CHAPTER-5

COMPLEX NUMBERS

05 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	If α and β are different complex numbers with |β|= 1 then find $\left\|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\|$	5
2.	Find the real number of x and y if (x-iy) (3+5i) is the conjugate of -6-24i	5
3.	If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$	5
4.	If α and β are different complex numbers with $\|\beta\|$ = 1 ; then find $\left\|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\|$.	5
5.	If $a^2+b^2+c^2=1, b+ic=(1+a)z,$ Prove that $\frac{a+ib}{1+c}=\frac{1+iz}{1-iz}$	5
6.	If z_1 and z_2 are any two complex numbers, then Prove that $(\|z_1\|+\|z_2\|)\left\|\frac{z_1}{\|z_1\|}+\frac{z_2}{\|z_2\|}\right\| \leq 2(\|z_1\|+\|z_2\|)$	5
7.	Solve the equation $z^2+\|z\|=0$, where z is a complex number	5
8.	If a+ib=$\frac{(x+i\}^2}{2x^2+1}$, prove that $a^2+b^2=\frac{(x+1\}^2}{(2x^2+1)^2}$.	5
9.	If α and β are different complex number with $\|\beta\|$=1 , find $\left\|\frac{\beta-\ \alpha}{1-\ \bar{\alpha}\beta}\right\|$	5
10.	If $\|z_1\|=\|\ \|z_2\|\|=$……….$=\|z_n\|=1$, prove that $\|z_1+\ z_2 \ldots.+z_n\| = \left\|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\ \ldots\ldots\ldots+\frac{1}{z_n}\right\|$	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	Let ∝=a+ib,$\beta = x + iy$ It is given that $\lvert\beta\rvert$=1 Thau $\sqrt{x^2 + y^2}$=1 $\Longrightarrow x^2 + y^2$=1 $\left\lvert \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right\rvert = \left\lvert \frac{(x+iy)-(a+ib)}{1-(a+ib)(x+iy)} \right\rvert$ $= \left\lvert \frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)} \right\rvert$ $= \left\lvert \frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)} \right\rvert$ $= \frac{\lvert (x-a)+i(y-b) \rvert}{\lvert (1-ax-by)+i(bx-ay) \rvert}$ $= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-y)^2}}$ $= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$ $= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(x^2+y^2)-2ax-2by}}$ $= \frac{\sqrt{a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}}$ =1 Therefore $\left\lvert \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right\rvert$=1	5
2.	Let z=(x-iy)(3+5i) $\Longrightarrow z = 3x + 5xi - 3yi - 5yi^2$ $= 3x + 5xi - 3yi + 5y$ $= (3x + 5y) + i(5x - 3y)$ $\bar{z}$=$(3x + 5y) - i(5x - 3y)$ It is given that $\bar{z} = -6 - 24i$ Thus,$(3x + 5y) - i(5x - 3y)$=$-6 - 24i$ Equating the real and imaginary parts we obtain, 3x+5y=-6 ______ (1) 5x-3y=24 ______ (2) Multiplying equation (1) by 3 and equation (2) by 5 and then adding them we obtain, 9x+15y=-18 25x-15y=1 34x=102 $\Longrightarrow$ x=3 Putting the value of x in equation (1) we obtain 3(3)+5y=-6 $\Longrightarrow$ 9+5y=-6 $\Longrightarrow$ 5y=-6-9 $\Longrightarrow$ 5y=-15 $\Longrightarrow$ y=-3	5

	Thus the value of x and y are 3 and -3 respectively.	
3.	Given, $(x+iy)^3 = u+iv$ $x^3+(iy)^3+3\cdot x\cdot iy(x+iy) = u+iv$ $x^3+i^3y^3+3x^2yi+3xy^2i^2 = u+iv$ $x^3-iy^3+3x^2yi-3xy^2 = u+iv$ $(x^3-3xy^2)+i(3x^2y-y^3) = u+iv$ On equating real and imaginary parts, we get $u = x^3-3xy^2,\ v = 3x^2y-y^3$ $\frac{u}{x}+\frac{v}{y} = \frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$ $= \frac{x(x^2-3y^2)}{x}+\frac{y(3x^2-y^2)}{y}$ $= x^2-3y^2+3x^2-y^2$ $= 4x^2-4y^2$ $= 4(x^2-y^2)$ $\therefore \frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$ Hence proved.	5

4.	Let $\alpha = a + ib$ and $\beta = x + iy$ It is given that, $\lvert\beta\rvert = 1$ $\therefore \sqrt{x^2+y^2} = 1 \quad \Rightarrow x^2+y^2 = 1 \qquad \ldots (i)$ $\left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert = \left\lvert\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right\rvert = \left\lvert\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right\rvert$ $= \left\lvert\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right\rvert$ $= \frac{\lvert(x-a)+i(y-b)\rvert}{\lvert(1-ax-by)+i(bx-ay)\rvert} \qquad \left[\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}\right]$ $= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$ $= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$ $= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$ $= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \quad [\text{Using } (1)] \quad = 1 \qquad \therefore \left\lvert\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right\rvert = 1$	5
5.	Given $a^2+b^2+c^2 = 1 => b^2+c^2 = 1-a^2$ $\frac{1+iz}{1-iz} = \frac{1+i\frac{b+ic}{1+a}}{1-i\frac{b+ic}{1+a}}$ $= \frac{1+a-c+bi}{1+a-bi+c}$ $= \frac{(1+a-c+bi)(1+a+c+bi)}{(1+a+c-bi)(1+a+c+bi)}$ $= \frac{1+2a+2ib+a^2+2iab-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+2b(1+i)+a^2-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+2b(1+i)+a^2-c^2-b^2}{1+2a+2c+a^2+b^2+c^2+2ac}$ $= \frac{1+2a+a^2-1+a^2+2b(1+a)i}{1+2a+1+2c+2ac}$ $= \frac{a(1+a)+b(1+a)i}{(1+a)+c(1+a)}$	5

<table>
<tr><td></td><td>$= \frac{a+bi}{1+c}$</td><td></td></tr>
<tr><td>6.</td><td>We have,
$(|z_1|+|z_2|)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le (|z_1|+|z_2|)\left(\left|\frac{z_1}{|z_1|}\right|+\left|\frac{z_2}{|z_{2|}}\right|\right)$
$\le (|z_1|+|z_2|)\left(\frac{|z_1|}{|z_1|}+\frac{|z_2|}{|z_2|}\right)$
$\le (|z_1|+|z_2|)$(1+1)
=2$(|z_1|+|z_2|)$

Therefore
$(|z_1|+|z_2|)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2(|z_1|+|z_2|)$</td><td>5</td></tr>
<tr><td>7.</td><td>Let z=x+iy then $z^2+|z|=0$
$\Rightarrow(x+iy)^2+\sqrt{x^2+y^2}=0$
$\Rightarrow(x^2-y^2)+\sqrt{x^2+y^2}+2ixy=0$
$\Rightarrow(x^2-y^2)+\sqrt{x^2+y^2}=0$ ……(1)and 2xy=0…….(2)
⇒ x=0 or y=0
caseI when y=0 we gate x=0 from equation 1
there for z=0
caseII when x=0 aftersolving equation 1 we gate y=-1
therefore z=0+I or , z=0-i
Hence z=0,i and –i are solutions of $z^2+|z|=0$</td><td>5</td></tr>
<tr><td>8.</td><td>We have a+ib=$\frac{(x+i\}^2}{2x^2+1}$ …….1
$\Rightarrow \overline{a+\imath b} = \overline{\frac{(x+\imath\}^2}{2x^2+1}}$

⇒ a-ib=$\frac{(x-i\}^2}{2x^2+1}$……………2
Multiplying 1 and 2, we get
(a+ib) (a-ib)= $\frac{(x+i\}^2}{2x^2+1}$x$\frac{(x-i\}^2}{2x^2+1}$
⇒ $a^2+b^2=\frac{(x+1\}^2}{(2x^2+1)^2}$</td><td>5</td></tr>
<tr><td>9.</td><td>$givn\ \alpha\ and\ \beta$ are different complex number with $|\beta|$=1

$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right)$
$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{\bar{1}-\overline{\bar{\alpha}\beta}}\right)$
$=\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{\bar{1}-\alpha\ \bar{\bar{\beta}}}\right)$</td><td>5</td></tr>
</table>

	$= \left(\frac{\beta\,\overline{\beta} - \overline{\alpha}\beta - \overline{\beta}\alpha + \alpha\,\overline{\alpha}}{1 - \overline{\alpha}\beta - \overline{\beta}\alpha + \overline{\alpha}\beta\,\overline{\beta}\alpha}\right)$ $=1$	
10.	Solution: LHS $\lvert z_1 + z_2 + \cdots . + z_n \rvert = \left\lvert \frac{z_1\overline{z_1}}{\overline{z_1}} + \frac{z_2\overline{z_2}}{\overline{z_2}} + \ldots\ldots\ldots + \frac{z_n\overline{z_n}}{\overline{z_n}} \right\rvert$ $z\bar{z} = \lvert z \rvert^2 = \left\lvert \frac{\lvert z_1\rvert^2}{z_1} + \frac{\lvert z_2\rvert^2}{z_2} + \ldots\ldots\ldots + \frac{\lvert z_n\rvert^2}{\overline{z_n}_n} \right\rvert$ $= \left\lvert \frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \ldots\ldots\ldots + \frac{1}{\overline{z_n}} \right\rvert$	5