\hat{\jmath}-5\hat{k}$. Find a unit vector parallel to $\vec{a}-\vec{b}$	3
8.	Find the area of a parallelogram whose adjacent sides are given by vectors $\vec{a}=\hat{\imath}-\hat{\jmath}+3\hat{k}$, $\vec{b}=2\hat{\imath}-7\hat{\jmath}+\hat{k}$.	3
9.	$p\hat{\imath}-5\hat{\jmath}+6\hat{k}$ and $2\hat{\imath}-3\hat{\jmath}-q\hat{k}$ are collinear, find p, q	3
10.	Let the vectors $\vec{a},\vec{b},\vec{c}$ be given as $a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$, $b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$, $c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$. then show that $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$.	3
11.	If the position vectors of the vertices of a triangle are $\hat{\imath}+2\hat{\jmath}+3\hat{k}, 2\hat{\imath}+3\hat{\jmath}+\hat{k}$ and $3\hat{\imath}+\hat{\jmath}+2\hat{k}$, show that the triangle is an equilateral triangle.	3
12.	If vectors $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\vec{k},\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.	3
13.	If vectors vectors $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c}3\hat{\imath}+\hat{\jmath}=$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.	3
14.	For any vector $\vec{a}$,show that $\vec{a}=(\vec{a}\cdot\hat{\imath})\,\hat{\imath}+(\vec{a}\cdot\hat{\jmath})\,\hat{\jmath}+(\vec{a}\cdot\hat{k})\,\hat{k}$	3
15.	Using vectors find the area of triangle ABC with vertices A(1,2,3),B(2,-1,4) and C(4,5,-1).	3

<u>ANSWERS:</u>

Q. NO	ANSWER	MARKS
1.	**Ans:** $\vec{a} = 2\hat{i}+4\hat{j}-5\hat{k}$ $\vec{b} = \lambda\hat{i}+\hat{j}+3\hat{k}$ $\vec{a}+\vec{b} = (2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$ **Unit vector along** $\vec{a}+\vec{b} = \frac{\vec{a}+\vec{b}}{\left\lvert\vec{a}+\vec{b}\right\rvert}$ $= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}}$ $= \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+40}}$ $ATQ\ \vec{c}.(\vec{a}+\vec{b})=1$ $(\hat{i}+\hat{j}+\hat{k}).\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{(2+\lambda)^2+40}\right)=1$ $\frac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}}=1$ $2+\lambda+4=\sqrt{(2+\lambda)^2+40}$ *sq.both site* $\lambda^2+36+12\lambda=(2+\lambda)^2+40$ $\lambda=1$	**3**
2.	**Ans:** $\left\lvert\vec{a}\right\rvert=1, \left\lvert\vec{b}\right\rvert=1, \left\lvert\vec{c}\right\rvert=1,$ $\vec{a}+\vec{b}+\vec{c}=0 \quad (Given)$ $\vec{a}.(\vec{a}+\vec{b}+\vec{c})$ $\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$ $(\vec{a})^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$ $1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$ $\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1 ------- (i)$ *similiorly* $\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1 ------- (ii)$ *again* $\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1 ------- (iii)$ $adding (i),(ii) and (iii)$ $2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-3 \qquad [\vec{a}.\vec{b}=\vec{b}.\vec{a}$ $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$	**3**

3.	. **Ans:** $\vec{a}+\vec{b}+\vec{c}=0$ $\vec{a}+\vec{b}=-\vec{c}$ $(\vec{a}+\vec{b}).(-\vec{c})=-\vec{c}.(-\vec{c})$ $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=\vec{c}.\vec{c}$ $\lvert\vec{a}\rvert^2+2\vec{a}\vec{b}+\lvert\vec{b}\rvert^2=\lvert\vec{c}\rvert^2$ $\vec{a}.\vec{b}=\frac{49-9-25}{2}=\frac{15}{2}$ $\cos\theta=\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $=\frac{1}{2}$ $\theta=60$	**3**
4.	$\vec{b}_1$ is parallel to $\vec{a}$ $\Rightarrow \vec{b}_1$ = m$\vec{a}$ for some scalar m $\Rightarrow \vec{b}_1$ = m($3\hat{\imath}+4\hat{\jmath}+5\hat{k}$) = 3m$\hat{\imath}$ + 4m$\hat{\jmath}$ + 5m$\hat{k}$ …..(i) If $\vec{b}=\vec{b}_1+\vec{b}_2$ $\Rightarrow \vec{b}_2=\vec{b}-\vec{b}_1$ = (2-3m)$\hat{\imath}$ + (1-4m)$\hat{\jmath}$ + (-4-5m)$\hat{k}$ …..(ii) Also given that $\vec{b}_2$ is perpendicular to $\vec{a}$ $\Rightarrow \vec{b}_2.\vec{a}=0$ $\Rightarrow$ 3(2 - 3m) + 4(1 - 4m) + 5(- 4 - 5m) = 0 $\Rightarrow$ - 10 - 50m = 0 $\Rightarrow$ m = -1/5 Therefore, $\vec{b}_1=-\frac{1}{5}(3\hat{\imath}+4\hat{\jmath}+5\hat{k})$ And $\vec{b}_2=\frac{13}{5}\hat{\imath}+\frac{9}{5}\hat{\jmath}-3\hat{k}$ $\therefore \vec{b}=(-\frac{3}{5}\hat{\imath}-\frac{4}{5}\hat{\jmath}-\hat{k})+(\frac{13}{5}\hat{\imath}+\frac{9}{5}\hat{\jmath}-3\hat{k})=2\hat{\imath}+\hat{\jmath}-4\hat{k}$ is the required expression.	3
5.	Given $\hat{a}$ and $\hat{b}$ are unit vectors and $\hat{a}+\hat{b}$ is also a unit vector. $\Rightarrow \lvert\hat{a}\rvert=1, \lvert\hat{b}\rvert=1$ and $\lvert\hat{a}+\hat{b}\rvert=1$ We have $\lvert\hat{a}+\hat{b}\rvert^2=\lvert\hat{a}\rvert^2+\lvert\hat{b}\rvert^2+2\hat{a}.\hat{b}$ $\Rightarrow 1 = 1 + 1 + 2\hat{a}.\hat{b} \Rightarrow 2\hat{a}.\hat{b}=-1$ Also we have, $\lvert\hat{a}-\hat{b}\rvert^2=\lvert\hat{a}\rvert^2+\lvert\hat{b}\rvert^2-2\hat{a}.\hat{b} \Rightarrow \lvert\hat{a}-\hat{b}\rvert^2=1+1-(-1)=3$ $\Rightarrow \lvert\hat{a}-\hat{b}\rvert=\sqrt{3}$ i.e., the magnitude of the difference is $\sqrt{3}$	3
6.	To prove P intersects $\overrightarrow{AB}$ and $\overrightarrow{CD}$, we have to show that A,P,B are collinear and C,P,D are collinear $\overrightarrow{AP}$ = (1+2)$\hat{\imath}$ + (2-3) $\hat{\jmath}$ + (3-5) $\hat{k}$ = $3\hat{\imath}-\hat{\jmath}-2\hat{k}$ $\overrightarrow{PB}$ = (7-1)$\hat{\imath}$ + (0-2) $\hat{\jmath}$ + (-1-3) $\hat{k}$ = $6\hat{\imath}-2\hat{\jmath}-4\hat{k}$ $\Rightarrow \overrightarrow{PB}=2(3\hat{\imath}-\hat{\jmath}-2\hat{k})$ = 2AP $\Rightarrow$the vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are collinear.	3

	Since P is a common point to $\overrightarrow{AP}$ and $\overrightarrow{PB}$, the points A, P, B are collinear. Similarly, $\overrightarrow{CP}$ = (1+3) $\hat{\imath}$ + (2+2) $\hat{\jmath}$ + (3+5) $\hat{k}$ = $4\hat{\imath} + 4\hat{\jmath} + 8\hat{k}$ $\overrightarrow{PD}$ = (3-1) $\hat{\imath}$ + (4-2) $\hat{\jmath}$ + (7-3) $\hat{k}$ = $2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ $\Rightarrow \overrightarrow{CP} = 2(2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 2\overrightarrow{PD}$ $\Rightarrow$ the vectors $\overrightarrow{CP}$ and $\overrightarrow{PD}$ are collinear Since P is a common point to $\overrightarrow{CP}$ and $\overrightarrow{PD}$, the points C, P, D are collinear. i.e., P is a common point to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ and so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect at P.	
7.	$\vec{a} - \vec{b} = -2\hat{\imath} + \hat{\jmath} + 4\hat{k}$ $req.\ vector\ parallel\ to\ \ \vec{a} - \vec{b} = \frac{\vec{a}-\vec{b}}{\lvert \vec{a}-\vec{b} \rvert} = \frac{-2\hat{\imath}+\hat{\jmath}+4\hat{k}}{\sqrt{21}}$	3
8.	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$ $\lvert \vec{a} \times \vec{b} \rvert = 15\sqrt{2}$	3
9.	$p\hat{\imath} - 5\hat{\jmath} + 6\hat{k} = \alpha(2\hat{\imath} - 3\hat{\jmath} - q\hat{k})$ $\alpha = \frac{5}{3}$ on comparing $p = \frac{10}{3}, q = -\frac{18}{5}$	3
10.	For correct proof	3
11.	$\lvert\overrightarrow{AB}\rvert = \sqrt{6} = \lvert\overrightarrow{BC}\rvert = \lvert\overrightarrow{CA}\rvert$	3
12.	$\lambda = 8.$	3
13.	As $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$ $((2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}) \cdot (3\hat{\imath} + \hat{\jmath}) = 0$ $3(2-\lambda) + (2+2\lambda) = 0$ $\lambda = 8$	
14.	Let $\vec{a} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$ $\vec{a} \cdot \hat{\imath} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{\imath}$ = l $\vec{a} \cdot \hat{\jmath} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{\jmath}$ = m $\vec{a} \cdot \hat{k} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{k}$ = n RHS= $(\vec{a} \cdot \hat{\imath})\hat{\imath} + (\vec{a} \cdot \hat{\jmath})\hat{\jmath} + (\vec{a} \cdot \hat{k})\hat{k}$ $= l\hat{\imath} + m\hat{\jmath} + n\hat{k}$ $= \vec{a}$ =LHS	

15.	We know that Area of triangle = $\frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BA}\right|$ $\overrightarrow{BC}$=(4-2) $\hat{\imath}$ +(5+1) $\hat{\jmath}$ + (-1-4) $\hat{k}$ =2 $\hat{\imath}$ + 6$\hat{\jmath}$ -5$\hat{k}$ $\overrightarrow{BA} = -\hat{\imath}$ + 3$\hat{\jmath}$ - $\hat{k}$ A(1,2,3)  (2,-1,4) B C(4,5,-1) $\overrightarrow{BC} \times \overrightarrow{BA} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & -5 \\ -1 & 3 & -1 \end{bmatrix} = 9\hat{\imath} + 7\hat{\jmath} + 12\hat{k}$ $\left|\overrightarrow{BC} \times \overrightarrow{BA}\right| = \sqrt{81+49+144} = \sqrt{274}$ Area of triangle = $\frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BA}\right|$ $= \frac{1}{2}\sqrt{274}$ sq. units	