CHAPTER-5

CONTINUITY & DIFFERENTIABILITY

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	A potter made a mud vessel, where the shape of the pot is based on $f(x) = \|x-3\| + \|x-2\|$, where $f(x)$ represents the height of the pot.  (i) When $x > 4$ What will be the height in terms of x ? (ii) What is $\frac{dy}{dx}$ at $x = 3$ (iii) If the potter is trying to make a pot using the function $(x) = [x]$, will he get a pot or not? Why? Or When the x value lies between (2,3) then the function is= ?	4
2.	Reena started to read the notes on the topic ‘Differentiability’ which she has prepared in the class of Mathematics. She wanted to solve the questions based on this topic, which teacher gave as home work. She has written following matter in her notes: Let $f(x)$ be a real valued function, then its Left Hand Derivative (LHD) is: $$f'(a) = \frac{f(a-h) - f(a)}{-h}$$ Right Hand Derivative (RHD) is: $$f'(a) = \frac{f(a+h) - f(a)}{h}$$ Also, a function $f(x)$ is said to be differentiable at $= a$, if its LHD and RHD at $x = a$ exist and are equal. For the function $f(x) = \begin{cases} \|x-3\| , & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} , & x < 1 \end{cases}$ (i) Find the value of $f'(1)$. (ii) Find the value of $f'(2)$. (iii) Check the differentiability of the function at $x = 1$. Or Check the differentiability of the given function at $x = 1$. $$f(x) = \begin{cases} x^3 - 1 , & 1 < x < \infty \\ x - 1 , & -\infty < x \leq 1 \end{cases}$$	4

3.	If a real valued function $f(x)$ is finitely derivable at any point of its domain it is necessarily continuous at that point .but its converse need not be true . e.g Every polynomial ,constant functions are both continuous as well as differentiable and inverse trigonometric functions are continuous and differentiable in their domain etc. Based on the above information ,answer the following question , [i] Write the interval in which the function $f(x) = \cos^{-1} x$ is always continuous . [ii] Show that the function $f(x) = \begin{cases} 3x, & for\ x \le 0 \\ 0, & for\ x > 0 \end{cases}$ is continuous at x= 0 [iii]show that the function $f(x) = \|x-2\|, x \in R,$ is continuous at $x = 2$ **OR** show that $f(x) = \|\cos 2x\|$ is continuous at $x = \frac{\pi}{4}$	4
4.	Sometimes x and y are given as functions of one another variable , say $x = \emptyset(t), y = \varphi(t)$ are two functions and t is a variable . in such a case , a x and y are called parametric equations and t is called the parameter. To find the derivatives of parametric functions ,we use the following steps . (i) First, write the given parametric functions suppose $x = f(t)\ and\ y = g(t)$ where t is a parameter . (ii) Differentiate both functions separately w.r.t parameter t by using suitable formula, i.e, find $\frac{dx}{dt}$ & $\frac{dy}{dt}$ (iii). Divide the derivative of one function w.r.t parameter by the derivative of second function w.r.t parameter , to get required value, i.e $\frac{dy}{dx}$. Thus , $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}\ where\ f'(t) \neq 0$ On the basis of above information , answer the following questions. [i] if $x = \log t\ and\ y = \cos t$, ,then find $\frac{dy}{dx}$ [ii]if $x = at^3$ and $y = t^2 + 1$, then find $\frac{dy}{dx} at\ t = \frac{2}{3}$ [iii] if $x = \cos t + \sin t$ and $y = \sin t - \cos t$, ,then find $\frac{dy}{dx} at\ t = \frac{\pi}{2}$. **OR** If $x = 4\cos t$ and $= 8\tan t$, then find $\frac{dy}{dx} at\ t = \frac{\pi}{4}$.	4
5.	Differentiate y = $(\sin x)^x + \sin^{-1}\sqrt{x}$ with respect to x	
6.	If y = $\tan^{-1}\{ \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \}$, find $\frac{dy}{dx}$	
7.	Read the following text and answer the following questions on the basis of the same: If a relation between x and y is such that y cannot be expressed in terms of x, then y is called implicit function of x. Assume a function, y = $6x^2$-11e^y This function can be rewritten as y+11e^y = $6x^2$ But it is not possible to completely separate and represent it as a function of y. This type of function is known as an implicit function. To differentiate an implicit function, we consider y as a function of x and then we use the chain rule to differentiate any term consisting of y. Now to differentiate the above function, we differentiate directly w.r.t. x the entire function. This step basically indicates the use of chain rule.	

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<td>i.e.,
$\frac{dy}{dx}$ =d(11e^y)/dx = d(6x^2) /dx

$\frac{dy}{dx}$ +11e^y $\frac{dy}{dx}$ = 12x

$\frac{dy}{dx}$ (1+11e^y) = 12x

$\frac{dy}{dx}$ =12x /(1+11e^y)

Q1) If $x^3+ x^2y$+x y^2+ y^3=81, the find $\frac{dy}{dx}$. (1)
Q2) Find the slope of the tangent to the cure y= $x^2 + 6\,y^2 + xy$. (1)
Q3) Find $\frac{dy}{dx}$ at x=1, y=$\frac{\pi}{4}$ if $sin^2 x + cos\ xy = K$. (2)</td>
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<td>8.</td>
<td>Read the following text and answer the following questions on the basis of the same:

Reena started to read the notes on the topic 'differentiability' which she has prepared in the class of mathematics. She wanted to solve the questions based on this topic, which teacher gave as home work. She has written following matter in her notes:
Let f(x) be a real valued function, then its Left-Hand derivative (LHD) is:

L f ' (a) = $\lim_{h\to 0}\left(\frac{\text{f(a−h)− f(a)}}{-\text{h}}\right)^n$

Right Hand Derivative (RHD) is :

Rf '(a) = $\lim_{h\to 0}\left(\frac{\text{f(a+h)− f(a)}}{\text{h}}\right)^n$

Also, a function f(x) is said to be differentiable at x = a if its LHD and RHD at x = a exist and one equal.
|x-31, x ≥1
For the function, f(x) =$f(x) = \begin{cases} \lvert x-3\rvert, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

Q1) Find the value of f '(-1). (1)
Q2) Find the value of f '(2). (1)
Q3) Check the differentiability of function at x=1. (2)</td>
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<td>9.</td>
<td>A potter made a mud vessel, where the shape of the pot is based on $f(x) = \lvert x-3\rvert + \lvert x-2\rvert$,
where $f(x)$represents the height of the pot.</td>
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(i) When $x > 4$ What will be the height in terms of x ?

(ii) What is $\frac{dy}{dx}$at $x = 3$

(iii) If the potter is trying to make a pot using the function $(x) = [x]$, will he get a pot or not? Why?

Or

When the x value lies between (2,3) then the function is= ?

10. Reena started to read the notes on the topic ‘Differentiability’ which she has prepared in the class of Mathematics. She wanted to solve the questions based on this topic, which teacher gave as home work. She has written following matter in her notes: **4**

Let $f(x)$ be a real valued function, then its Left Hand Derivative (LHD) is:

$$f'(a) = \lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}$$

Right Hand Derivative (RHD) is:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

Also, a function $f(x)$ is said to be differentiable at $= a$, if its LHD and RHD at $x = a$ exist and are equal.

For the function $f(x) = \begin{cases} |x-3| , & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} , & x < 1 \end{cases}$

(i) Find the value of $f'(1)$.

(ii) Find the value of $f'(2)$.

(iii) Check the differentiability of the function at $x = 1$.

Or

Check the differentiability of the given function at $x = 1$.

$$f(x) = \begin{cases} x^3 - 1 , & 1 < x < \infty \\ x - 1 , & -\infty < x \leq 1 \end{cases}$$

11. Let f(x) be a real valued function then its **4**

left hand derivative(L.H.D): Lf'(a) = $\lim_{h\to 0} \frac{f|(a-h)- f(a)}{-h}$

Right hand derivative(R.H.D): Rf'(a) = $\lim_{h\to 0} \frac{f|(a-h)- f(a)}{h}$

Also a function is said to be differentiable at x = a if its L.H.D and R.H.D at x= a exist and are equal

	for the function f(x) = $\begin{cases} \lvert x-3 \rvert, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$ answer the following question (i) R.H.D of f(x) at x = 1 is (a) 1 (b) -1 (c) 0 (d) 2 (ii) L.H.D of f(x) at x = 1 is (b) 1 (b) -1 (c) 0 (d) 2 (iii) f(x) is non differentiable at (a) x = 1 (b) x = 2 (c) x = 3 (d) x = 4 (iv) find the value of $f'(2)$. (a) 1 (b) 2 (c) 3 (d) -1 (v) find the value of $f'(-1)$. (a) 2 (b) 1 (c) -2 (d) -1	
12.	A function f(x) is said to be continuous in an open interval (a,b), if it is continuous at every point in this interval. A function f(x) is said to be continuous in the closed interval [a,b], if f(x) is continuous in (a,b) and $\lim_{h\to 0} f(a+h)$ = f(a) and $\lim_{h\to 0} f(b-h)$ = f(b) If function f(x) = $\begin{cases} \frac{\sin(a+1)x + sinx}{x}, x < 0 \\ c, \quad x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, \ x > 0 \end{cases}$ is continuous at x = 0 then answer the following question (i) The value of a is (a) -3/2 (b) 0 (c) ½ (d) -1/2 (ii) The value of b is (a)1 (b) -1 (c) 0 (d) any real number (iii) The value of c is (a) 1 (b) 1/2 (c) -1 (d) -1/2 (iv) The value of a + c is (a) 1 (b) 0 (c) -1 (d) -2 (v) The value of c - a is (a) 1 (b) 0 (c) -1 (d) 2	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	(i) $2x-5$ (ii) Function is not differentiable. (iii) No, because it is not continuous. Or 1 .	4
2.	(i) -1 (ii) -1 (iii) $f(x)$ *is* differentiable at $x = 1$. or $f(x)$ *is not* differentiable at $x = 1$.	4
3.	$x \in [-1,1]$ ---	4
4.	(i) $-t\sin t$ (ii) $\frac{1}{a}$ (iii) -1 or $-4\sqrt{2}$	4
5.	$(\sin x)^x (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$	4
6.	$\frac{1}{2\sqrt{1-x^2}}$	4
7.	1) We have, $x^3 + x^2y + xy^2 + y^3 = 81$ Differentiating both sides w.r.t. x, we get $3x^2 + \left(2xy + x^2\frac{dy}{dx}\right) + \left(y^2 + 2xy\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx}$ $\Rightarrow (x^2+2xy+3y^2)\frac{dy}{dx} + (3x^2+2xy+y^2) = 0$ $\Rightarrow (x^2+2xy+3y^2)\frac{dy}{dx} = -(3x^2+2xy+y^2)$ $\Rightarrow \frac{dy}{dx} = \frac{-(3x^2+2xy+y^2)}{(x^2+2xy+3y^2)}$ 2) To find slope of curve y, we find $\frac{dy}{dx}$. We have, $y = x^2+6y^2+xy$ $\therefore \frac{dy}{dx} = \frac{d(x^2)}{dx}+\frac{d(6y^2)}{dx}+\frac{d(xy)}{dx}$ $\Rightarrow \frac{dy}{dx} = 2x+12y\frac{dy}{dx}+y+x\frac{dy}{dx}$ $\Rightarrow (1-x-12y)\frac{dy}{dx} = 2x+y$ $\Rightarrow \frac{dy}{dx} = \frac{2x+y}{(1-x-12y)}$ 3) . From the given equation $2\sin y\cos y.\frac{dy}{dx} - \sin xy.\left[x.\frac{dy}{dx}+y.1\right] = 0$ or $\frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin(xy)}$ $\frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}$	4

$$\left(\frac{dy}{dx}\right)_{\substack{x=1\\ y=\frac{\pi}{4}}} = \frac{\frac{\pi}{4}\sin 1.\frac{\pi}{4}}{\sin 2\frac{\pi}{4} - 1\sin 1.\frac{\pi}{4}} = \frac{\frac{\pi}{4}.\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{\pi}{}$$

8.	1) f ’(x)= $\frac{x}{2} - \frac{3}{2}$, $x < 1$ f ’(-1) = $\frac{-1}{2} - \frac{3}{2}$ = -2 2) f ’(x)= -1,1 ≤ x ≤ 3 f ’(-2) = -1 3) We have, $f(x) = \begin{cases} x-3, & x \geq 3 \\ 3-x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ L [illegible] $\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$ $= \lim_{h\to 0} \frac{1}{-h}\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]$ $= \lim_{h\to 0}\left(\frac{1+h^2-2h-6+6h+13-8}{-4h}\right)$ $= \lim_{h\to 0}\left(\frac{h^2+4h}{-4h}\right) = 1$ RHD at $x = 1 = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ $= \lim_{h\to 0} \frac{3-(1+h)-2}{h}$ $= \lim_{h\to 0} \frac{3-(1+h)-2}{h}$ $= \lim_{h\to 0} -\frac{h}{h} = -1$ **Since, LHD = RHD** **Thus, $f(x)$ is differentiable at $x = 1$.**	4
9.	(i) $2x - 5$ (ii) Function is not differentiable. (iii) No, because it is not continuous. Or 1 .	4
10.	(i) -1 (ii) -1 (iii) $f(x)$ *is* differentiable at $x = 1$.	4

	or $f(x)$ *is not* differentiable at $x = 1$.	
11.	(i) (b) (ii) (b) (iii) (c) (iv) (d) (v) (c)	4
12.	(i) (a) (ii) (d) (iii) (b) (iv) (c) (v) (d)	4