CHAPTER-11

THREE DIMENSIONAL GEOMETRY

05 MARKS TYPE QUESTIONS

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<tr><th>Q. NO</th><th>QUESTION</th><th>MARK</th></tr>
<tr><td>1.</td><td>

The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are (2, 3, 5) and (1, 4, 2) respectively.

Based on the above information, answer the following questions.

(i) If the line joining the positions of the helicopter and boat is perpendicular to the plane in which boat moves, then equation of plane is

(a) $x - y + 3z = 2$ (b) $x + y + 3z = 2$

(c) $x - y + 3z = 3$ (d) $x + y + 3z = 3$

(ii) If the soldier decides to shoot the boat at given instant of time, where the distance measured in metres, then what is the distance that bullet has to travel?

(a) $\sqrt{5}$ m (b) $\sqrt{8}$ m

(c) $\sqrt{10}$ m (d) $\sqrt{11}$ m

(iii) If the speed of bullet is 30 m/sec, then how much time will the bullet take to hit the boat after the shot is fired?

(a) 30 seconds (b) 1 second

(c) $\frac{1}{2}$ second (d) $\frac{\sqrt{11}}{30}$ seconds

(iv) At the given instant of time, the equation of line passing through the positions of helicopter and boat is

(a) $\frac{x}{1} = \frac{y}{-1} = \frac{z}{3}$ (b) $\frac{x-1}{1} = \frac{y-4}{-1} = \frac{z-2}{3}$

(c) $\frac{x}{1} = \frac{y}{1} = \frac{z}{-3}$ (d) $\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-2}{-3}$

(v) At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat for the bullet to hit the boat if soldier shoots the bullet along the line whose equation is $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$?

(a) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{3}{2}, \frac{5}{4}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right)$ (d) none of these

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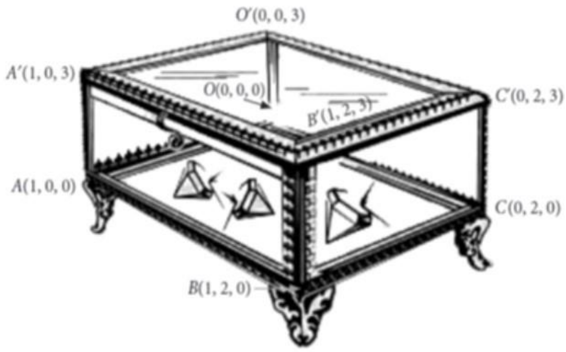
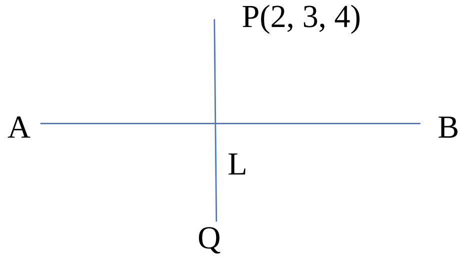
2.	In a diamond exhibition, a diamond is covered in cubical glass box having coordinates $O(0,0,0)$, $A(1,0,0)$, $B(1,2,0)$, $C(0,2,0)$, $O'(0,0,3)$, $A'(1,0,3)$, $B'(1,2,3)$ and $C'(0,2,3)$.  Based on the above information, answer the following questions. (i) Direction ratios of OA are (a) $<0, 1, 0>$ (b) $<1, 0, 0>$ (c) $<0, 0, 1>$ (d) none of these (ii) Equation of diagonal OB' is (a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (b) $\frac{x}{0}=\frac{y}{1}=\frac{z}{2}$ (c) $\frac{x}{1}=\frac{y}{0}=\frac{z}{2}$ (d) none of these (iii) Equation of plane $OABC$ is (a) $x=0$ (b) $y=0$ (c) $z=0$ (d) none of these (iv) Equation of plane $O'A'B'C'$ is (a) $x=3$ (b) $y=3$ (c) $z=3$ (d) $z=2$ (v) Equation of plane $ABB'A'$ is (a) $x=1$ (b) $y=1$ (c) $z=2$ (d) $x=3$	5 MARKS
3.	Find the coordinates of the image of the point (2, 3, 4) with respect to the line $\vec{r} = (2\hat{j} + 4\hat{k}) + \gamma\,(2\hat{\imath} + 4\hat{j} + 1\hat{k})$; where γ is a scalar. Also, find the distance of the image from the origin.	5
4.	An aeroplane is flying along the line $\vec{r} = \alpha\,(2\hat{\imath} + 3\hat{j} + 4\hat{k})$; where α is a scalar and another aeroplane is flying along the line $\vec{r} = (\hat{\imath} + \hat{j}) + \gamma\,(3\hat{j} + 2\hat{k})$; where γ is a scalar. At what points on the line should they reach, so that distance between them is shortest. Find the shortest possible distance between them.	5
5.	 A snake is crawling along the line $\vec{r} = 3\hat{\imath} + 2\hat{j} + 3\hat{k} + \lambda\,(\hat{\imath} - 2\hat{j} + 2\hat{k})$ and another snake is crawling along the line $\vec{r} = -4\hat{\imath} - 2\hat{k} + \mu\,(3\hat{\imath} - 2\hat{j} - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest ? Find the shortest possible distance between them.	5
6.	 The equation of motion of an airplane are $x = 4t$, $y = -4t$, $z = -2t$, where the time t is given in minutes and the co-ordinates of a moving point in km. What is the path of the airplane	5

	? At what distances will the rocket be from the starting point O(0,0,0) and from the following line in 10 minutes ? $\vec{r} = 40\hat{\imath} - 10\hat{\jmath} - 20\hat{k} + \lambda(10\hat{\imath} - 20\hat{\jmath} + 10\hat{k})$	
7.	Find the Vector equation of the line passing through the point P $(1, 2, -4)$ and perpendicular to the two lines : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.	5
8.	Find the angle between the lines whose direction cosines are given by the equations : $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$	5
9.	Show that the lines $r \rightarrow = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$; $r \rightarrow = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence, find their points of intersection.	5
10.	If a variable line in two adjacent positions has direction cosines l, m, n and $l + \delta l, m + \delta m, n + \delta n,$ show that the small angle $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$.	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	1. (i) (c) : Let $P(2, 3, 5)$ and $Q(1, 4, 2)$ be the positions of helicopter and boat respectively. Now, direction ratios of PQ are proportional to $1-2, 4-3, 2-5$, *i.e.*, $-1, 1, -3$. So, equation of plane passing through $Q(1, 4, 2)$ and perpendicular to PQ is $-(x-1)+(y-4)+(-3)(z-2)=0 \Rightarrow x-y+3z=3$ (ii) (d) : Required distance = Distance between P and Q $=\sqrt{(1-2)^2+(4-3)^2+(2-5)^2}=\sqrt{1+1+9}=\sqrt{11}$ m (iii) (d) : We know, Distance = Speed × Time $\therefore$ Required time $=\frac{\sqrt{11}}{30}$ seconds (iv) (b) : Equation of line PQ is $\frac{x-1}{1}=\frac{y-4}{-1}=\frac{z-2}{3}$. (v) (b) : Any point on the line $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$ is given by $(\lambda+1, -2\lambda+1, 3\lambda+2)$. Now, on substituting this point in the equation of plane $x-y+3z=3$, we get $(\lambda+1)-(-2\lambda+1)+3(3\lambda+2)=3$ $\Rightarrow \lambda+1+2\lambda-1+9\lambda+6=3 \Rightarrow 12\lambda=-3$ $\Rightarrow \lambda=\frac{-1}{4}$ Thus, the required point is $\left(\frac{-1}{4}+1, \frac{1}{2}+1, \frac{-3}{4}+2\right)$ *i.e.*, $\left(\frac{3}{4}, \frac{3}{2}, \frac{5}{4}\right)$.	
2.	2. (i) (b) : D.R.'s of OA are $<1-0, 0-0, 0-0>$, *i.e.*, $<1, 0, 0>$. (ii) (a) : Equation of diagonal OB' is $\frac{x-0}{1}=\frac{y-0}{2}=\frac{z-0}{3}$ *i.e.*, $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (iii) (c) : $OABC$ is xy-plane, therefore its equation is $z=0$. (iv) (c) : Plane $O'A'B'C'$ is parallel to xy-plane passing through $(0, 0, 3)$, therefore its equation is $z=3$. (v) (a) : Plane $ABB'A'$ is parallel to yz-plane passing through $(1, 0, 0)$, therefore its equation is $x=1$.	
3.	Let P(2, 3, 4) be the given point, L be the foot of perpendicular from 'P' to the given line AB  The coordinates of the general point on the given line are given by: $\frac{x-0}{2}=\frac{y-2}{4}=\frac{z-4}{1} = \gamma$, where γ is a scalar Let coordinates of L be $(2\gamma, 4\gamma+2, \gamma+4)$ Direction ratios of PL are $2\gamma-2, 4\gamma-1, \gamma$ Direction ratios of given line are 2, 4, 1 which is perpendicular to PL So, $2(2\gamma-2)+4(4\gamma-1)+\gamma=0$ i.e. $\gamma=8/21$ so coordinates of L are (16/21, 74/21, 92/21)	5

	let Q(a, b ,c) be the image of P(2, 3, 4), then L is mid-point of PQ. So, $\frac{a+2}{2}=\frac{16}{21}$, $\frac{b+3}{2}=\frac{74}{21}$, $\frac{c+4}{2}=\frac{92}{21}$ i.e. a = $\frac{-10}{21}$, b = $\frac{85}{21}$, c = $\frac{100}{21}$ so, image of P in the given line is $(\frac{-10}{21}, \frac{85}{21}, \frac{100}{21})$ Distance of point $(\frac{-10}{21}, \frac{85}{21}, \frac{100}{21})$ from origin is 6.27 approximately using distance formula.	
4.	The equations of two given straight lines in Cartesian form are: $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ …….. (i) and $\frac{x-1}{0}=\frac{y-1}{3}=\frac{z}{2}$ ………(ii) Lines are not parallel as direction ratios are not proportional. Let P and Q be the points on the straight lines (i) and (ii) respectively such that PQ is perpendicular to both of the lines, Let the coordinates of P be $(2\gamma, 3\gamma, 4\gamma)$ and Q be $(1, 3\beta+1, 2\beta)$ where β and γ are scalars Then the direction ratios of the line PQ will be $(2\gamma-1, 3\gamma-3\beta-1, 4\gamma-2\beta)$ Since, PQ is perpendicular to both (i) and (ii), so $2(2\gamma-1)+3(3\gamma-3\beta-1)+4(4\gamma-2\beta)=0$ …….(iii) $3(3\gamma-3\beta-1)+2(4\gamma-2\beta)=0$ ……..(iv) Solving (iii) and (iv) we get, $\gamma=\frac{7}{44}$, $\beta=\frac{-1}{44}$ Hence, coordinates of P are $(\frac{14}{44}, \frac{21}{44}, \frac{28}{44})$ and Q are $(1, \frac{129}{44}, \frac{-2}{44})$ The required shortest distance can be found by distance formula.	5
5.	The given lines are non-parallel lines. There is a unique line segment PQ which is at right angles to both the lines. Hence, shortest distance between the snakes = PQ The position vector of P lying on the line $\vec{r}=3\hat{\imath}+2\hat{\jmath}+3\hat{k}+\lambda(\hat{\imath}-2\hat{\jmath}+2\hat{k})$ is $(3+\lambda)\hat{\imath}+(2-2\lambda)\hat{\jmath}+(3+2\lambda)\hat{k}$ for some λ The position vector of Q lying on the line $\vec{r}=-4\hat{\imath}-2\hat{k}+\mu(3\hat{\imath}-2\hat{\jmath}-2\hat{k})$ is $(-4+3\mu)\hat{\imath}-2\mu\hat{\jmath}+(-2-2\mu)\hat{k}$ for some μ $\overrightarrow{PQ}=(-7+3\mu-\lambda)\hat{\imath}-(-2-2\mu+2\lambda)\hat{\jmath}+(-2-2\mu-3-2\lambda)\hat{k}$ Since, Pq is perpendicular to both the lines . So, $(-7+3\mu-\lambda)-(-2-2\mu+2\lambda)(-2)+(-2-2\mu-3-2\lambda)2=0$ $-7+3\mu-\lambda-4-4\mu+4\lambda-10-4\mu-4\lambda=0$ $-\lambda-5\mu=21$ (i) $(-7+3\mu-\lambda)3-(-2-2\mu+2\lambda)(-2)+(-2-2\mu-3-2\lambda)(-2)=0$ $-21+9\mu-3\lambda-4-4\mu+4\lambda+10+4\mu+4\lambda=0$ $5\lambda+9\mu=15$ (ii) Solving equn. (i) and (ii) $\lambda=33/2$ and $\mu=-15/2$ The position vector of the points at which they should be so that the distance between them is the shortest are $(39\hat{\imath}-62\hat{\jmath}+70\hat{k})/2$ and $(-53\hat{\imath}+30\hat{\jmath}+26\hat{k})/2$ $\overrightarrow{PQ}=(-92\hat{\imath}+92\hat{\jmath}+44\hat{k})/2=(-46\hat{\imath}+46\hat{\jmath}+22\hat{k})$ The shortest distance = $\mid\overrightarrow{PQ}\mid=\sqrt{2116+2116+484}=\sqrt{4716}$ $=2\sqrt{1179}$ Unit	5
6.	x= 4t, y = -4t , z = -2t, Or, x/4 = t, -y/4 = t , z/-2 = t	5

	So, x/4 = y/-4 = z/-2 Direction Ratios are 4,-4,-2 When t = 10 seconds, the airplane will be at the points (40,-40,-20) Distance from the origin in 10 minutes = $\sqrt{1600+1600+400}$ = $\sqrt{3600}$ = 60 km Distance of the point (40,-40,-20) from the given line = I (a_2- a_1)X$\overleftarrow{b}$I/I $\overleftarrow{b}$I = I -30$\hat{j}$ X ((10$\hat{\imath}$ -20 $\hat{j}$ +10 $\hat{k}$)I/I(10$\hat{\imath}$ -20 $\hat{j}$ +10 $\hat{k}$)I = I -300 $\hat{\imath}$ + 300 $\hat{k}$ I/ I(10$\hat{\imath}$ -20 $\hat{j}$ +10 $\hat{k}$)I =300 $\sqrt{2}$ /10 $\sqrt{6}$ = 10 $\sqrt{3}$ km	
7.	Let, the direction ratios of the line be (a, b, c) then the equation of the line will be $\vec{r} = \hat{\imath} + 2\hat{j} - 4\hat{k} + (a\hat{\imath} + b\hat{j} + c\hat{k})$ ……………(i) Equations of the given lines are : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Line given in equation (i) is perpendicular to these lines so that $3a - 16b + 7c = 0$ & $3a + 8b - 5c = 0$. Solving above by cross – multiplication method we get , $\frac{a}{80-56} = \frac{b}{21+15} = \frac{c}{24+48}$ $\frac{a}{24} = \frac{b}{36} = \frac{c}{72} = k(let)$ Hence, $a = 2k, b = 3k, c = 6k$ So that required vector equation of line is: $\vec{r} = \hat{\imath} + 2\hat{j} - 4\hat{k} + (2\hat{\imath} + 3\hat{j} + 6\hat{k})$.	1 1 1 1 1
8.	The relation between direction – cosines are given by $3l + m + 5n = 0$ ……..(1) $6mn - 2nl + 5lm = 0$ ………(2) From, (1) we get, $m = -3l - 5n$ …………..(3) Putting this value in (2) we get, $6(-3l-5n)n - 2nl + 5l(-3l-5n) = 0$ $\Longrightarrow l^2 + 3ln + 2n^2 = 0 \Longrightarrow (l+n).(l+2n) = 0$ $\Longrightarrow l = -n \; or \; l = -2n$ Now, if $l = -n, then\; m = -2n$ using (3) and if $l = -2n, then\; m = n.$ using (3) Thus the direction ratios of two lines are proportional to $\boldsymbol{-n, -2n, n}$ and $\boldsymbol{-2n, n, n}$ i.e. $\mathbf{1, 2, -1}$ and $\mathbf{-2, 1, 1}$ Hence, angle between these lines are given by $\cos\theta = \frac{1\times(-2)+2\times1+(-1)\times1}{\sqrt{1+4+1}\times\sqrt{4+1+1}} = \frac{-1}{6}$ $\Longrightarrow \theta = \cos^{-1}(\frac{-1}{6})$.	1 2 1 1
9.	$$\left\lvert\frac{-3+6}{3\sqrt{2}}\right\rvert = \left\lvert\frac{3\sqrt{2}}{2}\right\rvert$$	5
10.	$$(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$$	5