CHAPTER-13
LIMITS & DERIVATIVES
05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the values of a and b if $\lim_{x\to 2} f(x) and \lim_{x\to 4} f(x) exists.$ Where f(x)= $\begin{cases} x^2+ax+b, 0\le x<2 \\ 3x+2, 2\le x\le 4 \\ 2ax+5b, 4<x\le 8 \end{cases}$	5
2.	Let f(x)= $\begin{cases} \frac{k\cos x}{\pi-2x}, x\ne\frac{\pi}{2} \\ 3, x=\frac{\pi}{2} \end{cases}$ and if $\lim_{x\to\frac{\pi}{2}} f(x)=f(\frac{\pi}{2})$ Find the value of k	5
3.	(i) Evaluate $\lim_{x\to 0}\left(\frac{e^{\tan x}-1-x}{\sin(x^2)}\right)$. (ii) If $\lim_{x\to 0}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then find the values of a and b.	5(3+2)
4.	Differentiate $\tan\sqrt{x}$ with respect to x by using first principle.	5
5.	Let f(x)= $\frac{[x]^2+15[x]+56}{\cos(x+7)\cos(x+8)}$ where [,] denoted the greatest integer function. Then show that $\lim_{x\to -7} f(x)$ =0	5
6.	Find the derivative of $x^{\sin x}+x^{\cos x}$	5
7.	Using first principle, show that the differentiation of $\sin x$ with respect to x is $\cos x$?	5
8.	The distance f(t) in meters moved by a particle travelling in a straight line in t seconds is given by $f(t)=t^2+3t+4$. Find the speed of the particle at the end of 2 seconds?	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	a=3 b=-2	5
2.	k=6	5
3.	(i) $\lim_{x\to 0}\left(\frac{e^{\tan x}-1-x}{\sin(x^2)}\right)=\lim_{x\to 0}\left(\frac{1+\frac{\tan x}{1!}+\frac{\tan^2 x}{2!}+.....-1-x}{\sin(x^2)}\right)$ $=\lim_{x\to 0}\left(\frac{\frac{\tan x}{1!}+\frac{\tan^2 x}{2!}+.....-x}{\sin(x^2)}\right)=\lim_{x\to 0}\left(\frac{x+\frac{x^3}{3}+\frac{2x^5}{15}+.....+\frac{\tan^2 x}{2!}+.....-x}{\sin(x^2)}\right)$ $\lim_{x\to 0}\left(\frac{\frac{x^3}{3}+\frac{2x^5}{15}+.....+\frac{\tan^2 x}{2!}+.....}{\sin(x^2)}\right)=\lim_{x\to 0}\frac{x^2\left(\frac{x}{3}+\frac{2x^2}{15}+.....+\frac{\tan^2 x}{x^2 2!}+.....\right)}{x^2}$ $=\lim_{x\to 0}\left(\frac{x}{3}+\frac{2x^2}{15}+.....+\frac{\tan^2 x}{x^2 2!}+.....\right)=\frac{1}{2!}=\frac{1}{2}$ (ii) $\lim_{x\to 0}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4\Rightarrow \lim_{x\to 0}\left(\frac{x^2+x+1-(x+1)(ax+b)}{x+1}\right)=4$ $\Rightarrow \lim_{x\to 0}\left(\frac{x^2+x+1-(ax^2+bx+ax+b)}{x+1}\right)=4\Rightarrow \lim_{x\to 0}\left(\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}\right)=4$ Since the limit is a finite value, the deg.of Nr=Deg. Of Dr Hence coefficient of x^2 is $1-a=0\Rightarrow a=1$. And $1-a-b=4\Rightarrow 1-1-b=4\Rightarrow b=-4$.	5
4.	$f(x)=\tan\sqrt{x}$ $f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\tan\sqrt{x+h}-\tan\sqrt{x}}{h}=\lim_{h\to 0}\frac{\tan\sqrt{x+h}-\tan\sqrt{x}}{x+h-x}$ $=\lim_{h\to 0}\frac{\frac{\sin\sqrt{x+h}}{\cos\sqrt{x+h}}-\frac{\sin\sqrt{x}}{\cos\sqrt{x}}}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}$ $=\lim_{h\to 0}\frac{\sin\sqrt{x+h}\cos\sqrt{x}-\cos\sqrt{x+h}\sin\sqrt{x}}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$ $=\lim_{h\to 0}\frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$	5

	$= \lim_{h \to 0} \frac{\sin\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}-\sqrt{x}\right)} \cdot \lim_{h \to 0} \frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}}$ $= 1 \times . \lim_{h \to 0} \frac{1}{\left(\sqrt{x+h}+\sqrt{x}\right)\cos\sqrt{x+h}.\cos\sqrt{x}} = \frac{1}{2\sqrt{x}\cos^2\sqrt{x}} = \frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$	
5.	$\frac{[x]^2+15[x]+56}{cos(x+7)cos(x+8)} = \frac{([x]+7)([x]+8)}{cos(x+7)cos(x+8)}$ Therefore $\lim_{x \to -7^+} f(x) = \frac{0 \times 1}{cos(0) \times cos(-1)} = 0$ $\lim_{x \to -7^-} f(x) = \frac{(-1) \times 0}{cos(-1) \times cos(0)} = 0$	5
6.	Let u $= x^{sinx}$ and v = $(x)^{cosx}$ Taking logarithm , we get log u= (sinx) logx And log(v) = cosxlogx Differentiating the above functions with respect to x we get $\frac{du}{dx}$=u($\frac{sinx}{x}$ + cosxlogx) ……………….(1) $\frac{dv}{dx}$=v($\frac{cosx}{x} - sinxlog(x)$)…………….(2) Also $\frac{dy}{dx} = \frac{du}{dx} + \frac{du}{dx}$, using algebraic property of derivative function Adding (1) and (2) we get the required answer.	5
7.	This can be proved by first principles,	5
8.	$f(t) = t^2 + 3t + 4$ The speed of the particle at the end of 2 seconds is given by f ’(2). I e the derivative of f (t) at t = 2 We can use first principle here $f'(t) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$ $= \lim_{h \to 0} \frac{\{(2+h)^2 + 392 + h) + 4\} - \{2^2 + 3 \times 2 + 4\}}{h}$ $= \lim_{h \to 0} \frac{h^2 + 7h}{h} = \lim_{h \to 0} h + 7 = 7$ Or we can can use the standard result $f'(t) = 2t + 3$ $\therefore f'(2) = 2 \times 2 + 3 = 7.$	5