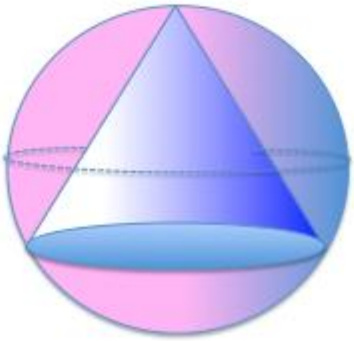
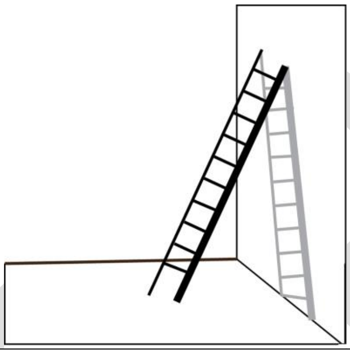
CHAPTER-6

APPLICATION OF DERIVATIVES

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Prove that the volume of the largest cone that can be inscribed in a sphere of radius ‘a’  is $\frac{8}{27}$ of the volume of the sphere.	4
2.	Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is (a) strictly increasing (b) strictly decreasing	4
3.	Find the equation of the tangent line to the curve $y=x^3-4x$ at the point where $x=2$.	4
4.	A ladder 12 meters long is leaning against a wall. The bottom of the ladder is sliding away from the wall at a rate of 2 m/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 meters from the wall? 	4
5.	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.  (i) What is the rate of growth of plant? (1 mark) (ii) On which day the plant attain the maximum height. (1 mark) (iii) What is the maximum height of the plant? (2 mark) **OR** What is the height of the plant after two days? (2 mark)	4
6.	A real estate company is going to build a new apartment complex. The land	4

they have purchased can hold at most 5000 apartments. Also, if they make x apartments, then the maintenance costs for the building, landscaping etc., would
be as follows:

Fixed cost = Rs 40,00,000

Variable cost =Rs $(140x – 0.04x^2)$

If C(x) denote the maintenance cost function, then $C(x) = 40,00,000 + 140x – 0.04x^2$

(i) Find the intervals in which the function C(x) is strictly increasing/strictly decreasing.

(ii) Find the points of local maximum/local minimum, if any, in the interval (0, 5000) as
well as the points of absolute maximum/absolute minimum in the interval [0, 5000].
Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

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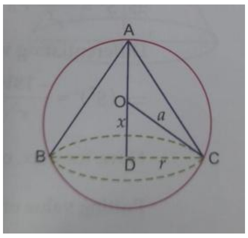
ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let OD = x and DC = r and height = h h= AD = AO +OD = a+x……………………..(1)  in Δ ODC $a^2 = r^2+x^2$…………………………………(2) volume of cone $V = \frac{1}{3}\pi r^2 h$, $V(x) = \frac{1}{3}\pi(a^2-x^2)(a+x)$ $V'(x) = \frac{1}{3}\pi(a-3x)(a+x)$ $V''(x) = \frac{1}{3}\pi[(a+x)(-3) +(a-3x)(1)]$ For maximum/minimum value $V'(x) = 0$ X = -a or $x = \frac{a}{3}$ Neglecting x = -a So, volume is maximum when $x = \frac{a}{3}$. Putting $x = \frac{a}{3}$ in equ^n(1) and (2) $h = \frac{4a}{3}$ and $r^2 = \frac{8a^2}{9}$ Volume of cone = $\frac{8}{27}(\frac{4}{3}\pi a^3)$ Volume of cone = $\frac{8}{27}$(volume of sphere).	4
2.	$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ $f'(x) = 6x^3-12x^2-90x = 6x(x+3)(x-5)$ critical pts x = 0/-3/5 (a)f(x) is strictly increasing in $(-3,0) \cup (5,\infty)$ (b)f(x) is strictly decreasing in $(-\infty,-3) \cup (0,5)$.	4
3.	The slope of the tangent line is the derivative of the function evaluated at the given point. Given the function $y=x^3-4x$, its derivative is $\frac{dy}{dx}=3x^2-4$. Evaluate the derivative at $x=2$ so $\frac{dy}{dx}=3(2)^2-4=8$. The point of tangency is $(2,y(2))=(2,4)$. The equation of a line with slope m passing through point (x_1,y_1) is given by: $y-y_1= m(x-x_1)$. Substitute $m=8$ and $(x_1,y_1) =(2,4)$: $y-4=8(x-2)$ Simplify and write in slope-intercept form: $y=8x-12$ So, the equation of the tangent line is $y=8x-12$.	4
4.	Let x be the distance between the bottom of the ladder and the wall, and let y be the height of the ladder on the wall. Given $\frac{dx}{dt}$=2 m/s and x=5 m, we want to find $\frac{dy}{dt}$ By the Pythagorean theorem, $x^2+y^2=122$, so $y^2= \sqrt{144-x^2}$ Differentiate both sides with respect to time t: $2y\frac{dy}{dt}=-2x\frac{dx}{dt}$	4

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	Solve for $\frac{dy}{dt}$: $\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}$ Substitute x=5 and y= $\sqrt{144-x^2}$ $\frac{dy}{dt}=\frac{-5}{\sqrt{144-5^2}}\cdot 2$ $\frac{dy}{dt}=-\frac{10}{\sqrt{119}}$ m/s. So, when the bottom of the ladder is 55 meters from the wall, the top of the ladder is sliding down the wall at a rate of $-\frac{10}{\sqrt{119}}$m/s.	
5.	(i) As the height of the plant is given by $y=4x-\frac{1}{2}x^2$ The rate of growth of plant is given by differentiating the above equation w.r.t x $y'(x)=4-x$ (ii) Maximum value of the given equation is $y'(x)=0 \Rightarrow x=4$ So, the maximum height of the plant is on day x=4, (iii) $y(4)=4\times 4-\frac{1}{2}4^2=8$ So, maximum height of plant is 8 cm. OR The height of the plant after x=2 days is $\mathbf{y(2)=4\times 2-\frac{1}{2}2^2=6}$ So, after two days height of the plant is 6 cm.	4
6.	**(i)** We have C(x) = 40,00,000 + 140x – $0.04x^2$ ∴ C’(x) = 140 – 0.08x For finding critical points,put C’(x) = 0 Then 140 – 0.08x = 0 ⇒x = 1750 Clearly, from the problem statement we can see that we only want critical points that are in the interval [0, 5000] ∴ Intervals are (0, 1750), (1750,5000) In Interval (0, 1750), C’(x) = Positive C(x) is strictly increasing in [0, 1750] In Interval (1750,5000), C’(x) = Negative C(x) is strictly decreasing in [1750, 5000]. **(ii)**We have C(x) = 40,00,000 + 140x – $0.04x^2$ ∴C’(x) = 140 – 0.08x C’’(x) = - 0.08 $C''(x)\|_{x=1750}$= -0.08 < 0 ∴ C(x) is Maximum at x = 1750 and Local Maximum Value = C (1750)	4

	= 4122500 Clearly, from the problem statement we can see that we only want critical points that are in the interval [0, 5000] Now we have C (0) = 40, 00000 C (1750) = 4122500 C (5000) = 3700000 ∴ Absolute Maximum value of C(x)=4122500	

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