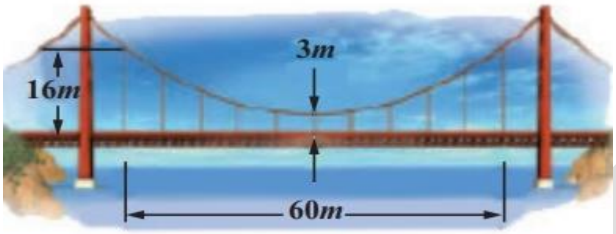
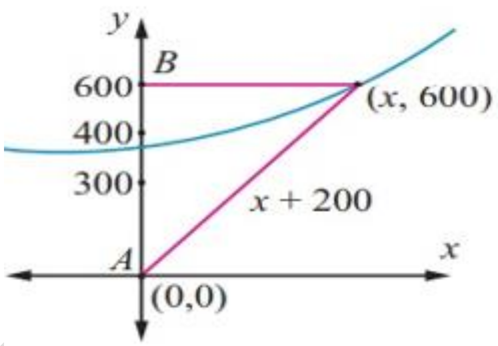
CHAPTER-11

CONIC SECTIONS

05 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.	5
2.	Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6malong this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.  	5
3.	Two coast guard stations are located 600 km apart at points *A*(0, 0) and *B*(0, 600) . A distress signal from a ship at *P* is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station *A* than it is from station *B* . Determine the equation of hyperbola that passes through the location of the ship.  	5
4.	A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.	5
5.	A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?	5
6.	An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.	5
7.	Prove that there is one and only one circle passing through 3 non collinear points. Hence find the equation of the circle passes through (0,0),(a,0) &(0,b). Where a,b≠0	5
8.	The girder of a railway bridge is a parabola with its vertex at the highest point, 10 metres above the ends. Its span is 100 metres	

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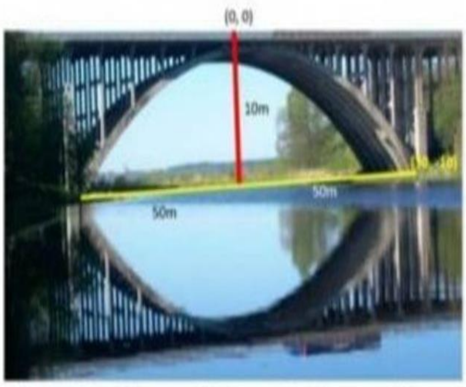

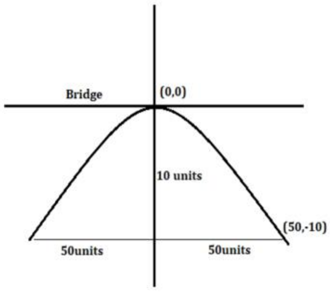

i)Find the co-ordinates of the focus of the parabola.
ii) Find the equation of bridge.
iii) Find the length of latus rectum.
iv) Find the height of the bridge at 20 metres from the mid point.
v)Find the equation of circle with center at focus of the parabola and passes through the vertex of the parabola.
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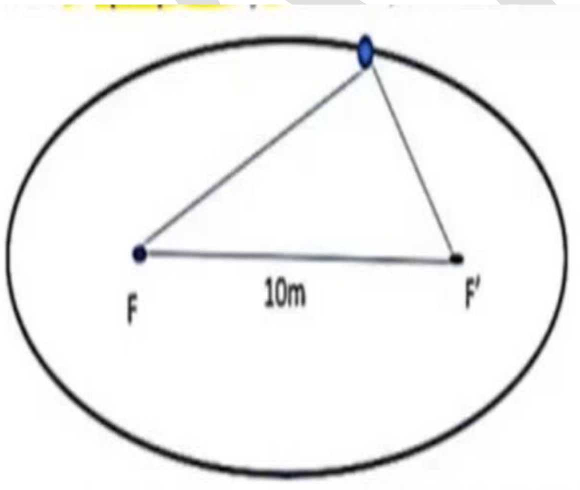
<tr><td>9.</td><td>
A farmer wishes to install 2 handpumps in his field for watering. The farmer moves in the field while watering in such a way that sum of distances between the farmer and each handpump is always 26 meters. Also, the distance between the hand pumps in 10 meters.

I) Curve along which the farmer moves?
ii) Two fixed points where hand pumps are placed
are called?
iii) Find the equation of the curve traced by the farmer?
(iv) find the eccentricity of curve along which the farmer moves.
(v) What is the radius of the circle having centre at (0,√11) and passing through.
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<tr><td>10.</td><td>Determine the equation of the hyperbola which satisfies the given conditions: Foci (0, ±13), the conjugate axis is of length 24.</td><td>5</td></tr>
<tr><td>11.</td><td>Find the coordinates of the foci, vertices, lengths of major and minor axes and the eccentricity of</td><td>5</td></tr>
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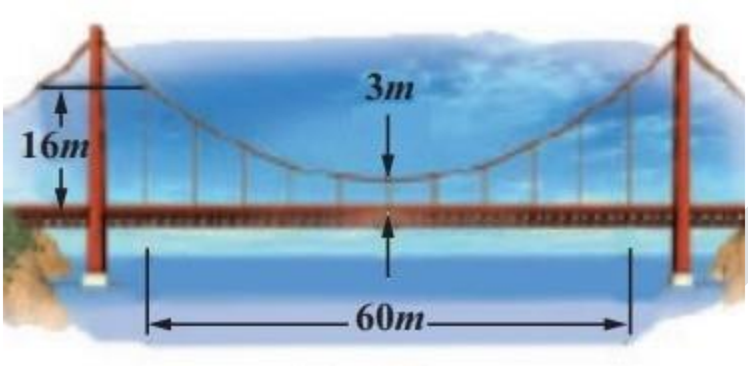
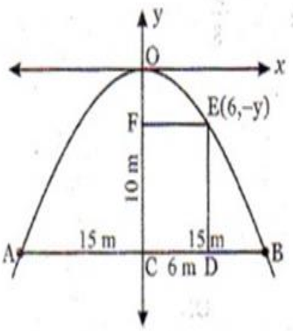
	the ellipse $9x^2 + 4y^2 = 36$.	
12.	Parabolic cable of a 60*m* portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6*m*along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.  	5
13.	Assume that water issuing from the end of a horizontal pipe, 7.5*m* above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	**SOLUTION** Given a bridge has a parabolic arch of 10 m high in the centre and 30 m wide at the bottom. Choose the vertex as the origin and the axis of the parabola as the y - axis. The parabola is open downward. AB - Width of the bottom = 30 m , CO - Height of the parabolic arch at the centre = 10 m. ∴ The coordinate of B is (CB , – OC) , B is (15 , – 10)  By our choice the equation of the parabola is $x^2 = -4ay$ -----(1) B (15 , – 10) is a point on the parabola. ∴ $15^2 = -4a \times -10 \Rightarrow 4a = \frac{15^2}{10}$ The equation of the parabola becomes $x^2 = -\frac{15^2}{10} y$ -----(2) D is a point 6m from the centre. DE is the height of the parabolic arch at a distance of 6m from the centre. Let the coordinates of E be (6 , –y). Substituting in equation (2) we get $6^2 = -\frac{15^2}{10} \times -y \Rightarrow y = \frac{36 \times 10}{225} = \frac{360}{225} = 1.6 \Rightarrow OF = 1.6$ Required height DE = CF = CO – OF ⇒ DE = 10 – 1.6 = 8.4 ∴ The height of the arch at a distance 6m from the centre is 8.4 m.	1+1+1+1+1

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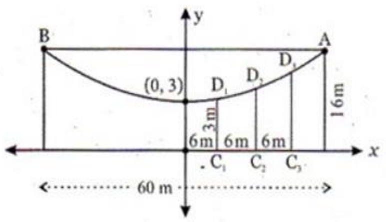
SOLUTION

From the given data the vertex of the parabola is 3m above the x - axis. Choose the axis of the parabola as y – axis. The parabola is open upward with vertex (0 , 3). Hence the equation of the parabola is

$$(x - 0)^2 = 4a(y - 3)$$

$$x^2 = 4a(y - 3) \qquad \text{-----}(1)$$



From the figure A (30 , 16) is a point on the parabola. Substituting in equation (1) we get

$$30^2 = 4a(16 - 3) \Rightarrow \frac{900}{13} = 4a$$

∴ The equation of the parabola becomes $x^2 = \frac{900}{13}(y - 3)$ -----(2)

Given the first vertical cable $C_1 D_1$ is placed at a distance 6 m from the vertex. Therefore the coordinates of D_1 are $D_1(6, y)$. D_1 is a point on the parabola (2)

$$\therefore \quad 6^2 = \frac{900}{13}(y - 3)$$

$$\frac{36 \times 13}{900} = y - 3 \Rightarrow y = \frac{36 \times 13}{900} + 3 = 3.52$$

The height of the first vertical cable $C_1 D_1$ = 3.52 m.

The second vertical cable $C_2 D_2$ is placed at a distance 12 m from the vertiex. Therefore coordinates of D_2 are $(12, x)$. But D_2 is a point on the parabola (2).

Substituting in equation (2) we get

$$12^2 = \frac{900}{13}(y - 3) \Rightarrow \frac{144 \times 13}{900} = y - 3$$

$$\Rightarrow \frac{144 \times 13}{900} + 3 = y \Rightarrow y = 2.08 + 3 = 5.08$$

∴ The height of the second vertical cable is $C_2 D_2$ = 5.08 m.

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Since the center is located at (0, 300) , midway between the two foci ,which are the coast guard stations, the equation is

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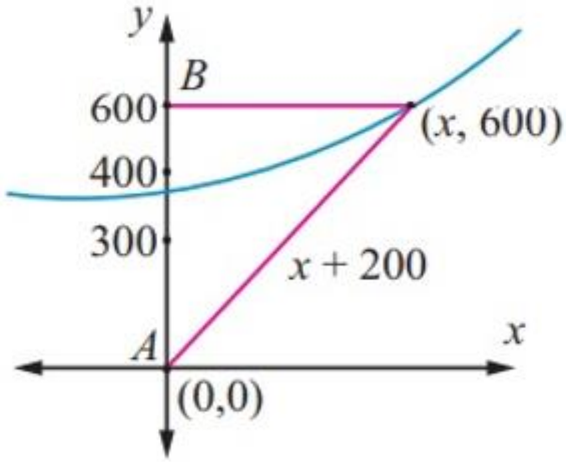
$$\frac{(y-300)^2}{a^2}-\frac{(x-0)^2}{b^2}=1.$$



To determine the values of *a* and *b* , select two points known to be on the hyperbola and substitute each point in the above equation.

The point (0, 400) lies on the hyperbola, since it is 200 km further from Station *A* than from station *B* .

$$\frac{(400-300)^2}{a^2}-\frac{0}{b^2}=1\ \frac{100^2}{a^2}=1,\ a^2=10000.$$

There is also a point $(x, 600)$ on the hyperbola such that $600^2 + x^2 = (x + 200)^2$.

$360000 + x^2 = x^2 + 400x + 40000$

$x = 800$

Substituting in (1), we have $\frac{(600-300)^2}{10000}-\frac{(800-0)^2}{b^2} = 1$

$$9-\frac{640000}{b^2} = 1$$

$$b^2 = 80000$$

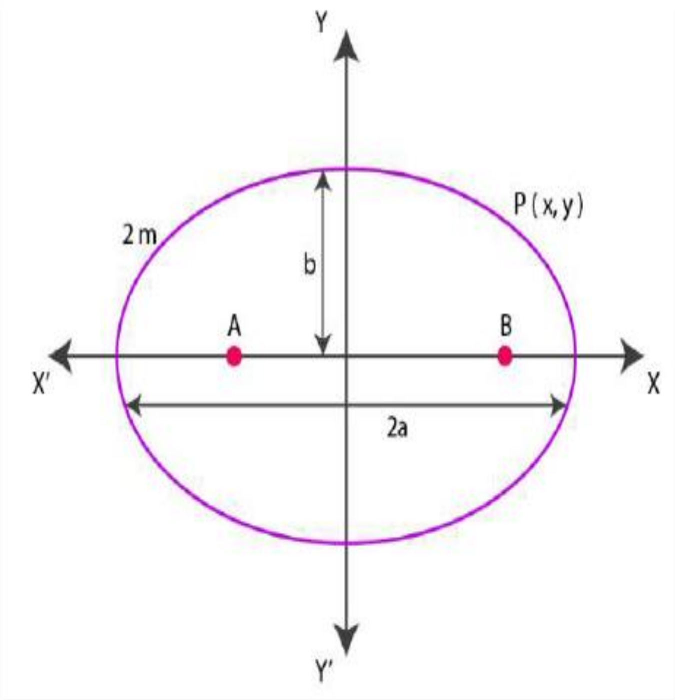
Thus the required equation of the hyperbola is

$$\frac{(y-300)^2}{10000}-\frac{x^2}{80000}=1$$

The ship lies somewhere on this hyperbola. The exact location can be determined using data from a third station.

4.	Let A and B be the positions of the two flag posts and P(x, y) be the position of the man. So, PA + PB = 10.	5

The diagrammatic representation of the ellipse is as follows:



The equation of the ellipse is in the form of $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

So, $2a = 10$

$a = 10/2$

Distance between the foci, $2c = 8$

$c = 8/2$

$= 4$

By using the relation, $c = \sqrt{(a^2 - b^2)}$, we get,

$4 = \sqrt{(25 - b^2)}$

$16 = 25 - b^2$

$b^2 = 25 -1$

$= 9$

$b = 3$

Hence, equation of the path traced by the man is $x^2/25 + y^2/9 = 1$

5.

Solution Let the vertex be at the lowest point and the axis vertical. Let the coordinate axis be chosen as shown in Fig 10.32.

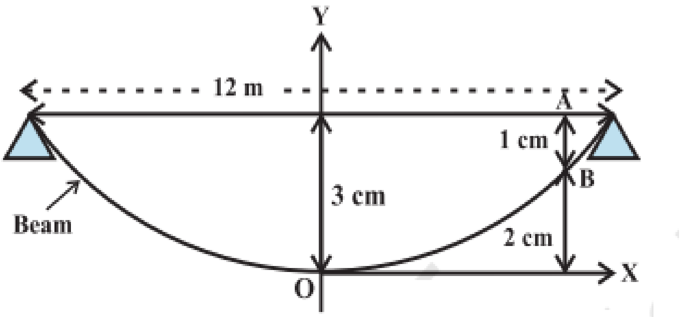


Fig 10.32

The equation of the parabola takes the form $x^2 = 4ay$. Since it passes through $\left(6, \frac{3}{100}\right)$, we have $(6)^2 = 4a\left(\frac{3}{100}\right)$, i.e., $a = \frac{36 \times 100}{12} = 300$ m

Let AB be the deflection of the beam which is $\frac{1}{100}$ m. Coordinates of B are $(x, \frac{2}{100})$.

Therefore $x^2 = 4 \times 300 \times \frac{2}{100} = 24$

i.e. $x = \sqrt{24} = 2\sqrt{6}$ metres

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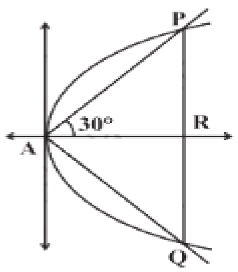
6.

As shown in the figure APQ denotes the equilateral triangle with its equal sides of length l (say).



Here $AP = l$

So $AR = l \cos 30^\circ$

$= l \frac{\sqrt{3}}{2}$

Also, $PR = l \cos 30^\circ = \frac{l}{2}$

Thus $l\frac{\sqrt{3}}{2}, \frac{l}{2}$ are the coordinates of the point P lying on the parabola $y^2 = 4ax$.

Therefore, $\frac{l^2}{4} = 4a\frac{l\sqrt{3}}{2}$

$\Rightarrow l = 8a\sqrt{3}$

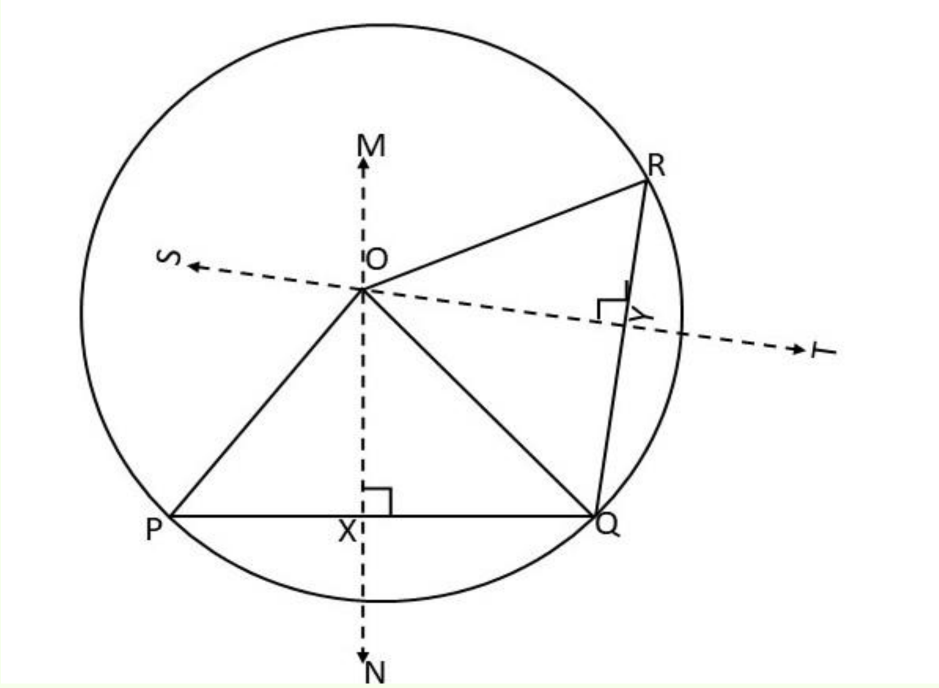
Thus, $8\sqrt{3}$ is the required length of the side of the equilateral triangle inscribed in the parabola $y^2 = 4ax$.

7.

Given : Three non - collinear points, P,Q **and** R.

To prove : There is one and only one circle passing through the points P,Q, **and** R.

Construction: Draw line segments PQ **and** QR. **Draw perpendicular bisectors MN and** ST **of** PQ **and** RQ **respectively. Since** P,Q,R **are not collinear,** MN **is not parallel to** ST **and will intersect, at the point** O. **Join** OP,OQ **and** OR **(Fig).**



Proof :

As O **lies on** MN**, the perpendicular bisector of** PQ**,**

In △OXP **and** △OXQ, **we have**

OX=OX **[Common]**

∠OXP=∠OXQ **[Right angles]**

XP=XQ **[**MN **is the perpendicular bisector]**

∴△OXP≅△OXQ **[By SAS congruence Rule]**

OP=OQ **[CPCT]**

Similary, △OYQ≅△OYR

and OQ=OR **[CPCT]**

∴OP=OQ=OR=r(suppose)

Taking O **as the centre and** r **as the radius, draw a circle** C(O,r) **which will pass through** P,Q**, and** R.

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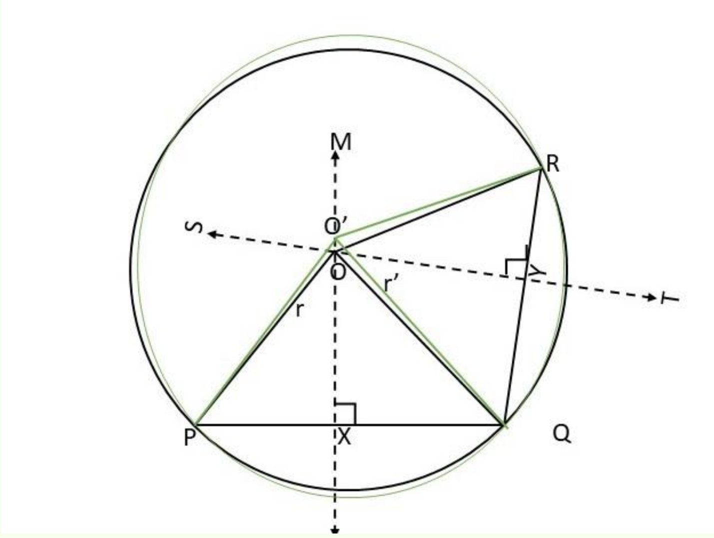

If possible, suppose there is another circle C(O′,r′) **passing through** P,Q **and** R. **Then** O′ **will lie on the perpendicular bisector** MN **of** PQ **and** ST **of** QR.

Since two lines cannot intersect at more than one point, O′ **must coincide with** O.
Since OP=r,O′P=r′.
We have, r=r′
Hence, C(O′,r′)=C(O,r)

Hence, there is one and only one circle passing through the three non-collinear points P,Q **and** R**.**
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i)Bridge form a downward parabola with vertex at (0,0) and passing through (50,-10) let the equation of parabola $x^2 = -4ay$ ....(i)
since is passes through (50,-10), then 50 ^ 2 = - 4a(- 10) hence a = 62.5
then focus is given by (0, - a) = (0, - 62.5)

(ii) Equation of girder of bridge will obtained by substituting the value of a in equation (i)we get x ^ 2 = - 4(62.5)
x ^ 2 = - 250y ,....(ii) which is the required equation.

(iii) Now height of bridge at 20m from mid-point is obtained by putting x = 20 in equation (ii),

20 ^ 2 = - 250y

y = - 400/250 = - 1.6
now height of bridge at 20 m distance from mid point is given by = 10 - 1.6 = 8.4m
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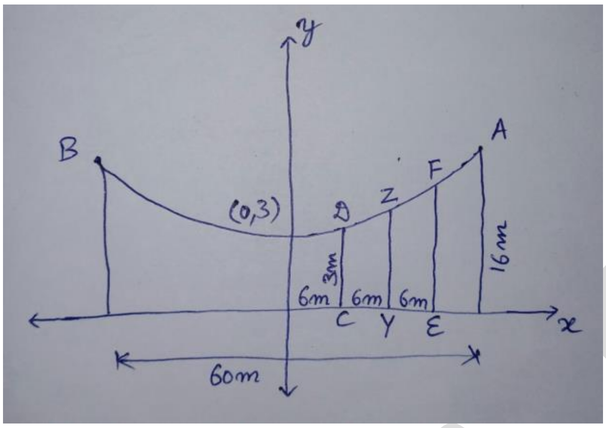
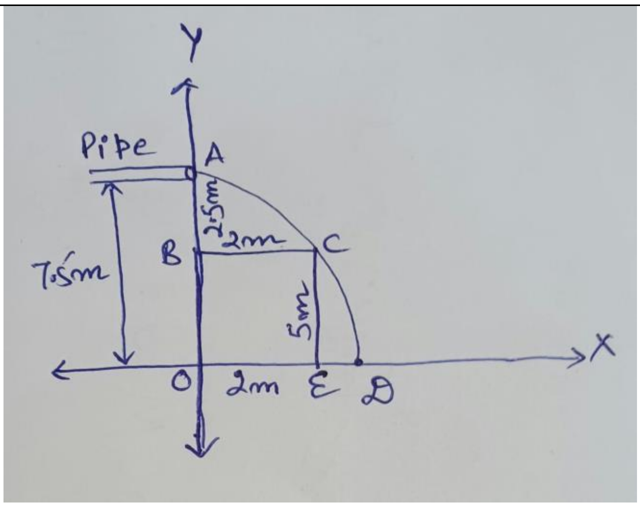
	OR (iv) Centre of circle is (0, -62.5) and radius is given by the distance between (0, -62.5) and (0,0) $r = \sqrt{(0 + 0)^2 + (0 + 62.5)^2} = 62.5$unit equation of circle $(x + 0)^2 + (y + 0)^2 = (62.5)^2$ $x^2 + (62.5)^2 + 2(62.5) \cdot x + y^2 = (62.5)^2$ $x^2 + 125x + y^2 = 0$	
9.	I) A farmer wishes to install 2 handpumps in his field for watering. The farmer moves in the field while watering in such a way that sum of distances between the farmer and each handpump is always 26 meters. Also, the distance between the hand pumps in 10 meters.such shape is ellipse. ii) According to the definition of ellipse, the fixed points are called foci. iii) The fixed distance is given as: $2a = 26m$ $a = 13m$ The distance between foci is given as 10 m. The equation of the curve traced by the farmer can be written as: $(x^2)/(a^2) + (y^2)/(b^2) = 1$ $(x^2)/((13)^2) + (y^2)/((12)^2) = 1$ $(x^2)/169 + (y^2)/144 = 1$. iv)$e = 5/13$ v) Now, as the centre of the circle is given by $(0, \sqrt{11})$ Since foci lie on a circle, to find the distance between the foci and the centre, we have: $r = \sqrt{(25 + 11)}$,$r = 6$	5
10.	Given that: Foci (0, ±13), Conjugate axis length = 24 It is noted that the foci are on the y-axis. Therefore, the equation of the hyperbola is of the form: $(y^2/a^2) - (x^2/b^2) = 1$ ………(1) Since the foci are (0, ±13), we can get C = 13 It is given that, the length of the conjugate axis is 24, It becomes 2b = 24 b= 24/2 b= 12	5

	And, we know that $a^2 + b^2 = c^2$ To find a, substitute the value of b and c in the above equation: $a^2 + 12^2 = 13^2$ $a^2 = 169-144$ $a^2 = 25$ Now, substitute the value of a and b in equation (1), we get $(y^2/25)-(x^2/144) = 1$, which is the required equation of the hyperbola.	
11.	Given, $9x^2 + 4y^2 = 36$. Dividing both sides by 36, we get $$x^2/4 + y^2/9 = 1$$ Observe that the denominator of y^2 is larger than that of x^2. Hence, the major axis is along the y-axis. Now, comparing it with the standard equation, we get, $a^2 = 4$ or $a = 2$ and $b^2 = 9$ or $b = 3$ Also, $c^2 = a^2 – b^2$ Or, $c = \sqrt{(a^2 – b^2)} = \sqrt{(9 – 4)} = \sqrt{5}$ And, $e = c/a = \sqrt{5}/3$ Therefore, • The foci are $(0, \sqrt{5})$ and $(0, – \sqrt{5})$. • Vertices are (0, 3) and (0, – 3) • Length of the major axis = 6 • Length of the minor axis = 4 • Eccentricity = $\sqrt{5}/3$	5
12.	The vertex of the parabola is 3m above the x – axis. Let the axis of the parabola is y-axis The parabola is open upward with vertex (0, 3). Hence the equation of the parabola is $(x – 0)^2 = 4a (y – 3)$ ……(i) From the figure A(30,16) is a point on the parabola. Substituting in equation (i) we get $30^2 = 4a (16 – 3)$ $4a = 900/13$ Therefore equation of the parabola becomes $x^2 = 900/13(y – 3)$……..(2) Given the first vertical cable CD is placed at a distance 6m from the vertex. Therefore the	5

	coordinates of D are (6, y). D is a point of parabola (2) Therefore $6^2 = 900/13\ (y - 3)$ $y = 3.52$ CD = 3.52m The second vertical cable YZ is placed at a distance 12m from the vertex. Therefore coordinates of Y is (12, x). But Y is a point on the parabola (2) Substituting in equation (2) we get $12^2 = 900/13(y - 3)$ $y = 5.08$ YZ = 5.08m 	
13.	Water coming from the horizontal pipe is in parabolic path and 7.5 m above the ground. Vertex of parabola is at the mouth of the pipe A. Therefore, the coordinates of the vertex is A (0, 7.5) The parabola is open downward with axis along y-axis Therefore, the equation of the parabola is $(x - 0)^2 = -4a(y - 7.5)$ $x^2 = -4a\ (y - 7.5)$ B is a point 2.5m below the pipe at which the distance of the path of the water flow is 3m. AB = 2.5m BC = 3m OA = 7.5m, OB = 5m Hence the coordinates of C is (3, 5), which is on the parabola. Therefore $3^2 = -4a\ (5 - 7.5)$ Which implies $4a = 9/25$ Therefore the equation of the parabola becomes $x^2 = -9/2.5\ (y - 7.5)$ Put y = 0 to find the distance at which strikes the ground. $x^2 = -9/2.5\ (0 - 7.5)$ $x = 3\sqrt{3}$ Therefore water strikes the ground at a distance $3\sqrt{3}\ m$ beyond the vertical.	5

	Y Pipe A 2.5m B 2m C 7.5m 5m X O 2m E D	