CHAPTER-10
VECTORS
04 MARKS TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Team A,B,C went for playing a tug of war game. Teams A, B, C, have attached a rope to a mental ring and its trying to pull the ring into their own area(learn areas shown below). Team A pulls with force $F1=4\hat{i}+0\hat{j}$KN Team B →$F2= 2\imath+\hat{4}\ \hat{j}$KN Team C →$F3=-3\hat{i}–3\hat{j}$KN Based on the above information, answer the following. 1. Which team will win the game? a. Team B b. Team A c. Team C d. No one 2. What is the magnitude of the teams combined force? a. 7 KN b. 1.4 KN c. 1.5 KN d. 2 KN 3. In what direction is the ring getting pulled? a. 2.0 radian b. 2.5 radian c. 2.4 radian d. 3 radian 4. What is the magnitude of the forces of Team B? a. $2\sqrt{5}$ KN b. 6 KN c. 2 KN d. $\sqrt{6}$KN 5. How many KN force is applied by Team A? a. 5 KN b. 4 KN c. 2 KN d. 16 KN	4
2.	A class XII student appearing for a competitive examination was asked to attempt the following questions. Let a , b and c be three non zero vectors. 1. If a and b are such that $\mid a + b \mid = \mid a - b \mid$ then a. a _|_ b b. a ||b c. a = b d. None of these 2. If -a = $\hat{i}-2\hat{j}$, -b =$2\hat{i}+\hat{j}+3\hat{k}$then evaluate (2-a + -b)·[(-a + -b)×(-a −2-b)] a. 0	4

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	b. 4 c. 3 d. 2 3. If a and b are unit vectors and θ be the angle between them then $\vert\vec{a} - \vec{b}\vert$ = a. $\sin^2\theta$ b. $2\sin\theta$ c. $2\cos\theta$ d. $\cos^2\theta$ 4. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} . \vec{b} = \vec{a} . \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is π 6 then $\vec{a}$ = a. $2 (\vec{b} \times \vec{c})$ b. $-2 (\vec{b} \times \vec{c})$ c. $\pm 2 (\vec{b} \times \vec{c})$ d. $2 (\vec{b} \pm \vec{c})$ 5. The area of the parallelogram If -a = $\hat{i} - 2\hat{j}$, -b = $2\hat{i} + \hat{j} + 3\hat{k}$ as diagonals is a. 70 b. 35 c. √70/2 d. √70	
3.	Rohan is walking around a triangular park. The vertices of the park are given by the position vectors ($-\hat{j}-2\hat{k}$), ($3\hat{i}+\hat{j}+4\hat{k}$) and ($5\hat{i}+7\hat{j}+\hat{k}$). Show that the park is in right triangular shape. Also find its other two angles.	4
4.	On the week days, every morning Piya first drops her son to his school and then she goes to her office. Let her house, the school and the office are represented by the position vectors ($-2\vec{a}+3\vec{b}+5\vec{c}$), ($\vec{a}+2\vec{b}+3\vec{c}$) and ($7\vec{a}-\vec{c}$). Show that for any $\vec{a}$, $\vec{b}$ and $\vec{c}$ the house, the school and the office are on the same straight path.	4
5.	$\vec{a}$, $\vec{b}$, and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$, and $\vec{c}$.	4
6.	Show that points $(2, -1, 3), (3, -5, 1)$ *and* $(-1, 11, 9)$ are collinear by vector method	4
7.	Using vectors find the area of a triangle ABC with vertices A(1,2,3), B=(2,-1,4) and C(4,5,-1).	4
8.	Dot product of a vector with vectors $\hat{\imath} - \hat{\jmath} + \hat{k}$, $2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ are respectively 4,0 and 2. Find the vectors.	4
9.	Two vectors $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$, represent the two sides vectors AB and AC respectively of triangle ABC . Find the length of the median through A.	4
10.	Show that each of the given three vectors is a unit vector $\frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$,$\frac{1}{7}(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k})$, $\frac{1}{7}(6\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$.Also show that they are mutually perpendicular to each other.	4

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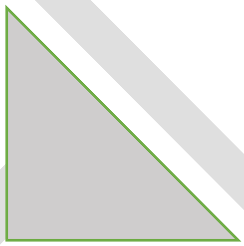
ANSWERS:

Q. NO	ANSWER	MARKS
1	1. a. Team B 2. b. 1.4 KN 3. c. 2.4 KN 4. a. 2√5 KN 5. b. 4 KN	**4**
2	1. a 2. A 3. B 4. C 5.C	**4**
3	Let the position vectors of the vertices A, B and C of the triangular park is $\vec{a} = 0\hat{i} - \hat{j} - 2\hat{k}$ $\vec{b} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{c} = 5\hat{i} + 7\hat{j} + \hat{k}$  $\overrightarrow{AB} = \vec{b} - \vec{a} = (3\hat{i}+\hat{j}+4\hat{k}) - (0\hat{i}-\hat{j}-2\hat{k}) = 3\hat{i} + 2\hat{j} + 6\hat{k}$ $\overrightarrow{BC} = \vec{c} - \vec{b} = (5\hat{i}+7\hat{j}+\hat{k}) - (3\hat{i}+\hat{j}+4\hat{k}) = 2\hat{i} + 6\hat{j} - 3\hat{k}$ $\overrightarrow{CA} = \vec{a} - \vec{c} = (0\hat{i}-\hat{j}-2\hat{k}) - (5\hat{i}+7\hat{j}+\hat{k}) = -5\hat{i} - 8\hat{j} - 3\hat{k}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(9+4+36)} = \sqrt{49} = 7$ $\lvert\overrightarrow{BC}\rvert = \sqrt{(4+36+9)} = \sqrt{49} = 7$ $\lvert\overrightarrow{CA}\rvert = \sqrt{(25+64+9)} = \sqrt{98} = 7\sqrt{2}$ $\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{BC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert} = \frac{(3\hat{i}+2\hat{j}+6\hat{k}).(2\hat{i}+6\hat{j}-3\hat{k})}{7\times7} = \frac{6+12-18}{49} = 0$ $\therefore \theta = \frac{\pi}{2}$ Therefore, the park is in right triangular shape. Again, $\cos\alpha = \frac{\overrightarrow{CA}.\overrightarrow{AB}}{\lvert\overrightarrow{CA}\rvert\lvert\overrightarrow{AB}\rvert} = \frac{(-5\hat{i}-8\hat{j}-3\hat{k}).(3\hat{i}+2\hat{j}+6\hat{k})}{7\sqrt{2}\times7} = \frac{-15-16-18}{49\sqrt{2}} = \left\lvert\frac{-1}{\sqrt{2}}\right\rvert$ $\therefore \alpha = \frac{\pi}{4}$ Hence the third angle is $\pi - (\frac{\pi}{2} + \frac{\pi}{4}) = \frac{\pi}{4}$ Therefore the other two angles are $\frac{\pi}{4}, \frac{\pi}{4}$ i.e., 45° and 45°	4
4	The position vectors of the house, the school and the office are $\vec{A} = (-2\vec{a}+3\vec{b}+5\vec{c})$, $\vec{B} = (\vec{a}+2\vec{b}+3\vec{c})$ and $\vec{C} = (7\vec{a}-\vec{c})$ $\overrightarrow{AB} = \vec{B} - \vec{A} = (\vec{a}+2\vec{b}+3\vec{c}) - (-2\vec{a}+3\vec{b}+5\vec{c}) = 3\vec{a} - \vec{b} - 2\vec{c}$ $\overrightarrow{AC} = \vec{C} - \vec{A} = (7\vec{a}-\vec{c}) - (-2\vec{a}+3\vec{b}+5\vec{c}) = 9\vec{a} - 3\vec{b} - 6\vec{c}$ To prove that $\vec{A}, \vec{B}, \vec{C}$ are collinear we need to prove that $\overrightarrow{AB} \times \overrightarrow{AC} = 0$ We know that $\vec{a}\times\vec{b} = (a_2b_3-b_2a_3)\hat{i} + (a_3b_1-a_1b_3)\hat{j} + (a_1b_2-a_2b_1)\hat{k}$	4

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	$\therefore\ \overrightarrow{AB} \times \overrightarrow{AC}$ = $(3\vec{a} - \vec{b} - 2\vec{c}\ ) \times (9\vec{a} - 3\vec{b} - 6\vec{c})$ = $(6\text{-}6)\vec{a}$ + $(-18+18)\vec{b}$ + $(-9+9)\vec{c}$ = 0 $\therefore \vec{A}\ ,\vec{B}\ ,\vec{C}$ are collinear Hence the house, the school and the office are on the same straight path.	
5	Since $\vec{a}, \vec{b}$, and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes So, $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$ and $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$ Let $\alpha, \beta, and\ \gamma = inclination\ of\ \vec{a} + \vec{b} + \vec{c}\ with\ \vec{a}, \vec{b}$, and $\vec{c}\ resp.$ Then $cos\alpha = \frac{\lvert\vec{a}\rvert}{\lvert\vec{a}+\vec{b}+\vec{c}\rvert}, cos\beta = \frac{\lvert\vec{b}\rvert}{\lvert\vec{a}+\vec{b}+\vec{c}\rvert}, cos\gamma = \frac{\lvert\vec{c}\rvert}{\lvert\vec{a}+\vec{b}+\vec{c}\rvert}$ so $\cos\alpha = \cos\beta = \cos\gamma \rightarrow\ \alpha = \beta = \gamma$	4
6	Let , $A(2,-1,3), B(3,-5,1)\ and\ C(-1,11,9)$ $\overrightarrow{OA} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}), \overrightarrow{OB} = (3\hat{\imath} - 5\hat{\jmath} + \hat{k}), \overrightarrow{OC} = (-\hat{\imath} + 11\hat{\jmath} + 9\hat{k})$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (3\hat{\imath} - 5\hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) = (\hat{\imath} - 4\hat{\jmath} - 2\hat{k})$ Similarly, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (-\hat{\imath} + 11\hat{\jmath} + 9\hat{k}) - (3\hat{\imath} - 5\hat{\jmath} + \hat{k}) = (-4\hat{\imath} + 16\hat{\jmath} + 8\hat{k}) =$ $-4\left((\hat{\imath} - 4\hat{\jmath} - 2\hat{k})\right) = -\overrightarrow{AB}$ $\overrightarrow{AB}, \overrightarrow{BC}$ are collinear ⋙ $A, B\ and\ C\ are\ collinear.$	4
7	$area = \frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{BA}\rvert = \frac{1}{2}\sqrt{274}.$	4
8	Forming three equations $x - y + z = 4, 2x + y - 3z = 0$ and $x + y + z = 2$ and solving to find the vector $2\hat{\imath} - \hat{\jmath} + \hat{k}$.	4

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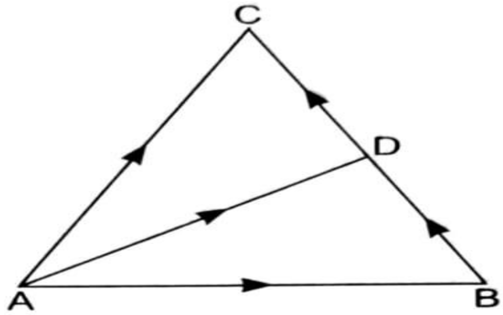
9	1. Given $\overrightarrow{AB} = \hat{j} + \hat{k}$, $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$  Using triangle law of vectors $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$ $\Rightarrow \overrightarrow{BC} = 3\hat{i} - \hat{j} + 4\hat{k} - \hat{j} - \hat{k} = 3\hat{i} - 2\hat{j} + 3\hat{k}$ Also $\overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC}$ [D is mid–point of BC $= \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$ In ΔABD, applying triangle law of vectors $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ $= \hat{j} + \hat{k} + \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k} = \frac{3}{2}\hat{i} + \frac{5}{2}\hat{k}$ Length of median $= \lvert\overrightarrow{AD}\rvert = \sqrt{\frac{9}{4} + \frac{25}{4}} = \frac{1}{2}\sqrt{34}$ units	4
1	Let, $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$; $\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$ $\lvert\vec{a}\rvert = \frac{1}{7}\sqrt{4+9+36} = 1$; $\lvert\vec{b}\rvert = \frac{1}{7}\sqrt{9+36+4} = 1$; $\lvert\vec{c}\rvert = \frac{1}{7}\sqrt{36+4+9} = 1$ $\vec{a}\cdot\vec{b} = \frac{1}{49}(6 - 18 + 12) = 0$, $\vec{b}\cdot\vec{c} = \frac{1}{49}(3\hat{i} - 6\hat{j} + 2\hat{k})\cdot(6\hat{i} + 2\hat{j} - 3\hat{k})$ $= \frac{1}{49}(18 - 12 - 6) = 0$ $\vec{c}\cdot\vec{a} = \frac{1}{49}(6\hat{i} + 2\hat{j} - 3\hat{k})(2\hat{i} + 3\hat{j} + 6\hat{k})$ $= \frac{1}{49}(12 + 6 - 18) = 0$ Hence, the given vectors are mutually perpendicular to each other.	4

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