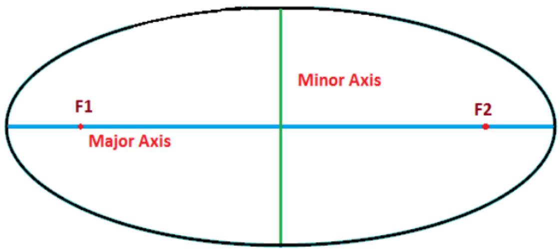
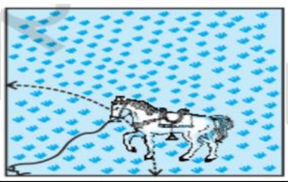
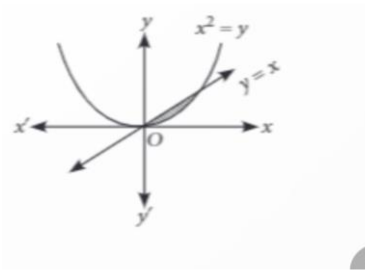
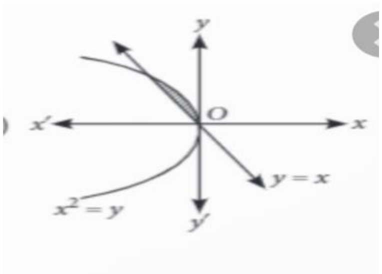
CHAPTER-8

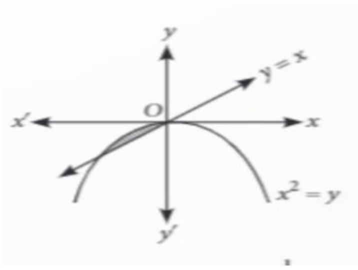
APPLICATION OF INTEGRALS

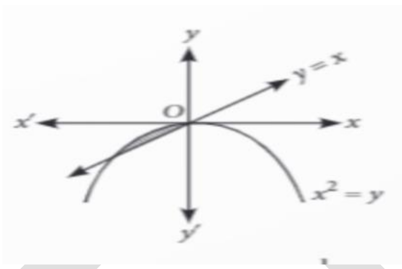
05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the area of the circle $x^2+y^2 =16$ exterior to the parabola.	5
2.	Find the area of the region lying above X-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.	5
3.	Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.	5
4.	Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$	5
5.	Using integration, find the area of region bounded by the triangle whose vertices are (-1, 0),(1, 3) and (3, 2).	5
6.	Show that the area cut off by a parabola in first quadrant and ordinate is one third of the corresponding rectangle formed by that ordinate and its distance from the vertex using integration.	5
7.	Find the area of the region bounded by the curve y = tan x, tangent to the curve at point at x = π/4 and the x-axis using integration.	5
8.	A particle is moving as a elliptical curve, whose horizontally maximum distance is 8 km and vertically maximum distance is 6 km.  Then find the area covered by the particle.	5
9.	**A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope (see Fig.).** **Find the area of that part of the field in which the horse can graze by using integration.** 	5
10.	Consider the following equation of curves $x^2 = y \ and \ y = x$ On the basis of above information, answer the following questions (i) The point(s) of intersection of both the curves is (are) (a) $(0,0), (2,2)$ (b) $(0,0), (1,1)$ (c) $(0,0), (-2,-2)$ (d) $(0,0), (-1,-1)$ (ii) Area bounded by the curves is represented by which of the following graphs?	5

(a) 

(b) 

(c) 

(d) 

(iii) The value of the integral $\int_0^1 x\,dx$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(iv) The value of the integral $\int_0^1 x^2\,dx$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(v) The value of area bounded by the curves $x^2 = y\ and\ y = x$ is

(a) $\frac{1}{6}sq.unit$ (b) $\frac{1}{3}sq.unit$

(c) $\frac{1}{2}sq.unit$ (d) $1sq.unit$

11. Location of three branches of a bank is representedby the three points $A(-2,0), B(1,4) and\ c(2,3)$ asshown in figure. **5**

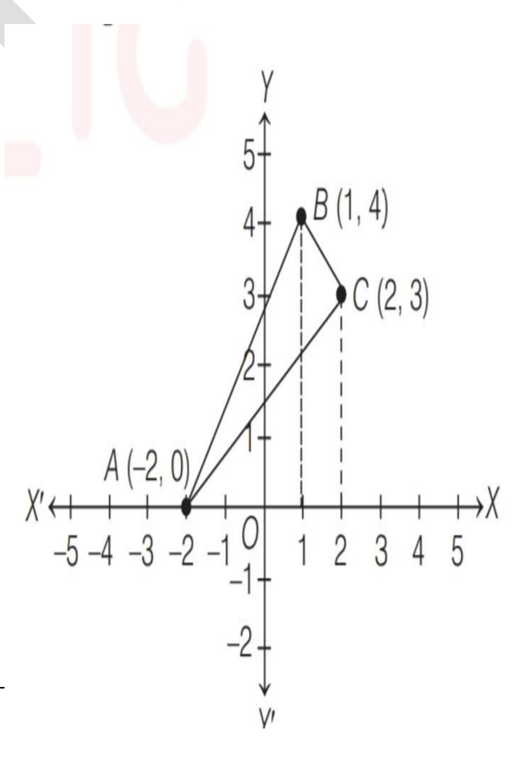
(i) Equation of line AB is

(a) $y = \frac{4}{3}(x+2)$

(b) $y = \frac{4}{3}(x+1)$

(c) $y = \frac{4}{5}(x+2)$

(d) $y = \frac{4}{5}(x+1)$



(ii) Equation of line BC is

(a) $y = x + 5$

(b) $y = -x + 5$

(c) $y = x + 4$

(d) $y = -x + 4$

(iii)Area of region ABCD is4 $sq\ units$

(a) $19\ sq\ units$

(b) $\frac{19}{2}\ sq\ units$

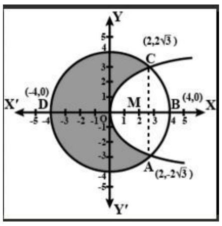
(c) $17\ sq\ units$

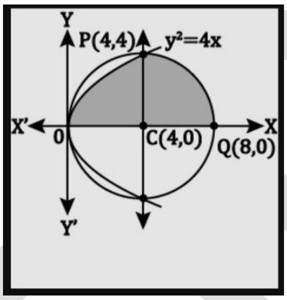
(d) $6\ sq\ units$

(iv)Area of $\Delta ADC$is

	(a) 3 $sq\ units$ (b) 4 $sq\ units$ (c) 6 $sq\ units$ (d) 5 $sq\ units$ (v) Area of ΔABC is (a) 7 $sq\ units$ (b) $\frac{3}{2}$ $sq\ units$ (c) 5 $sq\ units$ (d) $\frac{7}{2}$ $sq\ units$	

ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>

The given equation are $x^2 + y^2 = 16$ and $y^2 = 6x$
Area bounded by the circle and the parabola =
= 2[area (OADO) + area(ADBA)]
$=2 [\int_0^2 \sqrt{6x}\, dx + \int_2^4 \sqrt{16 - x^2}\, dx]$
by finding the integration we will get $\frac{4}{3}[4\pi + \sqrt{3}]$ sq.unit.
area of the circle $\pi r^2 = \pi 4^2 = 16\pi$
required Area = $16\pi - \frac{4}{3}[4\pi + \sqrt{3}]$
$= \frac{4}{3}[8\pi - \sqrt{3}]$ Ans.</td><td>5</td></tr>
<tr><td>2.</td><td>

Given:
$Y^2=4x$
x2+y2=8x
⇒x2−8x+y2=0
⇒x2−2×4×x+y2=0
⇒x2−2×4×x+42−42+y2=0
⇒(x−4)2+y2=42

So, circle has centre 4,0 & radius=4
Equation of circle is
x2+y2=8x
Putting y2=4x
⇒x2+4x=8x
⇒x2−4x=0
⇒x(x−4)=0
⇒x=0 & x=4

For x=0</td><td>5</td></tr>
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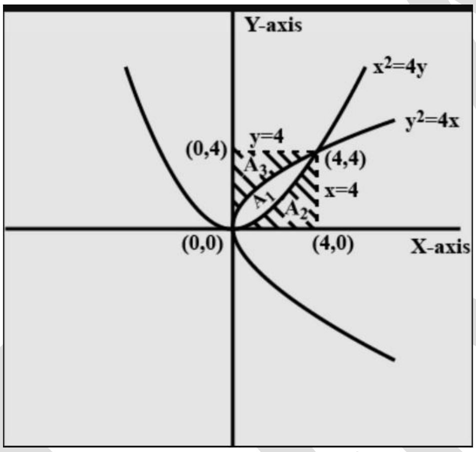
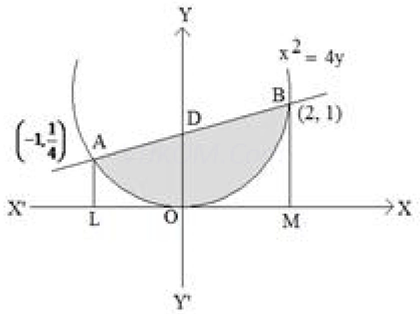
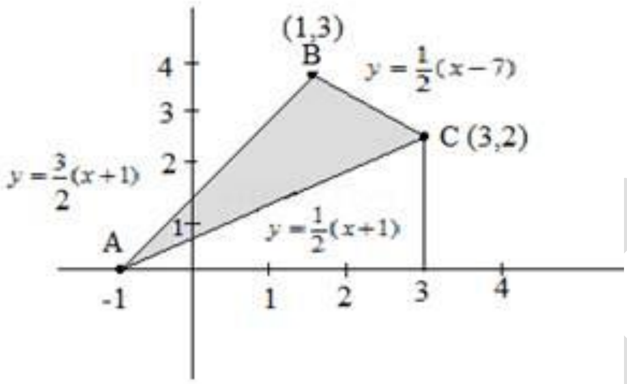
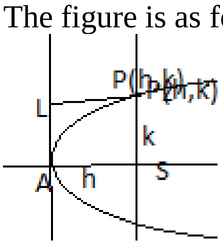
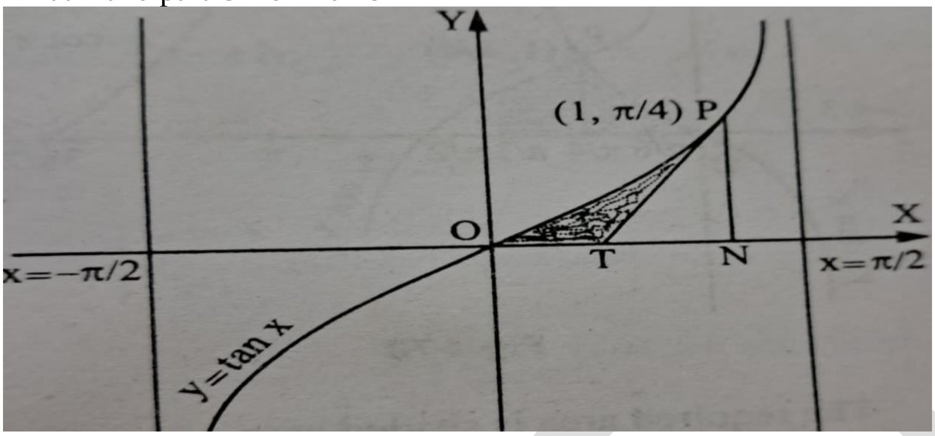
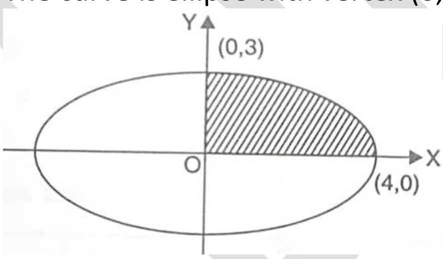
	Y^2 =4x =y =0 Point is (0,0) For x=4 Y^2 =4x =16 Y =±4 So, point is (4,4) And (4,−4) Since, point P is in 1st quadrant, so, the coordinates of P = (4,4) Equation of the curves in first quadrant is Parabola: y=2√x Circle: (x−4)2+y2=42 ⇒y=√42−(x−4)2 Area Required=AreaOPCO+AreaPCQP =∫40y1 dx+∫84y2dx where y1=2√x and y2=√42−(x−4)2 so by solving we gwt the required area $\frac{4}{3}(8+3\pi)$. ANS.	
3.	 To prove A1= A2= A3 = $\frac{A}{3}$ whetre A is the area of the square. A=4*4= 16 sq. unit A1 = $\int_0^4 \frac{x^2}{4}dx = \frac{16}{3}$ sq. unit A2 = $\int_0^4 \frac{y^2}{4}dy = \frac{16}{3}$ sq. unit A3= A - (A1+A2) =16- 2*$\frac{16}{3}$ =$\frac{16}{3}$ sq. unit So A1=A2=A3=$\frac{16}{3}$ sq. unit.	5
4.	$x^2 = 4y$ -------(1) $x = 4y - 2$ -----(2)	5

	 Req. area = $\int_{-1}^{2}\frac{1}{4}(x+2)\,dx-\frac{1}{4}\int_{-1}^{2}x^2\,dx$ $=\frac{9}{8}\,sq\ unit$	
5.	 A (-1, 0) B (1, 3) C (3, 2) Equation of AB $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ $y-0=\frac{3-0}{1+1}(x+1)$ $y=\frac{3}{2}(x+1)$ Similarly Equation of BC $y=\frac{-1}{2}(x-7)$ Equation of AC $=\frac{1}{2}(x+1)$ $Area\ \Delta ABC=\int_{-1}^{1}\frac{3}{2}(x+1)\,dx+\int_{1}^{3}\frac{1}{2}(x-7)\,dx$ $-\int_{-1}^{3}\frac{1}{2}(x+1)\,dx$ $=4\,sq.\ unit$	5
6.	The figure is as follows  The equation of parabola is $y^2 = 4ax$ Let ordinate be drawn through P(h,k)	5

	Then $k^2 = 4ah$ or $k=2\sqrt{(ah)}$ So area of triangle = k.h = $2h\sqrt{(ah)}$ Now area of triangle = $\int_0^h y\,dx = \int_0^h 2\sqrt{a\sqrt{x}dx} = \frac{1}{3}(2h\sqrt{ah})$ = one third of area of triangle	
7.	The required area is as shown in shaded portion bounded by curve y = tan x , tangent PT at P and part OT of x-axis  Now $\frac{dy}{dx} = sec^2 x = 2$ at x = π/4 Also y = tan π/4 = 1 at point P(π/4 , 1) Equation of tangent is y – 1 = 2(x – π/4) y = 2x + 1 -π/2 When y = 0 then x = π/2 – ½ = OT So, TN = ON – OT = π/4 - π/4 +1/2 = ½ Required area is given by Area OPNO - Area of ΔPTN $= \int_o^{\pi/4} \tan x\, dx - \frac{1}{2}\, TN.PN$ = log√2 – ¼ = 1/2 (log2 – ½)	5
8.	Equation of the curve is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ The curve is ellipse with vertex (0, 0)  The area of the region bounded by the given ellipse = 4 × Area of the ellipse in the first quadrant Reqd. area = 4 $\int_0^4 y\,dx$ = 4 $\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\; dx$ = 12 π sq. units	5
9.	Equation of the curve is $x^2 + y^2 = 25$ Reqd. area = area of the circle in the first quadrant. = $\int_0^5 y\,dx$ = $\int_0^5 \sqrt{25 - x^2}\; dx$	5

		$=\frac{25\pi}{4}$ sq. units	
10.	(i)	(b)We have $x^2 = y \dots\dots (1) and \quad x = y \dots\dots..(2)$ From eq (1) and (2), $x^2 = x \Rightarrow x^2 - x = 0$ $\Rightarrow x(x-1) = 0 \Rightarrow x = 0,1$ $from\ Eq.(2) y = 0,1$ ∴Required points of intersection are (0,0) ,(1,1).	5
	(ii)	(a)	
	(iii)	(c) $\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$	
	(iv)	(b) $\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$	
	(v)	(a) Required area $=\int_0^1 x dx - \int_0^1 x^2 dx$ $=\frac{1}{2} - \frac{1}{3} = \frac{1}{6} sq\ units$	
11.	(i)	(a) Equation of line AB is given by $(y-0) = \frac{4-0}{1+2}(x+2) \Rightarrow y = \frac{4}{3}(x+2)$	5
	(ii)	(b) Equation of line BC is given by $(y-4) = \frac{3-4}{2-1}(x-1) \Rightarrow y = -x+5$	
	(iii)	(b) Area of the region ABCD = Area of ΔABE + Area of region BCDE $=\int_{-2}^{1} \frac{4}{3}(x+2)dx \int_1^2 (-x+5)dx$ $=\frac{4}{3}\left[\frac{x^2}{2} + 2x\right]_{-2}^{1} + \left[-\frac{x^2}{2} + 5x\right]_1^2$ $=\frac{4}{3}\left[\frac{1}{2} + 2 - 2 - 2 + 4\right] + \left[-2 + 10 + \frac{1}{2} - 5\right]$ $=\frac{4}{3}.\frac{9}{2} + \left(\frac{1}{2} + 3\right)$ $=6 + \frac{7}{2} = \frac{19}{2}\ sq\ units$	
	(iv)	(c) Equation of line AC is given by $(y-0) = \frac{3-0}{2+2}(x+2) \Rightarrow y = \frac{3}{4}(x+2)$ Area of $\Delta ADC = \int_{-2}^{2}\left(\frac{3}{4}(x+2)\right)dx$ $=\frac{3}{4}\left[\frac{x^2}{2} + 2x\right]_{-2}^{2} \Rightarrow \frac{3}{4}(2+4-2+4) \Rightarrow \frac{3}{4}.8 = 6\ sq\ units$	
	(v)	(d) Area of $\Delta ABC = (iii) - (iv)$ $= \frac{19}{2} - 6 = \frac{7}{2} sq\ units$	