CHAPTER-5

CONTINUITY & DIFFERENTIABILITY

05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	For what value of k is the following function continuous at $= -\frac{\pi}{6}$? $f(x) = \{\frac{x + \cos \cos x}{x + \frac{\pi}{6}}, \quad if\ x \neq -\frac{\pi}{6}\ k \quad , \quad if\ x = -\frac{\pi}{6}$	5
2.	Find $\frac{dy}{dx}$, if $y = (\tan \tan x)^{\cot x} + (\cot \cot x)^{tantan\, x}$	5
3.	Find $\frac{dy}{dx}$, if $y = e^{(\sin\theta)^2}\ (\ 2.\tan^{-1}\sqrt{\frac{1-x}{1+x}}\ )$.	5
4.	$y = x^{x^{x^{x......\infty}}}$ then prove that $x.\frac{dy}{dx} = \frac{y^2}{1-y\log x}$	5
5.	Find the values of a and b so that the function $f(x) = x^2 + 3x + a \quad , \text{ if } x \leq 1$ $bx + 2 \quad , \text{ if } x > 1$ is differentiable at x = 1	
6.	If x = sin t and y = sin pt , prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$	
7.	Find $\frac{dy}{dx}, if\ y = e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$.	
8.	If x cos(a+y) =cos y, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence, show that $\sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$.	
9.	For what value of k is the following function continuous at $= -\frac{\pi}{6}$? $f(x) = \begin{cases} \frac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}, & if\ x \neq -\frac{\pi}{6} \\ k, & if\ x = -\frac{\pi}{6} \end{cases}$	5
10.	Find $\frac{dy}{dx}$, if $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$	5
11.	If x = cost(3-2cos^{2}t) and y = sint(3-2sin^{2}t), then find the value of $\frac{dy}{dx}$ at t = $\frac{\pi}{4}$	5
12.	If y= log(x + $\sqrt{x^2 + a^2}$), then show that $(x^2 + a^2)\frac{d^2y}{dx^2}$ + x$\frac{dy}{dx}$ = 0	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\dfrac{2\left(\frac{\sqrt{3}}{2} sin\, sin\, x + cos\, cos\, x \,.\, \frac{1}{2}\right)}{\left(x+\frac{\pi}{6}\right)} = \dfrac{2\left(cos\, cos\, \frac{\pi}{6}\ sin\, sin\, x + cos\, cos\, x\ \ sin\frac{\pi}{6}\right)}{\left(x+\frac{\pi}{6}\right)}$ $= \dfrac{2sin\left(x+\frac{\pi}{6}\right)}{\left(x+\frac{\pi}{6}\right)} = 2 \quad \Rightarrow\ k = 2$	5
2.	$\dfrac{dy}{dx} = (tan\, tan\, x\,)^{cot\, x} cosec^2 x(1 - log\, log\ \text{□□□}\ tan\, x\ )$ $+ (cot\, cot\, x\,)^{tantan\, x}\ sec^2 x(log\, log\ \ cot\, cot\, x\ - 1\,)$	5
3.	$e^{(sin\theta)^2}\ cos^{-1} x\ \left[sin2x - \frac{1}{cos^{-1} x\sqrt{1-x^2}}\right]$	5
4.	Correct proof should be there without step missing	5
5.	a = 3, b = 5	5
6.	dx/dt = cos t , dy/dt = p cos pt dy/dx = p cos pt / cos t $\cos t \frac{dy}{dx} = p \cos pt$ $\cos t \frac{d^2y}{dx^2} + \frac{dy}{dx}(-\sin t)\frac{dt}{dx} = -p^2 \sin pt \frac{dt}{dx}$ $\cos^2 t \frac{d^2y}{dx^2} + \frac{dy}{dx}(-\sin t) = -p^2 \sin pt$ $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$	5
7.	Putting $x = \cos 2\theta$ in $\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$, we get $2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ *i.e.,* $2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$ $= 2\tan^{-1}(\tan\theta)$ $= 2\theta = \cos^{-1}x$ Hence, $y = e^{\sin^2 x}\cos^{-1}x$ or $\log y = \sin^2 x + \log(\cos^{-1}x)$ or $\frac{1}{y}\times\frac{dy}{dx} = 2\sin x\cos x + \frac{1}{\cos^{-1}x}\times\frac{-1}{\sqrt{1-x^2}}$ $= \sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}$ or $\frac{dy}{dx} = e^{\sin^2 x}\cos^{-1}x\left[\sin 2x - \frac{1}{\ldots}\right]$	5

8.	Given, $x\cos(a+y)=\cos y$ or $x=\dfrac{\cos y}{\cos(a+y)}$ On differentiating both sides *w.r.t.* y, we get $\dfrac{dx}{dy}=\dfrac{\cos(a+y)\dfrac{d}{dy}\cos y-\cos y\dfrac{d}{dy}\cos(a+y)}{\cos^2(a+y)}$ **1** [By using quotient rule of derivative] $=\dfrac{\cos(a+y)\times(-\sin y)+\cos y\times\sin(a+y)}{\cos^2(a+y)}$ ½ $=\dfrac{\sin(a+y)\cos y-\cos(a+y)\sin y}{\cos^2(a+y)}$ or $\dfrac{dx}{dy}=\dfrac{\sin(a+y-y)}{\cos^2(a+y)}=\dfrac{\sin a}{\cos^2(a+y)}$ $[\because \sin A\cos B-\cos A\sin B=\sin(A-B)]$ or $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$ ...(i) ½ Again, on differentiating both sides of Eq. (i) *w.r.t.* x, we get $\dfrac{d^2y}{dx^2}=\dfrac{1}{\sin a}\dfrac{d}{dx}\cos^2(a+y)$ $=\dfrac{1}{\sin a}\times\dfrac{d}{dy}\cos^2(a+y)\times\dfrac{dy}{dx}$ **1** $=\dfrac{1}{\sin a}\times 2\cos(a+y)$ $[-\sin(a+y)]\times\dfrac{dy}{dx}$ $=-\dfrac{2\sin(a+y)\cos(a+y)}{\sin a}\times\dfrac{dy}{dx}$ **1** $\therefore \dfrac{d^2y}{dx^2}=\dfrac{-\sin 2(a+y)}{\sin a}\dfrac{dy}{dx}$ $\therefore \sin a\dfrac{d^2y}{dx^2}+\sin 2(a+y)\dfrac{dy}{dx}=0$	5

9.	$$\lim_{x\to-\frac{\pi}{6}} \frac{2\left(\frac{\sqrt{3}}{2}\sin x + \cos x \,.\frac{1}{2}\right)}{\left(x + \frac{\pi}{6}\right)} = \lim_{x\to-\frac{\pi}{6}} \frac{2\left(\cos\frac{\pi}{6}\sin x + \cos x \;\; sin\frac{\pi}{6}\right)}{\left(x + \frac{\pi}{6}\right)}$$ $$= \lim_{x\to-\frac{\pi}{6}} \frac{2sin\left(x+\frac{\pi}{6}\right)}{\left(x + \frac{\pi}{6}\right)} = 2 \quad \Rightarrow \; k = 2$$	5
10.	$$\frac{dy}{dx} = (\tan x)^{cot\,x} cosec^2 x(1 - \log\tan x) + (\cot x)^{\tan x} sec^2 x(\log\cot x - 1)$$	5
11.	Given that: x = cost(3-2cos^{2}t) on differentiating w.r.t 't' we get dx/dt = d/dt(cost(3-2cos^{2}t)) = cost(4costsint) –sint(3-2cos^{2}t) = 4cos^{2}tsint-3sint + 2cos^{2}tsint = 6cos^{2}tsint – 3sint..................(i) Now, y = sint(3-2sin^{2}t) on differentiating w.r.t 't' we get dy/dt = d/dt(sint(3-2sin^{2}t) = sint(-4sintcost) + cost(3 – 2sin^{2}t) = -4sin^{2}tcost + 3cost – 2sin^{2}tcost = 3cost – 6sin^{2}tcost................(ii) eq. (ii) / (i) dy/dx = $\frac{3cost-6sin^2tcost}{6cos^2tsint-3sint}$ = $\frac{3cost(1-2sin^2t)}{3sint(2cos^2t-1)}$ = $\frac{3costcos2t}{3sintcos2t}$ = cott at t = $\frac{\pi}{4}$ dy/dx = cot$\frac{\pi}{4}$ = 1	5
12.	Given, $y = log[x + \sqrt{(x^2+a^2)}]$ Differentiating both side w.r.t x, we get, $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)\left[\because \frac{d}{dx}(logx) = \frac{1}{x}d(x)\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{2x}{2\sqrt{x^2+a^2}}\right)\left[\because \frac{d}{dx}\left(\sqrt{x^2=a^2}\right) = \frac{2x}{2\sqrt{x^2+a^2}}\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right)$ $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$ $\Rightarrow \frac{dy}{dx}(\sqrt{x^2+a^2}) = 1$	5

	Again differentiating both side w.r.t x, we get $\Rightarrow \left(\sqrt{x^2+a^2}\right)\frac{d}{dx}\left(\frac{dy}{dx}\right)+\frac{dy}{dx}\frac{d}{dx}\left(\sqrt{x^2+a^2}\right)=\frac{d(1)}{dx}$ $\Rightarrow \left(\sqrt{x^2+a^2}\right)\frac{d^2y}{dx^2}+\frac{2x\frac{dy}{dx}}{2\sqrt{x^2+a^2}}=0\left[\because \frac{d}{dx}\left(\sqrt{x^2+a^2}\right)=\frac{2x}{2\sqrt{x^2+a^2}}\right]$ $\Rightarrow \left(\sqrt{x^2+a^2}\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{x}{\sqrt{x^2+a^2}}=0\ \left[multiplying\ \sqrt{x^2+a^2}\ both\ side\right]$ Hence, Proved	