

CHAPTER-2
RELATIONS & FUNCTIONS
04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	<p>Let $A = \{1, 2\}$ and $B = \{2, 4, 6\}$. Let $f = \{(x, y) : x \in A, y \in B \text{ and } y > 2x + 1\}$.</p> <p>Write f as a set of ordered pairs. Show that f is a relation but not a function from A to B</p>	4
2.	<p>Let R^+ be the set of all positive real numbers. Let $f : R^+ \rightarrow R : f(x) = \log x$. Base e</p> <p>Find</p> <p>(i) Range (f)</p> <p>(ii) $\{x : x \in R^+ \text{ and } f(x) = -2\}$.</p> <p>(iii) Find out whether $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in R$.</p>	4
3.	<p>Let $A = \{5, 7\}$ and $B = \{9, 13\}$</p> <p>Let $R = \{(x, y) : x \in A, y \in B, x - y = \text{odd integer}\}$</p> <p>Show that R is an empty relation from set A to set B</p>	4
4.	<p>Find the domain range of $f(x) = \frac{1}{\sqrt{x - [x]}}$</p>	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	<p>Given: $A = \{1, 2\}$ and $B = \{2, 4, 6\}$</p> <p>$f = \{(x, y): x \in A, y \in B \text{ and } y > 2x + 1\}$</p> <p>Putting $x = 1$ in $y > 2x + 1$, we get</p> $y > 2(1) + 1$ $\Rightarrow y > 3$ <p>and $y \in B$</p> <p>this means $y = 4, 6$ if $x = 1$ because it satisfies the condition $y > 3$</p> <p>Putting $x = 2$ in $y > 2x + 1$, we get</p> $y > 2(2) + 1$ $\Rightarrow y > 5$ <p>this means $y = 6$ if $x = 2$ because it satisfies the condition $y > 5$.</p> <p>$\therefore f = \{(1, 4), (1, 6), (2, 6)\}$</p> <p>$(1, 2), (2, 2), (2, 4)$ are not the members of 'f' because they do not satisfy the given</p> <p>condition $y > 2x + 1$</p> <p>Firstly, we have to show that f is a relation from A to B.</p> <p>First elements = 1, 2</p> <p>All the first elements are in Set A</p> <p>So, the first element is from set A</p> <p>Second elements in F = 4, 6</p> <p>All the second elements are in Set B</p>	

So, the second element is from set B

Since the first element is from set A and second element is from set B

Hence, F is a relation from A to B.

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

Now, we have to show that f is not a function from A to B

$$f = \{(1, 4), (1, 6), (2, 6)\}$$

Here, 1 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

2. Given that $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that $f(x) = \log x$

To find: (i) Range of f

Here, $f(x) = \log x$

We know that the range of a function is the set of images of elements in the domain.

\therefore The image set of the domain of $f = \mathbb{R}$

Hence, the range of f is the set of all real numbers.

To find: (ii) $\{x : x \in \mathbb{R}^+ \text{ and } f(x) = -2\}$

We have, $f(x) = -2 \dots (a)$

And $f(x) = \log x \dots (b)$

From eq. (a) and (b), we get

$$\log x = -2$$

Taking exponential both the sides, we get

$$\Rightarrow x = e^{-2}$$

$$\therefore \{x : x \in \mathbb{R}^+ \text{ and } f(x) = -2\} = \{e^{-2}\}$$

To find: (iii) $f(xy) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$

We have,

	$f(xy) = \log(xy)$ $= \log(x) + \log(y)$ [Product Rule for Logarithms] $= f(x) + f(y)$ [$\because f(x) = \log x$] $\therefore f(xy) = f(x) + f(y)$ holds.	
3.	$A = \{5,7\}, B = \{9,13\}$ $A \times B = \{(5,9), (5,13), (7,9), (7,13)\}$ Every relation is a subset of $A \times B$ $R = \{(x,y): x \in A, y \in B, x - y = \text{An odd integer}\}$ Now $(5 - 9) = -4, (5 - 13) = -8, (7 - 9) = -2, (7 - 13) = -6$ We see that every ordered pair (x, y) of $A \times B, x - y$ is not odd, i.e, no $(x, y) \in R$ Hence, R is an empty relation.	
4.	We have, $f(x) = \frac{1}{\sqrt{x-[x]}}$ Domain of f : We know that $0 \leq x - [x] < 1$ for all $x \in R$ and $x - [x] = 0$ for $x \in Z$ $\therefore 0 < x - [x] < 1$ for all $x \in R - Z$ $\Rightarrow f(x) = \frac{1}{\sqrt{x-[x]}}$ exists for all $x \in R - Z$ Hence, Domain (f) = $R - Z$ Range of f : We have $0 < x - [x] < 1$ for all $x \in R - Z$ $\Rightarrow 0 < \sqrt{x - [x]} < 1$ for all $x \in R - Z$ $\Rightarrow 1 < \frac{1}{\sqrt{x-[x]}} < \infty$ for all $x \in R - Z$ $\Rightarrow 1 < f(x) < \infty$ for all $x \in R - Z$ Range (f) = $(1, \infty)$	