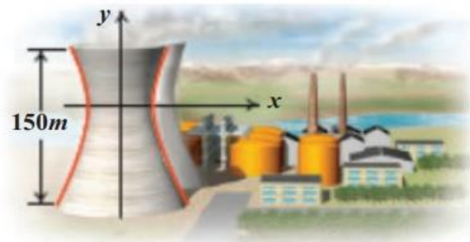
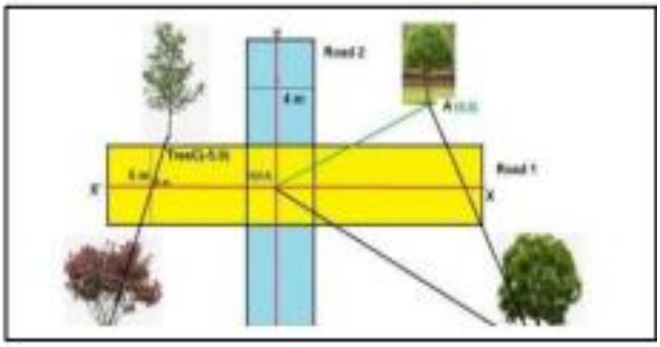
CHAPTER-11

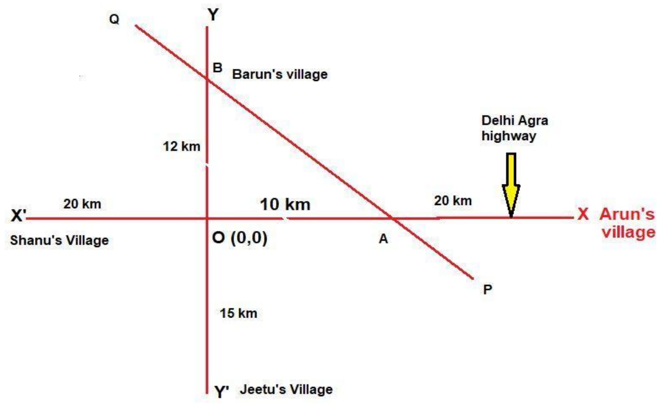
CONIC SECTIONS

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	A search light has a parabolic reflector (has a cross section that forms a ‘bowl’). The parabolic bowl is 40 cm wide from rim to rim and 30 cm deep. The bulb is located at the focus . (i) What is the equation of the parabola used for reflector? (ii) How far from the vertex is the bulb to be placed so that the maximum distance covered?	4
2.	Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. 	4
3.	In a park Road 1 and road 2 of width 5 m and 4 m are crossing at center point O(0, 0) as  shown in the figure . For trees A, B, C and D are situated in four quadrants of the Cartesian system of coordinate. The coordinates of the trees A, B, C and D are (6, 8), (12, 5), (-5, 0) and (-3, -4) respectively. Based on the above information answer the following questions: I) What is the distance of Tree C from the Origin? a. 5 m b. 10 m c. 15 m d. 25 m ii. What is the equation of line AB? a. 2x + y = 22 b. x - 2y = -6 c. x + 2y - 22 = 0 d. x + 2y = 6 iii. What is the slope of line CD? a. 2/1 b. -2 c. -1/2 d. 3/2 iv. What is the distance of point B from the origin? a. 13 m b. 15 m c. 12 m d. 5 m	4

4.

Villages of Shanu and Arun's are 50 km apart and are situated on Delhi Agra highway as shown in the following picture. Another highway YY' crosses Agra Delhi highway at O(0,0). A small local road PQ crosses both the highways at pints A and B such that OA=10 km and OB =12 km. Also, the villages of Barun and Jeetu are on the smaller high way YY'. Barun's village B is 12 km from O and that of Jeetu is 15 km from O.



Based on the above information answer the following questions:

i.What are the coordinates of A?

a. (10, 0) b. (10, 12) c. (0,10) d. (0,15)

ii. What is the equation of line AB?

a. 5x + 6y = 60 b. 6x + 5y = 60 c. x = 10 d. y = 12

iii. What is the distance of AB from O(0, 0)?

a. 60 km b. 60/√61 km c. √61 km d. 60 km

iv. What is the slope of line AB?

a) 6 /5 . b. 5 /6 c. −6 /5 d. 10/ 12

4

5.

The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m.

Based on above information answer the following questions

i)equation of parabola is

A)$6x^2=625y$ B) $4x^2=625y$ C) $6x^2=125y$ D) none of these

ii)Focus of the parabola is

A)$\frac{625}{6}$ B) $\frac{625}{24}$ c) $\frac{125}{24}$ D) none of these

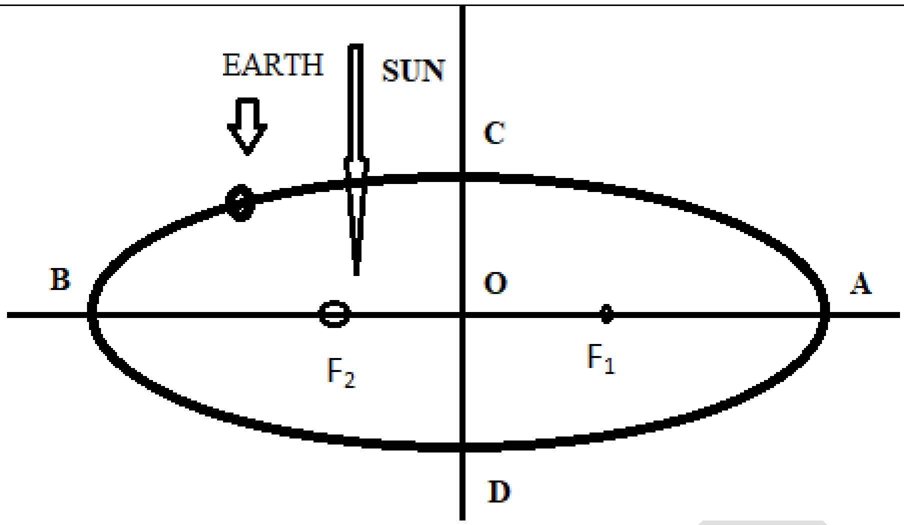
iii) length of latusrectum of the parabola is

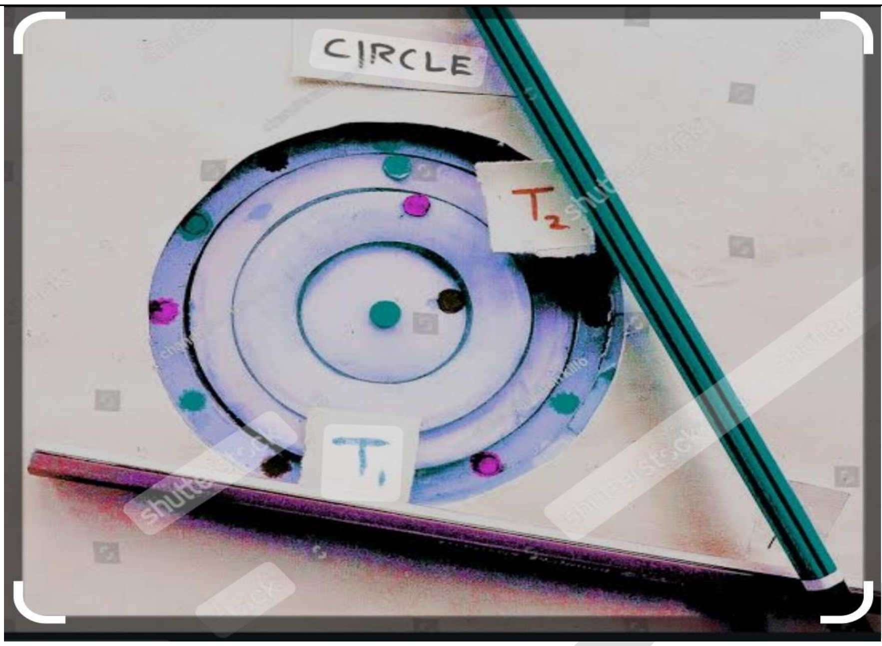
A)$\frac{625}{6}$ B) $\frac{625}{24}$ c) $\frac{125}{24}$ D) none of these

iv) length of the supporting wire attached to the the roadway 18m from the middle is

A)7.11 B)8.11 C)9.11 D)none of these

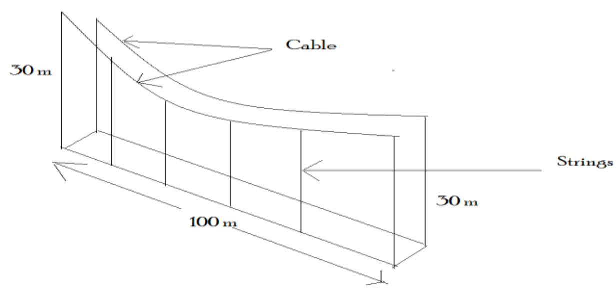
4

6.	 Earth moves around sun in an elliptical path as shown in above fig, Sun at focus F_2. Suppose coordinate of two points on elliptical path is (4,3) and (6,2). Based on above information answer the following questions i) equation of ellipse is A) $x^2/52 + y^2/13=1$ B) $x^2/13 + y^2/52=1$ C) $x^2/12 + y^2/13=1$ D) None of these ii) coordinate of sun is $A)(-\sqrt{13}, 0)$ $B)(-\sqrt{52}, 0)$ $C)(-\sqrt{39}, 0)$ D) None of these iii) length of latusrectum is A) $338/\sqrt{52}$ B) $339/\sqrt{52}$ C) $440/\sqrt{52}$ D) none of these iv) length of major axis is A) $3\sqrt{52}$ B) $2\sqrt{52}$ C) $4\sqrt{52}$ D) None of these	4
7.	A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated the centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. I) Construct the figure as per given dimensions. ii) Which kind of parabola is forming in this case? iii) Length of latus rectum? iv) How far from the centre is the deflection 1 cm?	4
8.	A line is a tangent to a circle if the length of perpendicular drawn from centre of the circle to the is equal to the radius of the circle. As in below figure two tangents are there.	

<table>
<tr><td></td><td>

Based on the above information, answer the following questions:

i)Find the equation of circle with centre C(- 2.3) and which touches the line x - y + 7 = 0.
ii) If the line y = √3 x + k touches the circle $x^2 + y^2 = 16$ then find the value of x - 2y + 3 = 0
iii) Find the equations of tangents to the circle x ^ 2 + y ^ 2 = 5 which are parallel to the line x-2y+3=0
iv) Find the equations of tangents to the circle x ^ 2 + y ^ 2 - 6x + 4u - 12 = 0 which are perpendicular to the line 5x-12y+1=0.</td><td></td></tr>
<tr><td>9.</td><td>The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m.



(a) Find the value of ‘a’ in the standard equation.
(b) Find the length of a supporting wire attached to the roadway 18 m from the middle.</td><td>4</td></tr>
<tr><td>10.</td><td>Due to heavy storm an electric wire got bent as shown in figure. It followed a mathematical</td><td>4</td></tr>
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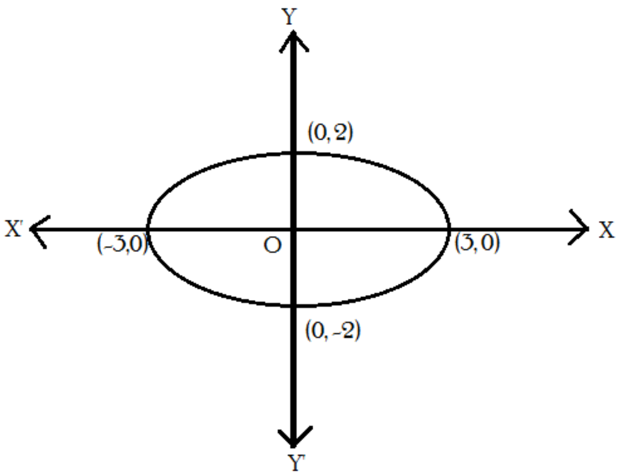
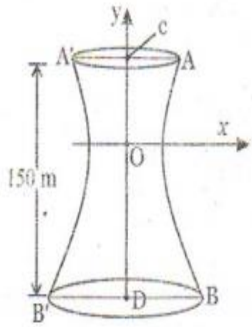
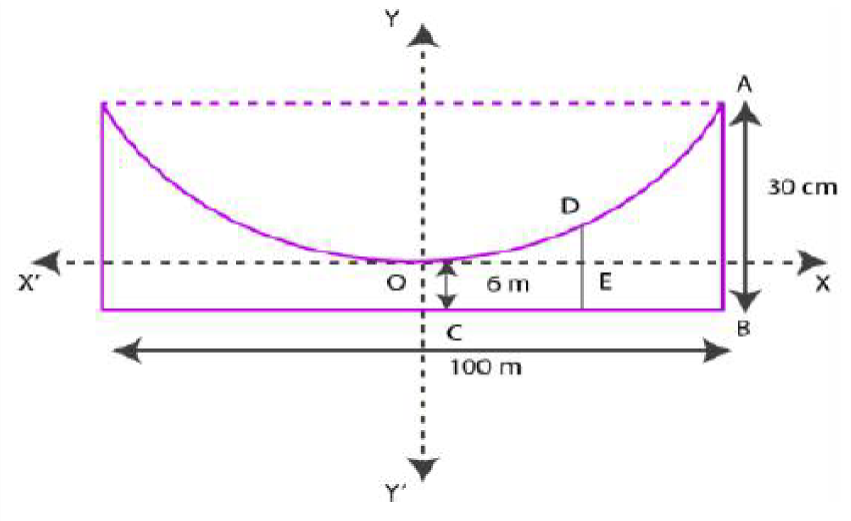
	shape.  (a) Write the equation of the shape of the curve. (b) Find the length of the latus rectum of the shape.	
11.	Determine the equation of the hyperbola which satisfies the given conditions: Foci $(0, \pm 13)$, the conjugate axis is of length 24.	4
12.	Determine the foci coordinates, the vertices, the length of the major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $(x^2/49) + (y^2/36) = 1$	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let the vertex be (0,0) The equation of the parabola is $y^2 = 4ax$ Since the diameter is 40 cm and the depth is 30 cm , the point (30,20) lies on the parabola $20^2 = 4 \times a \times 30$ 400=120a a=$\frac{400}{120} = \frac{10}{3}$ so, equation of the parabola is $y^2 = \frac{40}{3}x$ (ii)The bulb is at focus (0, a) . Hence the bulb is at a distance of 10/3 cm from the vertex.	1+1+1+1

2.	**SOLUTION**  The equation of the hyperbola is $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ -----(1) Height of the tower DC = 150 m Let the distance of the top of the tower from the centre of the hyperbola be OC = a ∴ The distance of the bottom of the tower from the centre of the hyperbola is OD = 150 − a Given the distance of the top of the tower from the centre of the hyperbola = half the distance of the bottom of the tower from the centre of the hyperbola. $a = \frac{150 - a}{2} \Rightarrow 2a = 150 - a$ $\Rightarrow 3a = 150 \Rightarrow a = \frac{150}{3} = 50$ **To find the diameter of the top of the tower. That is to find AA′.** From the figure the coordinates of A are A(x, 50). But A(x, 50) is a point on the hyperbola $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ $\therefore \frac{x^2}{30^2} - \frac{50^2}{44^2} = 1 \Rightarrow \frac{x^2}{30^2} = 1 + \frac{50^2}{44^2}$ $x^2 = 30^2\left(\frac{44^2 + 50^2}{44^2}\right) \Rightarrow x = \frac{30}{44} \times \sqrt{44^2 + 50^2}$ $\Rightarrow x = \frac{15}{22} \times \sqrt{1936 + 2500} = \frac{15}{22} \times \sqrt{4436} = \frac{15}{22} \times 66.60 = 45.41$ ∴ Diameter of the top of the tower = 45.41 m **Next to find the diameter of the bottom of the tower BB′.** The coordinates of B are (x, −100). But B is a point on the hyperbola $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ $\therefore \frac{x^2}{30^2} - \frac{100^2}{44^2} = 1 \Rightarrow \frac{x^2}{30^2} = 1 + \frac{100^2}{44^2}$ $\Rightarrow x^2 = 30^2\left(\frac{44^2 + 100^2}{44^2}\right) = \frac{30^2}{44^2}(1936 + 10000) \Rightarrow x = \frac{30}{44} \times \sqrt{11936}$ $\Rightarrow x = \frac{15}{22} \times \sqrt{11936} \Rightarrow x = \frac{15}{22} \times 109.25 \Rightarrow x = 74.49$ m. ∴ The diameter of the bottom of the tower = 74.49 m Hence diameter of the top of the tower = 45.41 m Diameter of the bottom of the tower = 74.49 m	1+1+1+1
3.	i) a ii)c iii)b iv) a	4
4.	1. (a) (10, 0) 2. (b) 6x + 5y = 60	4

	3. (b) 60/√ 61 km 4. (c) -6/5	
5.	Diagrammatic representation is as follows:  Here, AB and OC are the longest and the shortest wires, respectively, attached to the cable. DF is the supporting wire attached to the roadways, 18m from the middle. So, AB = 30m, OC = 6m, and BC = 50m. The equation of the parabola is of the from $x^2 = 4ay$ (as it is opening upwards).	4

The coordinates of point A are (50, 30 -6) = (50, 24)

Since A(50, 24) is a point on the parabola.

$y^2 = 4ax$

$(50)^2 = 4a(24)$

$a = (50×50)/(4×24)$

$= 625/24$

Equation of the parabola, $x^2 = 4ay = 4×(625/24)y$ or $6x^2 = 625y$

The x coordinate of point D is 18.

Hence, at x = 18,

$6(18)^2 = 625y$

$y = (6×18×18)/625$

= 3.11(approx.)

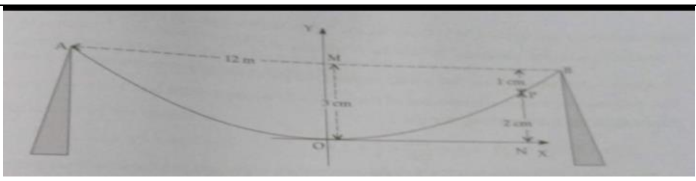
Thus, DE = 3.11 m

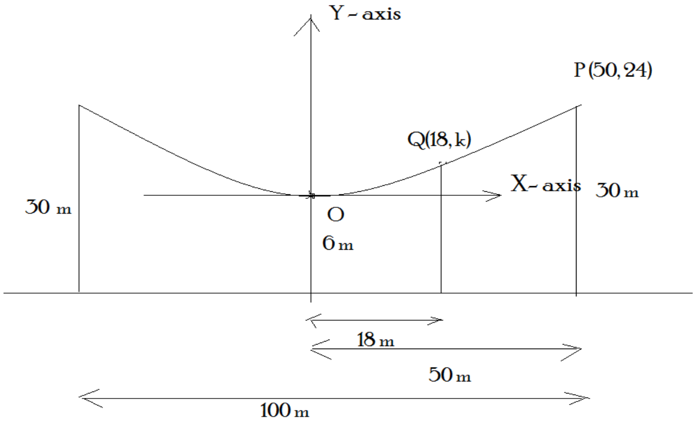
DF = DE +EF = 3.11m +6m = 9.11m

i)A
ii)B
iii)A
iv)C

6.	Major axis on the x-axis and passes through the points (4, 3) and (6, 2). Since the major axis is on the x-axis, the equation of the ellipse will be the form $x^2/a^2 + y^2/b^2 = 1$.... (1) [Where 'a' is the semi-major axis.] The ellipse passes through points (4, 3) and (6, 2). So by putting the values x = 4 and y = 3 in equation (1), we get, $16/a^2 + 9/b^2 = 1$ .... (2) Putting, x = 6 and y = 2 in equation (1), we get, $36/a^2 + 4/b^2 = 1$ .... (3) From equation (2) $16/a^2 = 1 - 9/b^2$ $1/a^2 = (1/16\ (1 - 9/b^2))$ .... (4) Substituting the value of $1/a^2$ in equation (3) we get, $36/a^2 + 4/b^2 = 1$ $36(1/a^2) + 4/b^2 = 1$	4

	$36[1/16 (1 - 9/b^2)] + 4/b^2 = 1$ $36/16 (1 - 9/b^2) + 4/b^2 = 1$ $9/4 (1 - 9/b^2) + 4/b^2 = 1$ $9/4 - 81/4b^2 + 4/b^2 = 1$ $-81/4b^2 + 4/b^2 = 1 - 9/4$ $(-81+16)/4b^2 = (4-9)/4$ $-65/4b^2 = -5/4$ $-5/4(13/b^2) = -5/4$ $13/b^2 = 1$ $1/b^2 = 1/13$ $b^2 = 13$ Now substituting the value of b^2 in equation (4) we get, $1/a^2 = 1/16(1 - 9/b^2)$ $= 1/16(1 - 9/13)$ $= 1/16(1 - 9/13)$ $= 1/16((13-9)/13)$ $= 1/16(4/13)$ $= 1/52$ $a^2 = 52$ Equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$ By substituting the values of a^2 and b^2 in above equation we get, $x^2/52 + y^2/13 = 1$ i)A ii)B iii)B iv)B	

7.	i)  ii) Upward Parabola iii) 1200 iv) Take the vertex O of the parabola as origin and the axis of the parabola as y-axis. The give is of third standard form, its equation is x^{2}, =4ay Given AB = 12 m and OM = 3 cm = 3/ 100 m. As M is mid-point of AB, MB =6m Then the coordinates of B are(6,3/100) Since B lies on the parabola, 6^{2}=4x 36 × 100 /12 = 300=a Let P be the point on the parabola whose deflection is 1 cm, then NP = 2 cm = 2/100m Let ON = x metres, then the coordinates of Parabola, 2 100 Since P lies on the parabola, we get x^2=4 × 300 × 2 /100 Hence, the points of the beam where the deflection is 1 cm are at a distance of 2√6 metres from	4
8.	i)r = perpendicular distance of (-2, 3) from the line (1) r = |- 2 - 3 + 7/√(1 ^ 2 + (- 1) ^ 2)) = 2/√2 = √2 The equation of the required circle is (x + 2) ^ 2 + (y - 3) ^ 2 = (√2)^ 2 or x ^ 2 + y ^ 2 + 4x - 6y + 11 = 0 ii) K=8, -8 iii) x-2y+5=0, x-2y-5=0 iv) 12x+5y+39=0, 12x+5y-91=0	4
9.	(a) Since given cable is in the form of an upward parabola.	4

	 Equation of parabola is $x^2 = 4ay$ Since the vertex of parabola is 6 m above the ground level, therefore top of the longest wire is 30 – 6 = 24 m above the vertex. Co-ordinates of top point of longest wire is P(50, 24). Also point P lies on the parabola $x^2 = 4ay$ So, $(50)^2 = 4a(24)$ $\therefore$ a = 625/24 (b)Let Q(18, k) be any point on the parabola $x^2 = 4a$ $\therefore (18)^2 = \frac{625}{6} k$ So, k = 3.11 (apprx.)	
10.	(a)The name of the path is ellipse, here a = 3, b = 2 The equation of the curve is $x^2/3^2 + y^2/2^2 = 1$ That is $x^2/9 + y^2/4 = 1$ (b)The length of the latus rectum = $2b^2/a$ = 2x4/3 = 8/3	4
11.	Given that: Foci (0, ±13), Conjugate axis length = 24 It is noted that the foci are on the y-axis. Therefore, the equation of the hyperbola is of the form: $(y^2/a^2)-(x^2/b^2) = 1$ …(1)	4

	Since the foci are (0, ±13), we can get C = 13 It is given that, the length of the conjugate axis is 24, It becomes 2b = 24 b= 24/2 b= 12 And, we know that $a^2 + b^2 = c^2$ To find a, substitute the value of b and c in the above equation: $a^2 + 12^2 = 13^2$ $a^2 = 169-144$ $a^2 = 25$ Now, substitute the value of a and b in equation (1), we get $(y^2/25)-(x^2/144) = 1$, which is the required equation of the hyperbola.	
12.	The given equation is $(x^2/49) + (y^2/36) = 1$ It can be written as $(x^2/7^2) + (y^2/6^2) = 1$ It is noticed that the denominator of $x^2/49$ is greater than the denominator of the $y^2/36$ On comparing the equation with $(x^2/a^2) + (y^2/b^2) = 1$, we will get a= 7 and b = 6 Therefore, $c = \sqrt{(a^2 - b^2)}$ Now, substitute the value of a and b $\Rightarrow \sqrt{(a^2 - b^2)} = \sqrt{(7^2 - 6^2)} = \sqrt{(49-36)}$ $\Rightarrow \sqrt{13}$ Hence, the foci coordinates are $(\pm \sqrt{13}, 0)$ Eccentricity, $e = c/a = \sqrt{13}/ 7$ Length of the major axis = 2a = 2(7) = 14 Length of the minor axis = 2b = 2(6) =12 The coordinates of the vertices are $(\pm 7, 0)$ Latus rectum Length= $2b^2/a = 2(6)^2/7 = 2(36)/7 = 72/7$	4