CHAPTER-4

DETERMINANTS

03 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Area of a triangle whose vertices are (x_1,y_1),(x_2, y_2) and (x_3, y_3) is given by the determinant $\Delta=\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ Since, area is a positive quantity, so we always take the absolute value of the determinant Δ. Also, the area of the triangle formed by three collinear points is zero. Based on the above information, answer the following questions (i) Find the area of the triangle whose vertices are (-2, 6), (3, -6) and (1, 5). (a) 30 sq. units (b) 35 sq. units (c) 40 sq. units (d) 15.5 sq. units ii. If the area of a triangle ABC, with vertices A (1, 3), B (O, 0) and C (k, 0) is 3 sq. units, then a value of k is (a) 2 (b) 3 (c) 4 (d) 5	3
2.	A Boy Monty brought 2 Bags, 1 Pen and 3 pencils and Paid 25 rupees, in same shop Nihar bought 3 bags, 2 pens & 1 Pencil and Paid 40 rupees and Pabitra brought 1 Bag, 3 pens & 2 Pencil and paid 30 rupees. Multiply by matrix method	3
3.	Using the property of determinants and without expanding, prove that: $$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$.	3
4.	If $A=\begin{bmatrix} 4 & 2 & 5 \\ 2 & 0 & 3 \\ -1 & 1 & 0 \end{bmatrix}$, then find the determinant of $3AA^{-1}$.	3
5.	Find the matrix X such that $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$	3
6.	Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & sin\propto \\ 0 & sin\propto & -cos\propto \end{bmatrix}$.	3
7.	Find the adjoint of the matrix A $=\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A(adj A) = $\|A\|$ I_3 .	3
8.	Let $A = \begin{bmatrix} 2 & 3 \\ 1 & ,2 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB. Hence solve the equation 2x + y = 4, 3x +2y = 1	3
9.	The monthly incomes of two brothers Sirish and Srijan are in the ratio 3:4 and the monthly expenditures are in the ratio 5:7. Each brother saves Rs. 15000 per month..	3

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	 Using matrix find their monthly income	
10.	On his birthday Rahul decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got Rs. 10 more. However if there were16 children more, everyone would have got Rs. 10 less.  Using matrix method the number of children and amount ditrbuted by Rahul.	3
11.	Show that the points (a + 5, a – 4), (a – 2, a + 3) and (a, a) do not lie on a straight line for any value of a.	3
12.	A school wants to awards its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6000. Three times the award money for Hard work added to that given for Honesty amounts to Rs. 11000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and justify can we find the award money for each value, using matrix method?	3
13.	Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs. x each, Rs. y each and Rs. z each for three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 2200. School B wants to spend Rs. 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total 53amount for one prize on each value is Rs. 1200, using matrices, find the award money for each value.	3
14.	If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3× 3 matrix A, then find the sum of all values of α for which $\lvert A\rvert$+1 = 0.	3
15.	Let $(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Show that $[F(\alpha)]^{-1} = F(-\alpha)$.	3
16.	Gautam buys 4 pens, 3 bags and 2 instrument box and pays a sum of Rs.60. From the same shop, Vikram buys 2 pens, 4 bag and 6 instrument boxes and pays a sum of Rs.90. Also, Ankur buys 6 pen, 2 bags and 3 instrument boxes and pays a sum of Rs. 90.	3

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	Based on the above information, answer the following questions. (i) Convert the given above situation into a matrix equation of the form AX = B. (ii) Find$\|A\|$. (iii) FindA^{-1}.	
17.	Solve using matrix method 2x-y = 1, 3x+2y=5	3
18.	If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that A^2-5A+7I = O . Hence Find A^{-1}.	3
19.	Using cofactors of element of third columns evaluate $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$	3

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ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>According to statement
3p+2q+r=3000
2p+4q+3r=3500
P+q+r=1500
Converting the system of equations in matrix form , we get
$\begin{bmatrix}3 & 2 & 1\\2 & 4 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}3000\\3500\\1500\end{bmatrix}$
i.e AX=B
Where A=$\begin{bmatrix}3 & 2 & 1\\2 & 4 & 3\\1 & 1 & 1\end{bmatrix}$ X=$\begin{bmatrix}X\\Y\\Z\end{bmatrix}$
B=$\begin{bmatrix}3000\\3500\\1500\end{bmatrix}$
|A|= $\begin{bmatrix}3 & 2 & 1\\2 & 4 & 3\\1 & 1 & 1\end{bmatrix}$
=3(4-3)-2(2-1)+1(6-4)
=3×1-2×1+1×2=3$\cancel{-2}$+$\cancel{2}$=3
3≠0
X =A^{-1}B $A^{-1}=\frac{\text{adjA}}{|A|}$
adjA= [cofactors of A]^T
cofactors of A =$\begin{bmatrix}1 & 1 & -2\\-1 & 2 & -1\\2 & -7 & 8\end{bmatrix}$
adjA=$\begin{bmatrix}1 & -1 & 2\\1 & 2 & -7\\-2 & -1 & 8\end{bmatrix}$
$A^{-1}=\frac{\text{adjA}}{|A|}=\frac{\begin{bmatrix}1 & -1 & 2\\1 & 2 & -7\\-2 & -1 & 8\end{bmatrix}}{3}=\frac{1}{3}\begin{bmatrix}1 & -1 & 2\\1 & 2 & -7\\-2 & -1 & 8\end{bmatrix}$
X=A^{-1}B
$=\begin{bmatrix}\frac{1}{3} & \frac{-1}{3} & \frac{2}{3}\\\frac{1}{3} & \frac{2}{3} & \frac{-7}{3}\\\frac{-2}{3} & \frac{-1}{3} & \frac{8}{3}\end{bmatrix}\begin{bmatrix}3000\\2300\\1500\end{bmatrix}$
$=\begin{bmatrix}1000-1100+1000\\1000+2200-3500\\2000-1100+4000\end{bmatrix}=\begin{bmatrix}900\\-300\\900\end{bmatrix}$
p=900, q=-300, z=900</td><td>3</td></tr>
<tr><td>2.</td><td>Let the cost of 1 bag =x

And the cost of 1 pen =y

⇒3x+4y=257

⇒4x+3y=324

Equation (1) × 4: 12x+16y=257×4</td><td>3</td></tr>
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	Equation (2) × 3: 12x+9y=324×3 Subtract two equations; ⇒7y=56 ⇒y=8 ⇒x=75 ⇒total cost of 1 bag and 10 pens=x+10y=75+80=155	
3.	$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$ Applying the Sum Property of determinants, we have $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} =$ $\begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix}$ We know, if two rows or columns of a determinant are identical, then the value of the determinant is zero. Since, the two columns in both the determinants are identical, thus its determinant would be zero. $\Rightarrow \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} =0+0$ $\Rightarrow \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} =0$	3
4.	As $AA^{-1} = I \Rightarrow \left\|3AA^{-1}\right\| = \|3I\| = 9\|I\| = 9$	3
5.	$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ $X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} = \frac{1}{15-14} adj. \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} . \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ $= \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$	3
6.	$Let\ \|A\| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & sin\propto \\ 0 & sin\propto & -cos\propto \end{bmatrix}$ $= 1(-\cos^2\propto - \sin^2\propto) = -(\cos^2\propto + \sin^2\propto) = -1$ ⇨ A^{-1} exist. $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & sin\propto \\ 0 & sin\propto & -cos\propto \end{bmatrix}$	3

7.	Adj A= $\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$ Determinant A = 27 For correct proof	2 1
8.	$AB = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = 2I, \Rightarrow A\left(\frac{1}{2}B\right) = I \Rightarrow A^{-1} = \frac{1}{2}B = \frac{1}{2}\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix} = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$ Given system of equations is PX=Q, where $P = \begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix} = A^T$; $X = \begin{bmatrix}x\\ y\end{bmatrix}$; $Q = \begin{bmatrix}4\\ 1\end{bmatrix}$ $\therefore X = P^{-1}Q = (A^T)^{-1}Q = (A^{-1})^{T}Q = \begin{bmatrix}7\\ -10\end{bmatrix}$ $\therefore x = 7, y = -10$	3
9.	Let monthly income of Sirish and Srijan be 3x and 4x and their expenditure are 5y and 7y respectively $\therefore$ 3x - 5y = 15000, 4x - 7y = 15000 AX=B, where A = $\begin{bmatrix}3 & -5\\ 4 & -7\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\end{bmatrix}$, $B = \begin{bmatrix}15000\\ 15000\end{bmatrix}$ $A^{-1} = \begin{bmatrix}7 & -5\\ 4 & -3\end{bmatrix}$, $\therefore\ X = A^{-1}B = \begin{bmatrix}7 & -5\\ 4 & -3\end{bmatrix}\begin{bmatrix}15000\\ 15000\end{bmatrix} = \begin{bmatrix}30000\\ 15000\end{bmatrix}$, $\therefore$ income of Sirish = Rs. 90000, income of Srijan = Rs.120000	3
10.	Let number of children be x and amount for each student be Rs. Y So, $(x-8)(y+10) = xy \Rightarrow 5x - 4y = 40$ $(x+16)(y-10) = xy \Rightarrow 5x - 8y = -80$ AX=B, where A = $\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\end{bmatrix}$, $B = \begin{bmatrix}40\\ -80\end{bmatrix}$ $A^{-1} = \frac{-1}{20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}$, $\therefore\ X = A^{-1}B = \begin{bmatrix}32\\ 30\end{bmatrix}$ No. of students = 32, Amount given to each students = Rs 30	3
11.	Area of the triangle with the points (a + 5, a – 4), (a – 2, a + 3) and (a, a) as vertices is ½ $\begin{vmatrix}a+5 & a-4 & 1\\ a-2 & a+3 & 1\\ a & a & 1\end{vmatrix}$ = ½ [3a+15+2a-8-5a]=7/2 , non-zero value independent of a So points (a + 5, a – 4), (a – 2, a + 3) and (a, a) are not collinear.	 1 1 1
12.	Let x, y and z be the award money for Honesty, Regularity and Hard work. Then X+y+z=6000 3z+x=11000 z+x-2y=0 The equations can be presented as AX=B where A= $\begin{pmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{pmatrix}$,X= $\begin{pmatrix}x\\ y\\ z\end{pmatrix}$,B= $\begin{pmatrix}6000\\ 11000\\ 0\end{pmatrix}$ Det A=6 so the above equations have solutions.	3
13.	3x+2y+z=2200 ; 4x+y+3z=3100 ; x+y+z= 1200 A= $\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$; B= $\begin{bmatrix}2200\\ 3100\\ 1200\end{bmatrix}$; X= $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ A^{-1}= $1/5. \begin{bmatrix}2 & 1 & -5\\ 1 & -2 & 5\\ -3 & 1 & 5\end{bmatrix}$	½ ½ 1+½

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	So x=300,y=400,z=500	½
14.	Here, $\|B\| = \|A^{-1}\| = -1$ $or, \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = -1$ Or, $2\alpha^2 - 2\alpha - 24 = 0$ Sum of all values of $\alpha = 1$	3
15.	We have, $[F(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(-\alpha).$	3
16.	Let the cost of 1 pen, 1 bag and 1 instrument box be x, y and z respectively. Then, $4x + 3y + 2z = 60$ $2x + 4y + 6z = 90$ $6x + 2y + 3z = 70$ The above equations can be written as, $AX = B$ Where $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$ Now, $\|A\| = 50 \neq 0$ So, $A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$,	3
17.	x=1 y=1	3
18.	For verification Finding $A^{-1} = 1/7 \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	2 1
19.	Value =(x-y)(y-z)(z-x)	3

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