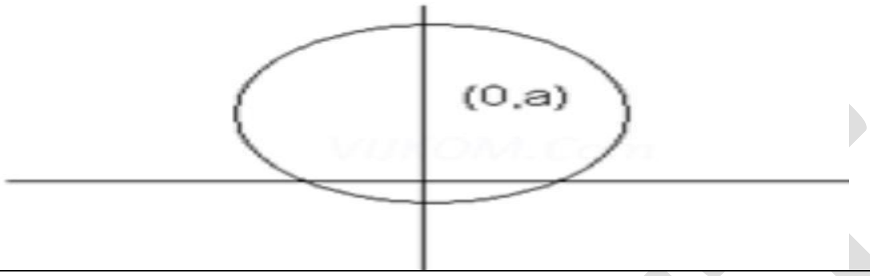
CHAPTER-9

DIFFERENTIAL EQUATIONS

05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Form the differential equation of the family of circles having centre on y-axis and radius 3 units. 	5
2.	Solve $(x^2 - y^2)dx + 2xydy = 0$	5
3.	$\frac{dy}{dx} + \frac{y}{x} = 0$ where 'x' denotes the percentage population in a city and 'y' denotes the area for living healthy life of population. Find the particular solution when x=100,y=1. Is higher density of population harmful? Justify your answer	5
4.	Solve the differential equation (xdy-ydx)y sin(y/x) = (ydx+xdy)x cos(y/x).	**5**
5.	Find the equation of a curve passing through $(0,\frac{\pi}{4})$and satisfying the differential equation Sinx.cosy dx + cosx.siny dy=0	**5**
6.	Find the particular solution of the differential equation $\frac{dy}{dx}$ +ycotx =2x+x^2cotx (x≠0) given that y=0 when x=$\frac{\pi}{2}$.	5
7.	Show that the differential equation $2ye^{\frac{x}{y}}dx+\left(y - 2xe^{\frac{x}{y}}\right)dy=0$ is homogeneous and find its particular solution, given that, x= 0 when y = 1	5
8.	In a lab, if in a culture, the bacteria count is 1,00,000.The number is increased by 10 % in 2 hours. In how many hours will the count reach 2, 00, 000, if the rate of growth of bacteria is proportional to the number present?	5
9.	The population of a village increase continuously at the rate proportional to the number of its inhabitants present at any time .If the population of the village was 20000 in 2018 and 25000 in the year 2023, what will be the population of the village in 2028?	5
10.	Find the particular solution of the differential equation $x\frac{dy}{dx}sin\left(\frac{y}{x}\right) + x - ysin\left(\frac{y}{x}\right) = 0$, given that $y(1) = \frac{\pi}{2}$.	5
11.	$(x^2 + y^2)dy = xydx$. If $y(1) = 1$ and $y(x_0) = e$, then find the value of x_0.	5
12.	Find the general solution of the differential equation $$ydx - (x + 2y^2)dy = 0$$	5
13.	Find the particular solution of the differential equation, satisfying the given condition $$(x + y)dy + (x - y)dx = 0 \ ; y = 1 \ when \ x = 1$$	5
14.	Find the particular solution of the differential equation $x\frac{dy}{dx}$= y -xtan$(\frac{y}{x})$, given that y = π/4 at x =1.	5
15.	Solve the differential equation	5

	xdy – ydx = $\sqrt{x^2 + y^2}$ dx, Given that y = 0, when x=1.	

ANSWERS:

Q. NO	ANSWER	MARKS
1.	As the centre of the circle lies on the y-axis. Let the center be (0,k). Thus the equation of the circles with center (0,k) and radius 3 is, $(x-0)^2 + (y-k)^2 = 3^2$ Differentiating and by solving we get $(x^2-9)y^2 + x^2 = 0$ Which is required solution.	
2.	$x^2 - y^2 = Cx$	
3.	$\frac{dy}{dx} + \frac{y}{x} = 0$ $\frac{dy}{dx} = -\left(\frac{y}{x}\right)$ Differentiating and by solving we get xy=100	
4.	sec(y/x) = cxy	
5.	cosy = secx/$\sqrt{2}$	
6.	The given equ. is a L.D.E. of the type $\frac{dy}{dx}$+Py=Q ,where P=cotx and Q=2x+x^2cotx. IF= $e^{\int cotxdx}$= sinx, General solution is given by ysinx=$\int(2x + x^2 cotx)sinxdx + c$ Gives ysinx=x^2sinx+c, Substituting y=0 and x=$\frac{\pi}{2}$ we get c=$\frac{-\pi^2}{4}$ Particular solution y=x^2 - $\frac{\pi^2}{4sinx}$	5
7.	This is of the form $\frac{dx}{dy}$=g$\left(\frac{x}{y}\right)$. Put x=vy ,then $\frac{dx}{dy}$=v+y$\frac{dv}{dy}$ Gives 2e^vdv=$\frac{-dy}{y}$ Integrate it, we get 2e^v= - log|y|+c Put v=$\frac{x}{y}$,we get General solution 2$e^{\frac{x}{y}}$+log|y|=c, Substituting x=0 and y=1 we get c=2 Particular solution 2$e^{\frac{x}{y}}$+log|y|=2.	5
8.	t =$\frac{2\log 2}{log\frac{11}{10}}$	5
9.	31250	5
10.	$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} \ldots (i)$	5

Let, $f(x,y) = \dfrac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)}$

$\therefore\ f(\lambda x, \lambda y) = \dfrac{\lambda y\sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x}{\lambda x\sin\left(\frac{\lambda y}{\lambda x}\right)} = \dfrac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} = f(x,y)$

Hence, f is a homogeneous function of degree 0.

Let, $y = vx$

Differentiating both sides w.r.t. x

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Hence, eq$^{\text{n}}$(i) becomes

$v + x\dfrac{dv}{dx} = \dfrac{vx\sin v - x}{x\sin v}$

$\Longrightarrow vx\dfrac{dv}{dx} = \dfrac{vx\sin v - x}{x\sin v} - v$

$\Longrightarrow x\dfrac{dv}{dx} = \dfrac{vx\sin v - x - vx\sin v}{x\sin v}$

$\Longrightarrow x\dfrac{dv}{dx} = \dfrac{-x}{x\sin v}$

$\Longrightarrow x\dfrac{dv}{dx} = -\dfrac{1}{\sin v}$

$\Longrightarrow \sin v\, dv = -\dfrac{dx}{x}$

$\Longrightarrow \int \sin v\, dv = -\int \dfrac{dx}{x}$

$\Longrightarrow -\cos v = -\log|x| + c$

$\Longrightarrow -\cos\left(\dfrac{y}{x}\right) = -\log|x| + c$

It is given that $y = \dfrac{\pi}{2}$ when $x = 1$

$\therefore -\cos\dfrac{\pi}{2} = -\log 1 + c$

$\Longrightarrow -0 = -0 + c$

$\Longrightarrow c = 0$

Hence, the complete sol$^{\text{n}}$ is :

$-\cos\left(\dfrac{y}{x}\right) = -\log|x|$

$\Longrightarrow \cos\left(\dfrac{y}{x}\right) = \log|x|$

11. (5 marks)

$(x^2 + y^2)dy = xydx$

$\Longrightarrow \dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$

Let, $f(x,y) = \dfrac{xy}{x^2 + y^2}$

$\therefore\ f(\lambda x, \lambda y) = \dfrac{\lambda x.\lambda y}{\lambda^2 x^2 + \lambda^2 y^2} = \dfrac{xy}{x^2 + y^2} = f(x,y)$

Hence, f is a homogeneous function of degree 0.

Let, $y = vx$

Differentiating both sides w.r.t. x

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Hence, eq^{n}(i) becomes

$$v + x\frac{dv}{dx} = \frac{x.vx}{x^2 + v^2x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{1 + v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1 + v^2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v - v(1 + v^2)}{1 + v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v - v - v^3}{1 + v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^3}{1 + v^2}$$

$$\Rightarrow \frac{(1 + v^2)dv}{v^3} = -\frac{dx}{x}$$

$$\Rightarrow \int \left[\frac{1}{v^3} + \frac{1}{v}\right] dv = -\int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2v^2} + \log|v| = -\log|x| + c$$

$$\Rightarrow -\frac{x^2}{2y^2} + \log\left|\frac{y}{x}\right| = -\log|x| + c$$

It is given that $y = 1$ when $x = 1$

$$\therefore -\frac{1}{2} + 0 = 0 + c$$

$$\Rightarrow c = -\frac{1}{2}$$

Hence, the complete sol^{n} is :

$$-\frac{x^2}{2y^2} + \log\left|\frac{y}{x}\right| = -\log|x| - \frac{1}{2}$$

Now, $x = x_0$ and $y = e$, then

$$-\frac{{x_0}^2}{2e^2} + \log\left|\frac{e}{x_0}\right| = -\log|x_0| - \frac{1}{2}$$

$$\Rightarrow -\frac{{x_0}^2}{2e^2} + \log\left|\frac{e}{x_0}\right| + \log|x_0| + \frac{1}{2} = 0$$

$$\Rightarrow -\frac{{x_0}^2}{2e^2} + \log\left|\frac{e}{x_0}.x_0\right| + \frac{1}{2} = 0$$

$$\Rightarrow -\frac{{x_0}^2}{2e^2} + \log|e| + \frac{1}{2} = 0$$

$$\Rightarrow -\frac{{x_0}^2}{2e^2} + 1 + \frac{1}{2} = 0$$

$$\Rightarrow -\frac{{x_0}^2}{2e^2} = -\frac{3}{2}$$

$$\Rightarrow {x_0}^2 = 3e^2$$

$$\Rightarrow x_0 = \pm\sqrt{3}e$$

12. The given differential equation can be rewritten as **[5]**

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

Which is a linear differential equation of the type

$$\frac{dx}{dy} + Px = Q, where\ P = \frac{-1}{y}\ \ and\ Q = 2y$$

	Therefore, I.F.=$e^{\int\frac{-1}{y}dy} = e^{-\log y} = \frac{1}{y}$ Hence, the solution of the given differential equation is $$x\frac{1}{y} = \int (2y)\left(\frac{1}{y}\right)dy + C$$ $$\frac{x}{y} = 2y + C$$ $$x = 2y^2 + Cy$$ Which is a general solution of the given differential equation.	
13.	The given differential equation can be rewritten as $$\frac{dy}{dx} = \frac{y-x}{y+x} = \frac{\frac{y}{x}-1}{\frac{y}{x}+1} = f\left(\frac{y}{x}\right)$$ Which is a homogeneous differential equation Now. Let $\frac{y}{x} = v \Rightarrow y = vx$ On differentiating wrt x on both sides, we get $$\frac{dy}{dx} = v + x\frac{dv}{dx} ------(ii)$$ Putting (ii) in (i), we get $$v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$ $$x\frac{dv}{dx} = \frac{v-1}{v+1} - v = \frac{v-1-v^2-v}{v+1}$$ $$\int \frac{v+1}{v^2+1}\, dv = -\int \frac{dx}{x}$$ $$\frac{1}{2}\int \frac{2v}{v^2+1}\, dv + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$$ $$\frac{1}{2}log(v^2+1) + tan^{-1}v = -\log x + C_1$$ $$log\left(\frac{y^2}{x^2}+1\right) + 2\, tan^{-1}\frac{y}{x} = -2\log x + 2C_1$$ $$\log(x^2+y^2) + 2\, tan^{-1}\frac{y}{x} = C$$ y=1, when x=1 so we get C=$\log 2 + \frac{\pi}{2}$ Hence, particular solution of the given differential equation is $$\log(x^2+y^2) + 2\, tan^{-1}\frac{y}{x} = \log 2 + \frac{\pi}{2}$$	5
14.	. :·Particular solution is given by x·sin($\frac{y}{x}$) = $\frac{1}{\sqrt{2}}$	5
15.	. y + $\sqrt{x^2+y^2} = x^2$ which is the required solution	5