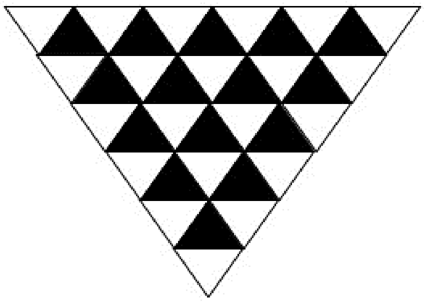
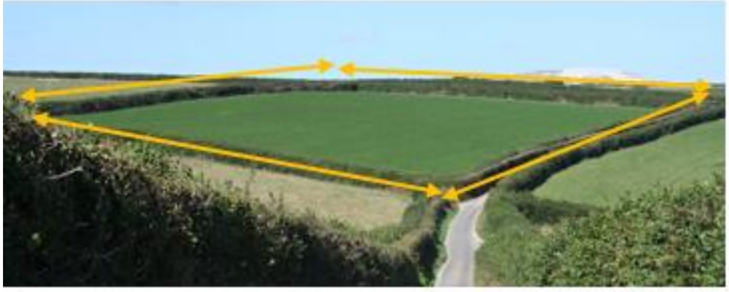
CHAPTER-4

DETERMINANTS

04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Two schools X and Y want to award their selected students on the values of Hard work, Honesty and Punctuality. The school X wants to award Rupees P each, Rupees q each and Rupees r each for the three respective values to its 3,2 and 1 students respectively with a total award money of Rupees 3000/- School wants to spend rupees 3500/- to award in 2,4 & 3 students on the respective values. The total amount of awards for one prize on each value is Rupees 1500/-. Using the concept of Determinants & matrices, Answer the following questions I) what is the award money for punctuality? a. 500 b. 300 c. 900 d. 1000 II) What is the award money for hard work? a. 200 b. 900 c.800 d. 500	4
2.	**Show that, using properties of determinants.** $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$	4
3.	 A triangular floral design is made up of 36 smaller equilateral triangles as shown in the figure. *Using the above information and the concept of determinants, answer the following questions.* (i) If the vertices of one of the smaller equilateral triangle are $(3,1),(9,3)$ and $(5,3)$, then the area of such triangle is (a) 4 sq. u (b) 6 sq. u (c) 10 sq. u (d) 8 sq. u (ii) What is the area of design? (a) 72 sq.u (b) 104 sq.u (c) 144 sq. u (d) 10 sq.u (iii) If the vertices of one of the smaller equilateral triangle are $(0,0),(3,\sqrt{3})$ $(3,-\sqrt{3})$, then the altitude of such triangle is ? (a) 4 u (b) 6 u (c) 3 u (d) 8 u (iv) If $(2,4),(2,6)$ are two vertices of smaller triangle and its area is $3\sqrt{3}$ sq. units , then the third vertex will lie on the line (a) $x+y=5$ (b) $x-y=5$ (c) $x=2\pm3\sqrt{3}$ (d) $2x+y=3$	4

4.	A missile launched to hit its target follows a parabolic path. Its velocity at any instant ‘t’ is given by $v(t) = at^2 + bt + c, 0 \le t \le 100,$ where a,b and c are constants. It has been found that the velocity at time t=3, t=6 and t=9 seconds are respectively 64,133 and 208 miles per second. If $\begin{bmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{bmatrix}^{-1} = \frac{1}{18}\begin{bmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{bmatrix}$, then answer the following questions. (i) Find the value of b+c. (ii) Find $v(t)$. (iii) Calculate the speed at time t=15 seconds. (iv) At what time the missile acquires a speed of 784 miles/sec?	4
5.	 Chandrayaan 3 is the third lunar exploration mission undertaken by the Indian Space Research Organisation (ISRO). It aims to further expand our understanding of the Moon's surface by deploying a Lander and a rover. During its launch stage, it's follows a definite trajectory and velocity of the rocket can be expressed as a function of time(t) as follows: **v(t) = 140at² + 3bt - 130c – M** where a, b and c are constants of unknown values and M accounts for the mass of the rocket which satisfies $4a + b - 2c + 58 = 0$ $2a + b - c + 35 = 0$ $-7a - 2b + 4c = 113$ Use the value of AB to solve the above system of equations and obtain the value of a, b and c.	4
6.	A trust invested some money in two type of bonds . The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs 2400 as interest . However, if trust had interchanged money in bonds they would have got Rs 100 less. Let the amount invested in first type and second type of bond be Rs x and Rs y. Based on the above information ,answer the following questions; (i) Write the equations in terms of x and y representing the given information. (ii) Write the matrix equation representing the given information. Find the amount invested by trust in first and second bond respectively.	4
7.	Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by10 m and breadth is decreasedby 20m, then its area will decreasey5300 m^2	4



Based on the information given above, answer the following questions :

i) The value of x(length of rectangular field)is

(a) 150 m b) 400 m c) 200 m d) 320 m

ii) The value of y (breadth of rectangular field) is

(a) 150 m b)200 m c) 430 m d) 350 m

iii) How much is the area of rectangular field?

a) 60000 sq m b)30000 sq m c) 3000 sq m d) 30000 m

iv) The equations in terms of x and y are

a. x+y =50, 3x-y = 550

b.x-y=50, 2x+y=550

c. x+y= 50, 2x+ y=550

d.x+y= 50, 2x+y=550

---

8. A factory produces three products every day. Their production on a particular day is 45 tones. It is found that production of third product exceeds the production of first product by 8 tons while production of first and third products is twice the production of second product. (.4)

1. If x, y and z respectively denotes the quantity (in tons) of first, second and third products produced, then construct the system of equation and write it in matrix form.
2. If $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix}2 & 2 & 2\\ 3 & 0 & -3\\ 1 & -2 & 1\end{bmatrix}$, then find the inverse of $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -1\\ 1 & -2 & 1\end{bmatrix}$
3. Find x : y : z

---

9. Three shopkeepers Ram Lal, Shyam Lal, and Ghansham are using polythene bags, handmade bags (prepared by prisoners), and newspaper envelopes as carrying bags. It is found that the shopkeepers Ram Lal, Shyam Lal, and Ghansham are using (20,30,40), (30,40,20), and (40,20,30) polythene bags, handmade bags, and newspaper envelopes respectively. The shopkeeper's Ram Lal, Shyam Lal, and Ghansham spent ₹250, ₹270, and ₹200 on these carry bags respectively. (4)

1. What is the cost of one polythene bag?
2. What is the cost of one handmade bag?
3. What is the cost of one newspaper bag?

Keeping in mind the environmental conditions, which shopkeeper is better?

10. A manufacturer makes three types of toys A, B and C. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

4

Types of Toys	Machines		
	I	II	III
A	20	10	10
B	10	20	30
C	5	25	15

The machines I, II and III are available for a maximum of 3 hours, 2 hours and 2 hours 30 minutes respectively. How can you find no of the three types of toys to be produced using determinants?

11. Let A be a matrix such that A$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ is a scalar matrix and $|3A|$=108 then what will be the value ofA^2.

4

12. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then find the inverse of the matrix $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$.

4

13. Three friends Rahul, Ravi and Rakesh went to a vegetable market to purchase vegetables. Rahul Purchased 1kg of each Potato,,Onion and Brinjalfor a total of Rs. 21. Ravi purchased 4kg Potato,3kgOnion and 2kgBrinjal for a total of Rs. 60. Rakesh purchased 6kg Potato,2kg Onion and 3kg Brinjal for a total of Rs. 70.

4

(i) If cost of potato, onion and brinjal are Rs. X, Y and Z respectively then convert above situation into system of linear equation.

(ii) Convert the sytem of equations in (i) in the form of AX=B.

(iii) Find the cost of potato ,onion and brinjal.

14. Gautam buys 5 pens , 3 bags and 1 instruments box and pays a sum of Rs 160. Vikram buys 2 pens , 1 bag and 3 instrument boxes and pays a sum of Rs. 190. Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs. 250.

4

(i) convert the given above situation into system of Linear equations.

(ii) Find IAI

(iii) Find A^{-1}

ANSWERS:

Q. NO	ANSWER	MARKS
1.	According to statement 3p+2q+r=3000 2p+4q+3r=3500 p+q+r=1500 Converting the system of equations in matrix form, we get $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3000 \\ 3500 \\ 1500 \end{bmatrix}$ i.e AX=B Where A= $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ X= $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ B= $\begin{bmatrix} 3000 \\ 3500 \\ 1500 \end{bmatrix}$ |A|= $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ =3(4-3)-2(2-1)+1(6-4) =3×1-2×1+1×2=3~~-2+2~~=3 3≠0 X = A^{-1}B $A^{-1} = \frac{\text{adjA}}{\|A\|}$ adjA= $[\text{cofactors of A}]^T$ cofactors of A = $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & -1 \\ 2 & -7 & 8 \end{bmatrix}$ adjA= $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -7 \\ -2 & -1 & 8 \end{bmatrix}$ $A^{-1} = \frac{\text{adjA}}{\|A\|} = \frac{\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -7 \\ -2 & -1 & 8 \end{bmatrix}}{3} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -7 \\ -2 & -1 & 8 \end{bmatrix}$ X= A^{-1}B $= \begin{bmatrix} \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{-7}{3} \\ \frac{-2}{3} & \frac{-1}{3} & \frac{8}{3} \end{bmatrix} \begin{bmatrix} 3000 \\ 2300 \\ 1500 \end{bmatrix}$ $= \begin{bmatrix} 1000 - 1100 + 1000 \\ 1000 + 2200 - 3500 \\ 2000 - 1100 + 4000 \end{bmatrix} = \begin{bmatrix} 900 \\ -300 \\ 900 \end{bmatrix}$ p=900, q=-300, z=900	4
2.	$R_1 \to R_1 + b.R_3$	4

	$L.H.S = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ **Taking common (1 + a^2 + b^2) from R_1** $= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ $R_1 \to R_1 - a.R_3$ $= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ **Taking (1 + a^2 + b^2) common from R_2** $= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ **Expending entry R_1** $= (1+a^2+b^2)^2[1(1-a^2-b^2+2a^2)-b(-2b)]$ $= (1+a^2+b^2)^2[1+a^2-b^2+2b^2]$ $= (1+a^2+b^2)^2(1+a^2+b^2)$ $= (1+a^2+b^2)^3$	
3.	(i) (a) 4 sq units (ii) (c) 144 sq. units (ii) (c) 3 units (iv) (c) $x = 2 \pm 3\sqrt{3}$	4
4.	$v(3) = 64, v(6) = 64 \, and \, v(6) = 133$ $\Rightarrow 9a+3b+c = 64; 36a+6b+c = 133 \, and \, 81a+9b+c = 208$ In matrix form $\begin{bmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{bmatrix} . \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 64 \\ 133 \\ 208 \end{bmatrix} \Rightarrow A.X = B \Rightarrow X = A^{-1}B$ $= \frac{1}{18} \begin{bmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{bmatrix} \begin{bmatrix} 64 \\ 133 \\ 208 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 20 \\ 1 \end{bmatrix} \Rightarrow a = 1/3; b = 20 \, and \, c = 1$ (i) $\Rightarrow b + c = 21$ (ii) $v(t) = \frac{1}{3}t^2 + 20t + 1$ (iii) $v(15) = 376 miles/\sec$ (iv) 27 seconds.	4
5.	$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & 1 & -1 \\ -7 & -2 & 4 \end{bmatrix}$ X = $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ B= $\begin{bmatrix} -58 \\ -35 \\ 113 \end{bmatrix}$ So,X = A^{-1} B Solving the above condition a= -3 ,b= -12 ,c = 17	4
6.	(i) As per given information : 10x/100 + 12y/100 = 2800	4

<table>
<tr><td></td><td>12x/100 + 10y/100 = 2700
After simplifying the equations are 5x +6y = 140000, 6x +5y = 135000
(ii) Let A = $\begin{bmatrix}5 & 6\\6 & 5\end{bmatrix}$,X = $\begin{matrix}x\\y\end{matrix}$ and B $\begin{pmatrix}140000\\135000\end{pmatrix}$
(iii) Given system cn be written as AX = B
Where $[A]$ = $\begin{bmatrix}5 & 6\\6 & 5\end{bmatrix}$ = 25-36=-11
⇨ A^{-1} exist.
Now, X = A^{-1} B
After solving we get , x = 10000 and y = 15000</td><td></td></tr>
<tr><td>7.</td><td>i b ii a iii b iv b</td><td>4</td></tr>
<tr><td>8.</td><td>4. By given information $x + y + z = 45,\ -x + z = 8,\ x - 2y + z = 0$
In matrix form $\begin{bmatrix}1 & 1 & 1\\1 & 0 & -2\\1 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}45\\8\\0\end{bmatrix}$
5. We know that $(A')^{-1} = (A^{-1})'$
$\therefore \begin{bmatrix}1 & 1 & 1\\1 & 0 & -1\\1 & -2 & 1\end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix}2 & 3 & 1\\2 & 0 & -2\\2 & -3 & 1\end{bmatrix}$
6. $\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{6}\begin{bmatrix}2 & 3 & 1\\2 & 0 & -2\\2 & -3 & 1\end{bmatrix}\begin{bmatrix}45\\8\\0\end{bmatrix} = \frac{1}{6}\begin{bmatrix}66\\90\\114\end{bmatrix} = \begin{bmatrix}11\\15\\19\end{bmatrix}$
$\therefore x : y : z = 11 : 15 : 19$</td><td>4</td></tr>
<tr><td>9.</td><td>Let the cost of one polythene bag, one handmade bag, one newspaper bag be R x,y, z respectively.
Then
20x+30y+40z=250 i e 2x+3y+4z=25
30x+40y+20z=270 i e 3x+4y+2z=27
40x+20y+30z=200 i e 4x+2y+3z=20
These can be written as
AX=B where
A= $\begin{pmatrix}2 & 3 & 4\\3 & 4 & 2\\4 & 2 & 3\end{pmatrix}$,X = $\begin{pmatrix}x\\y\\z\end{pmatrix}$,B= $\begin{pmatrix}25\\27\\20\end{pmatrix}$

Det A=16-3-40=-27,
Adj A= $\begin{pmatrix}8 & -1 & -10\\-1 & -16 & 8\\-10 & 8 & -1\end{pmatrix}$, A^{-1} =Adj A / det A
X= A^{-1} B=1/(-27) $\begin{pmatrix}8 & -1 & -10\\-1 & -16 & 8\\-10 & 8 & -1\end{pmatrix}\begin{pmatrix}25\\27\\20\end{pmatrix} = \begin{pmatrix}1\\11\\2\end{pmatrix}$
X=1,y=11,z=2
1. cost of one polythene bag=Rs 1
2. cost of one handmade bag=Rs 11
3. cost of one newspaper bag=Rs 2
newspaper bag is better for environment.</td><td>1
1
1
1</td></tr>
<tr><td>10.</td><td>Let no of the three types of toys be x, y, z.
20x+10y+5z=180, 10x+20y+25z=120,10x+30y+15z=120</td><td>½
½</td></tr>
</table>

	AX=B where A=$\begin{pmatrix}20 & 10 & 5\\ 10 & 20 & 25\\ 10 & 30 & 15\end{pmatrix}$, B=$\begin{bmatrix}180\\ 120\\ 120\end{bmatrix}$; X=$\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ detA=-1500 Adj A=$\begin{pmatrix}-450 & 0 & 150\\ 100 & 250 & -450\\ 100 & -500 & 300\end{pmatrix}$, A^{-1} =Adj A / det A X=A^{-1} B= 1/(-1500) $\begin{pmatrix}-450 & 0 & 150\\ 100 & 250 & -450\\ 100 & -500 & 300\end{pmatrix}\begin{bmatrix}180\\ 120\\ 120\end{bmatrix} = \begin{bmatrix}42\\ 40\\ 40\end{bmatrix}$ So x=42,y=40,z=40	½ ½ 1 1
11.	Let, $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$ According to the given condition, $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix} = \begin{bmatrix}\gamma & 0\\ 0 & \gamma\end{bmatrix}$ for some scalar γ. Or, $a = \gamma, \quad 2c + 3d = \gamma, \quad c = 0, \quad 2a + 3b = 0$ Therefore, $a = \gamma, b = \frac{-2\gamma}{3}, \ c = 0, \ d = \frac{\gamma}{3}$ $\lvert 3A \rvert = 108$ Or, $\lvert A \rvert = 12$ Also, $\lvert A \rvert = \frac{\gamma^2}{3}$ So, $\frac{\gamma^2}{3} = 12$ Or, $\gamma = \pm 6$ Therefore, $A = \begin{bmatrix}6 & -4\\ 0 & 2\end{bmatrix}$ When $\gamma = 6$ $A^2 = \begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$	4
12.	Given, $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$ Or, $\begin{bmatrix}1 & 1+2+3+\cdots+(n-1)\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$ Or, $\frac{n(n-1)}{2} = 78$ Or, $n = 13$ as $n \neq -12$ $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 13\\ 0 & 1\end{bmatrix} = A(Say)$ Therefore, $A^{-1} = \begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$	4

13.	(i) x +y+z= 21 , 4x+3y+2z=60 ,6x+2y+3z=70 $(ii)\begin{bmatrix}1 & 1 & 1\\4 & 3 & 2\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}21\\60\\70\end{bmatrix}$ (iv) x= Rs 5 , y= Rs 8 ,z= Rs. 8	1 1 2
14.	(i) 5x +3y+z= 160 , 2x+y+3z=190 ,x+2y+4z=250 (ii) IAI= -22 (iii) $A^{-1}=\frac{1}{22}\begin{bmatrix}2 & 10 & -8\\5 & -19 & 13\\-3 & 7 & 1\end{bmatrix}$	1 1 2