CHAPTER-8

BINOMIAL THEOREMS

05 MARK TYPE QUESTIONS

Q. NO	**QUESTION**	**MARK**
1.	Find $(a+b)^4-(a-b)^4$. Hence evaluate $$(\sqrt{3}+\sqrt{2})^4+(\sqrt{3}-\sqrt{2})^4$$	5
2.	Show that $9^{n+1}-8n-9$ is divisible by 64 , whenever n is positive integer.	5
3.	Consider the binomial theorem for positive integral index If x and a are real numbers, then $\forall\ n \in N$ $(x+a)^n = n_{C_0}x^n a^0 + n_{C_1}x^{n-1}a^1 + \cdots + n_{C_{n-1}}x^1 a^{n-1} + n_{C_n}x^0 a^n$ $(x+a)^n = \sum_{r=0}^{n} n_{C_r}\, x^{n-r} a^r$ Here, $n_{C_r} = n_{C_{n-r}}$ Putting x=1 and a=x, we get $(1+x)^n = n_{C_0} + n_{C_1}x^1 + \cdots + n_{C_{n-1}}x^{n-1} + n_{C_n}x^n$ Based on the above information answer the following questions i. Find the sum of coefficients of second term from beginning and fourth term from the end of the expansion of $\left(\sqrt[3]{x}-\sqrt[3]{a}\right)^7$ ii. If n is a positive integer, prove that $3^{3n}-26n-1$ is divisible by 676	5
4.	A student applies binomial theorem and deduces that $(x+a)^n+(x-a)^n = 2\left(n_{C_0}x^n a^0 + n_{C_2}x^{n-2}a^2 + n_{C_4}x^{n-4}a^4 ..\right)$ $(x+a)^n-(x-a)^n = 2\left(n_{C_1}x^{n-1}a^1 + n_{C_3}x^{n-3}a^3 + n_{C_5}x^{n-5}a^5 ..\right)$ Based on the above information, answer the following i. If the student has deduced right, find the value of $\left(1+2\sqrt{x}\right)^5 + \left(1-2\sqrt{x}\right)^5$ ii. Find the sum of number of digits in the expansions $\left(\sqrt{a}+\sqrt{b}\right)^6 + \left(\sqrt{a}+\sqrt{b}\right)^6$ and $\left(\left(x+\sqrt{y}\right)^7-\left(x-\sqrt{y}\right)\right)^7$ iii. With the help of above deduction evaluate $(0.99)^5+(1.01)^5$	5
5.	If O is the sum of odd terms and E is the sum of even terms in the expansion of $(x+a)^n$,then prove that $O^2-E^2=(x^2-a^2)^n$	5
6.	Find $(x+y)^5+(x-y)^5$. Hence, evaluate $(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$	5
7.	Using binomial theorem , prove that 6^n-5n always leaves remainder 1 when divided by 25	5
8.	If a and b are distinct integers , prove that a^n-b^n is divisible by a- b whenever $n \in N$	5
9.	Find the term independent of x in the expansion of $(2x^2+x+1)\left[\left(\frac{3x^2}{2}-\frac{1}{3x}\right)\right]^9$.	5

10.	Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290, and 30375 respectively .	5
11.	The sum of the coefficients of the first three terms in the expansion of $(x- 3/x^2)^m$ is 559, where m is the natural number . Determine the coefficient of expansion containing x^3.	5
12.	Find the value of r, If the coefficients of (r – 5)th and (2r – 1)th terms in the expansion of $(1 + x)^{34}$ are equal.	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$(a+b)^4-(a-b)^4$ $= 2({}^4C_1 a^3 b + {}^4C_3 ab^3)$ $= 2(4a^3b+4ab^3)$ $= 8ab(a^2+b^2)$ $a=\sqrt{3} \quad b=\sqrt{2}$ $= 8\sqrt{3}\times\sqrt{2}[(\sqrt{3})^2+(\sqrt{2})^2]$ $= 40\sqrt{6}.$	5
2.	$9^{n+1} = (1+8)^{n+1}$ $= 1 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 8^2 + \cdots + {}^{n+1}C_{n+1} 8^{n+1}$ $= 1 + (n+1)8 + 8^2[{}^{n+1}c_2 + \cdots + 8^{n-1}]$ $9^{n+1} - 8n - 9 = 64[{}^{n+1}c_2 + {}^{n+1}c_3 + \cdots + 8^{n-1}]$ $= 64k$	5
3.	i) $T_2 + T_5 = 7_{C_1} + 7_{C_4} = 7 + 35 = 42$ ii) $3^{3n} = 27^n$ $27^n = (1+26)^n = n_{C_0} + n_{C_1}26^1 + n_{C_2}26^2 \ldots + n_{C_{n-1}}26^{n-1} + n_{C_n}26^n$ $3^{3n} = 1 + 26n + 26^2\left(n_{C_2} + \cdots + n_{C_{n-1}}26^{n-3} + n_{C_n}26^{n-2}\right)$ $3^{3n} - 26n - 1 = 26^2k, k \in Z$ $3^{3n} - 26n - 1$ is divisible by 676	5
4.	i) $\left(1+2\sqrt{x}\right)^5 + \left(1-2\sqrt{x}\right)^5$ $= 2\left(5_{C_0}\left(2\sqrt{x}\right)^0 + 5_{C_2}\left(2\sqrt{x}\right)^2 + 5_{C_4}\left(2\sqrt{x}\right)^4\right)$ $= 2(1+40x+80x^2)$ ii)The number of digits in expansion of $\left(\sqrt{a}+\sqrt{b}\right)^6 + \left(\sqrt{a}+\sqrt{b}\right)^6$ is $\left(\frac{6}{2}+1\right)$terms= 4 terms The number of digits in expansion of $\left(\left(x+\sqrt{y}\right)^7 - \left(x-\sqrt{y}\right)\right)$is $\frac{7+1}{2}$ terms= 4 terms Sum=8 iii) $(0.99)^5 + (1.01)^5 = (1-0.01)^5 + (1+0.01)^5$ $= (1+0.01)^5 + (1-0.01)^5$ $= 2\left(5_{C_0}(0.01)^0 + 5_{C_2}(0.01)^2 + 5_{C_4}(0.01)^4\right)$ $= 2(1 + 10\times 0.0001 + 5\times 0.00000001)$ $= 2(1 + 0.001 + 0.00000005) = 2.0020001$	5
5.	A/Q $(x+a)^n = O + E$ So, $(x-a)^n$=O -E	5

	Multiplying, $O^2 - E^2 = (x^2 - a^2)^n$	
6.	$(x+y)^5 + (x-y)^5 = 2\{x^5 + C(5,2)x^3y^2 + C(5,4)xy^4\} = 2\{x^5 + 10x^3y^2 + 5xy^4\}$ $(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left\{(\sqrt{2})^5 + 10(\sqrt{2})^3 + 5\sqrt{2}\right\} = 58\sqrt{2}$	5
7.	We have $(1+a)^n = {}^nc_0 + {}^nc_1 a + {}^nc_2 a^2 + ... + {}^nc_n a^n$ putting a=5 $(1+5)^n = {}^nc_0 + {}^nc_1 5 + {}^nc_2 5^2 + ... + {}^nc_n 5^n$ implies $6^n - 5n = 1 + 25({}^nc_2 + 5\,{}^nc_3 + ... + 5^{n-2})$ $6^n - 5n = 25k + 1$ This shows that when divided by 25 $6^n - 5n$ leaves remainder 1	5
8.	$a^n - b^n = \{(a-b)+b\}^n$ $a^n - b^n = (a-b)\left\{(a-b)^{n-1} + {}^nc_1 (a-b)^{n-2} b + ... + {}^nc_{n-1} b^{n-1}\right\}$ Clearly RHS is divisible by a - b	5
9.	17/52	5
10.	a = 3, b=5, n= 6	5
11.	**The coefficients of the first three terms of (x- 3/x^2)^m are mC0, (-3) mC1, and 9 mC2..According to the question,** **559 = mC0 – 3 mC1 + 9 mC2** **⇒ 1 – 3m + (9m(m-1)/2) = 559** **m = 12** **After determining the third term of (x- 3/x^2)^m,** **12C$_r$(x)$^{12-r}$(-3/x^2)^r = T$_{r+1}$** **⇒ 12C$_r$(x) (x)** **12-r(-3)** **r.(x-2r)** **⇒ 12Cr(x)12-3r(-3)r** **Because we want to find the term with x3, 12 – 3r = 3, i.e., r = 3.** **Using r = 3 as a value** **⇒ 12C$_3$(x)^9(-3)^3 = -5940×^{3}** **As a result, the coefficient of x^3 is -5940.**	5
12.	**For the given condition, the coefficients of (r – 5)th and (2r – 1)th terms of the**	5

expansion $(1 + x)^{34}$ are $34C_{r-6}$ and $34C_{2r-2}$ respectively.

Since the given terms in the expansion are equal,

$34C_{r-6} = 34C_{2r-2}$

From this, we can write it as either

$r-6=2r-2$

(or)

$r-6=34 -(2r-2)$ [We know that, if $nCr = nCp$, then either $r = p$ or $r = n – p$]

So, we get either $r = – 4$ or $r = 14$.

We know that r being a natural number, the value of $r = – 4$ is not possible.

Hence, the value of r is14.