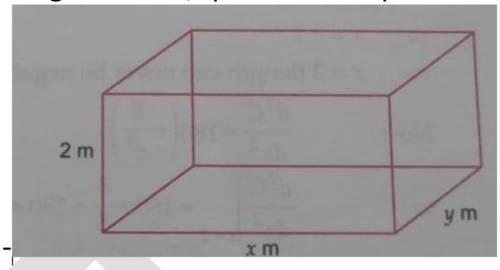
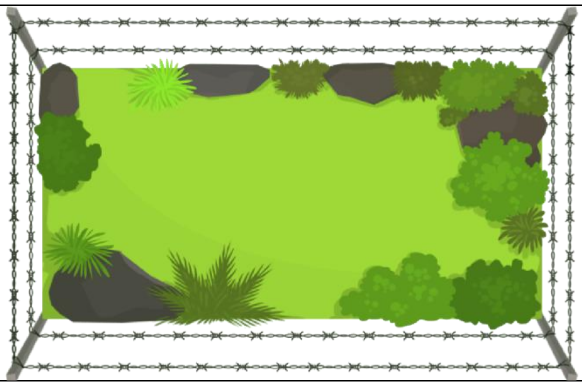
CHAPTER-6

APPLICATION OF DERIVATIVES

05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	On the request of villagers,a construction agency designs a tank with the help of an architect.Tank consists of rectangular base with rectangular sides,open at the top so that its depth is 2m and volume is $8m^3$ as shown below:-  (i)If x and y represent the length and breadth of its rectangular base,then the relation between the variables is (a) x+y = 8 (b) x.y = 4 (c) x+y = 4 (d) $\frac{x}{y}$ = 4 (ii) If construction of tank cost Rs.70 per sq.metre for the base and Rs.45 per sq.metre for sides,then making cost 'C' expressed as a function of x is (a) $C= 80+80(x+\frac{4}{x})$ (b) $C= 280x+280(x+\frac{4}{x})$ (c) $C= 280+180(x+\frac{4}{x})$ (d) $C= 70+70(x+\frac{4}{x})$ (iii)The owner of a construction agency is interested in minimizing the cost 'C' of whole tank,for this to happen the value of x should be (a) 4m (b) 3m (c) 1m (d) 2m (iv)For minimum cost 'C' the value of y should be (a) 1m (b) 3m (c) 2m (d) 4m (v)The Pradhan of village wants to know minimum coat.The minimum cost is Rs. (a)2000 (b) 4000 (c) 11000 (d) 1000	5
2.	A helicopter is flying along the curve represented by $y = x^2+7$.A soldier placed at (3,7) wants to shoot down the helicopter when it is nearest to him. (i)If(x_1,y_1) represents the position of helicopter on the curve $y = x^2+7$,when the distance D from soldier placed at S(3,7) is minimum,then the relation between x_1,y_1 is (a)$x_1 = y_1^2+7$ (b) $y_1 = x_1^2+7$ (c) $y_1 + x_1^2 =7$ (d) $y_1^2 + x_1 = 7$. (ii)The distance 'D' expressed as a function of x_1 is (a) $D = x_1^2-6x_1 +x_1^4$(b) $D = x_1^2-6x_1 +9+x_1^4$(c) $D^2 = x_1^2-6x_1 +9+x_1^4$(d) $D^2 = x_1^2+6x_1 -9+x_1^4$ (iii)The soldierat S wants to know the enemy helicopter is nearest to soldier,then the value y_1 should be (a) 4 (b) 3 (c) 8 (d) 5 (iv)When the enemy helicopter is nearest to soldier,then the value of D should be (a) 4 units (b) 5 units (c) $\sqrt{5}$ units (d) $\sqrt{7}$ units (v) The nearest position of helicopter from soldier is (a) $(1,\sqrt{5})$ (b) (1,8) (c) (1,7) (d) $(1,\sqrt{7})$	5
3.	A rectangular garden with an area of 60 m^2 is bounded by a straight fence along three sides. The fourth side is a wall of a building. Let *x* be the length of the garden parallel to the wall, and *y* be the width of the garden perpendicular to the wall. If the rate of change of *x* with respect to time is 2 m/s, at what rate is the width *y* changing when the width is 5 m?	5

		
4.	A cylindrical tank with a radius of 6 meters is being filled with water at a rate of 10 cubic meters per minute. How fast is the water level rising when the water is 4 meters deep?	5
5.	Read the following passage and answer the questions given below:  The relation between the height of the plant (y in cm) w.r.t exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight. (i) Find the rate of growth of the plant w.r.t sunlight. (ii) what will be the height of the plant after 2 days and 4 days (iii) Find the minimum number of days it will take for the plant to grow to the maximum, height? **OR** (iii) If the height of the plant is $\frac{7}{2}cm$. Then find the number of days it has been exposed to the sunlight	5
6.	Rama wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square 7piece of cardboard of side 40 cm.  **Based on the above information, answer the following question:** 1. If x cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 40 cm , then possible value of x will be	5

given by the interval

(A) $[0,20]$ (B) $(0,10)$ (C) $(0,20)$ (D) $(0,40)$

2. Volume of the open box formed by folding up the cutting corner can be expressed as

(A) $V = x(20 - 2x)(20 - 2x)$ (B) $V = x(40 - 2x)(40 - 2x)$

(C) $V = x(40 - 4x)(40 - 4x)$ (D) $V = 2x(20 - 2x)(20 - 2x)$

3. The values of x for which $\frac{dV}{dx} = 0$,are

(A) $20, \frac{20}{3}$ (B) $0, \frac{20}{3}$ (C) $0, \frac{10}{3}$ (D) $10, \frac{10}{3}$

4. Rama is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(A) 20 cm (B) $\frac{20}{3}$ cm (C) $\frac{10}{3}$ cm (D) 10 cm

5. The maximum value of the volume is

(A) $\frac{64000}{27} cm^3$ (B) $\frac{128000}{27} cm^3$

(C) $\frac{8000}{27} cm^3$ (D) $\frac{16000}{27} cm^3$

ANSWERS:

Q. NO	ANSWER	MARKS
1.	(i)option (b) x.y = 4 (ii) option (c) C= 280+180(x+$\frac{4}{x}$) (iii) option (d) 2 m (iv) option (c) 2 m (v) option (d) Rs 1000	5
2.	(i)option (b) $y_1 = x_1^2+7$ (ii) option (c) $D^2 = x_1^2-6x_1+9+x_1^4$ (iii) option (c) 8 (iv) option (c) $\sqrt{5}$ units (v) option (b) (1,8)	5
3.	Given: Area of the rectangular garden (*A*) = 60 m^2 Rate of change of *x* with respect to time ($\frac{dx}{dt}$) = 2 m/s Width of the garden (*y*) = 5m We want to find the rate of change of *y* with respect to time ($\frac{dy}{dt}$). The area of the rectangle is given by *A*=xy. Since *A*=60 m^2, we have the equation xy=60. Differentiate both sides of the equation with respect to time *t*: $\frac{\dot{d}}{d\tau}(xy) = \frac{d}{dt}(60)$. Using the product rule for differentiation: $\frac{dx}{dt}y + x\frac{dy}{dt}=0$. Given $\frac{dx}{dt}$=2, *y*=5, and xy=60, we can solve for $\frac{dy}{dt}$: $2\cdot5 + x\frac{dy}{dt}=0$. $10 + 5\frac{dy}{dt}=0$. $\frac{dy}{dt}=-2$m/s. So, when the width *y* is 5 m, the width is changing at a rate of −2 m/s.	
4.	Let *r* be the radius of the cylindrical tank and *h* be the height of the water. Given *r*=6 meters and $\frac{dv}{at}$=10 cubic meters per minute, we want to find $\frac{dh}{dt}$when *h*=4 meters. The volume *V* of a cylinder is $V=\pi r^2 h$. Differentiate *V* with respect to *t*: $\frac{dv}{at}=2\pi rh\frac{dr}{dt}+\pi r^2\frac{dh}{dt}$ Solve for $\frac{dh}{dt}$	

	$$\frac{dh}{dt} = \frac{\frac{dv}{dt} - 2\pi rh\frac{dr}{at}}{\pi r^2}$$ Substitute r=6, $\frac{dv}{at}$=10, h=4, and $\frac{dr}{dt}$=0 (since the radius is constant): $$\frac{dh}{dt} = \frac{10 - 2\pi(6)(4)(0)}{\pi(6)^2}$$ $\frac{dh}{dt}$= $\frac{10}{36\pi}$ So, the water level is rising at a rate of $\frac{10}{36\pi}$ meters per minute when the water is 4 meters deep.	
5.	(i) $\frac{dy}{dx} = 4 - x$ (ii) $y(2) = 4 \times 2 - \frac{1}{2}(2)^2 = 6cm$ $y(4) = 4 \times 4 - \frac{1}{2}(4)^2 = 8cm$ (iii) $\frac{dy}{dx} = 0 \Rightarrow x = 4 \Rightarrow no\ of\ \ days = 4$ and prove y is max at x=4 OR (iii) $\frac{7}{2} = 4x - \frac{1}{2}x^2$ $\Rightarrow x^2 - 8x + 7 = 0 \Rightarrow x = 1, 7, x \neq 7$ As maximum days are 4.	5
6.	1.c 2.b 3.a 4.b 5. b	5