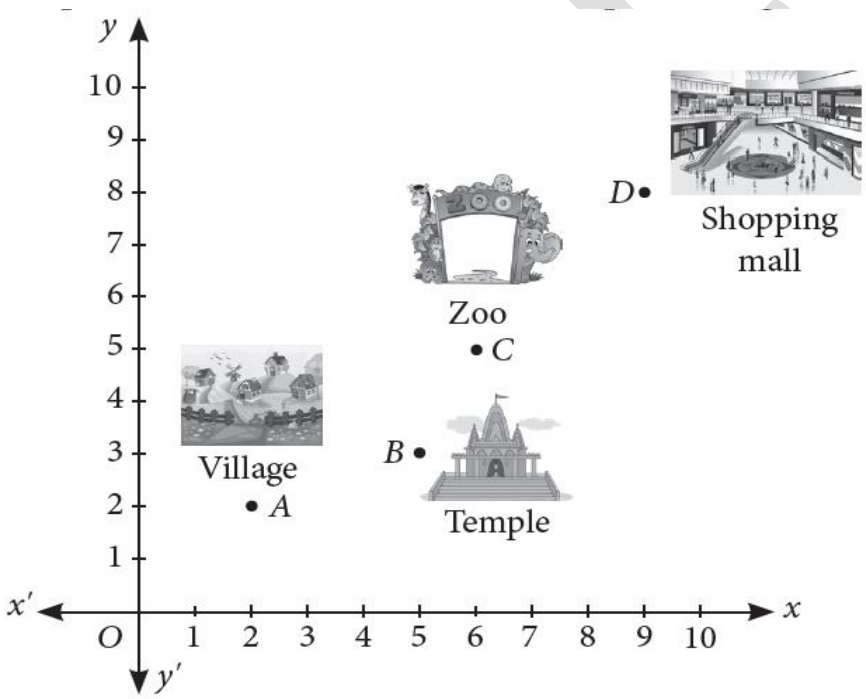
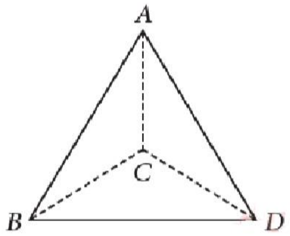
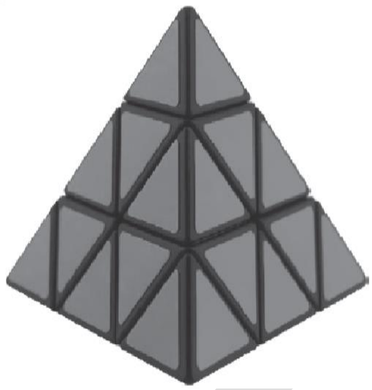
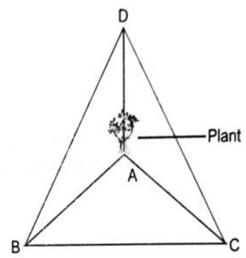
CHAPTER-10
VECTORS
05 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Let $\vec{a} = \hat{\imath} + 4j + 2\,\hat{k}$, $\vec{b} = 3\hat{\imath} - 2j + 7\,\hat{k}$ and $\vec{c} = 2\hat{\imath} - j + 4\,\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$	**5**
2.	If $\vec{a}$, , $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to the $\vec{a}$, , $\vec{b}$ and $\vec{c}$.	**5**
3.	Find the position vector of the point which divides the join of the points ($2\vec{a}$-$3\vec{b}$) and ($3\vec{a}$-$2\vec{b}$) in the ratio, (i) internally, (ii) externally.	5
4.	Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1 = 3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\vec{d}_2 = \hat{\imath}-3\hat{\jmath}+4\hat{k}$	5
5.	**CSB1:** Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.  Based on the above information, answer the following questions. **(i)** Position vector of B is (a) $3\hat{\imath} + 5\hat{\jmath}$ (b) $5\hat{\imath} + 3\hat{\jmath}$ (c) $-5\hat{\imath} - 3\hat{\jmath}$ (d) $-5\hat{\imath} + 3\hat{\jmath}$ **(ii)** Position vector of D is (a) $5\hat{\imath} + 3\hat{\jmath}$ (b) $3\hat{\imath} + 5\hat{\jmath}$ (c) $8\hat{\imath} + 9\hat{\jmath}$ (d) $9\hat{\imath} + 8\hat{\jmath}$ **(iii)** Find the vector $\overrightarrow{BC}$ in terms of $\hat{\imath}, \hat{\jmath}$ (a) $\hat{\imath} - 2\hat{\jmath}$ (b) $\hat{\imath} + 2\hat{\jmath}$ (c) $\widehat{2\imath} + \hat{\jmath}$ (d) $2\hat{\imath} - \hat{\jmath}$ **(iv)** Length of vector $\overrightarrow{AD}$ is (a) $\sqrt{67}$ units (b) $\sqrt{85}$ units (c) 90 units (d) 100 units **(v)** If $\vec{M} = 4j + 3k$, then its unit vector is (a) $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ (b) $\frac{4}{5}\hat{\imath} - \frac{3}{5}\hat{\jmath}$ (c) $-\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ (d) $-\frac{4}{5}\hat{\imath} - \frac{3}{5}\hat{\jmath}$	5
6.	**CSB2:**	5

	A building is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure.   Let its angular points are $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$ and $D(2, 3, 2)$ and G be the point of intersection of the medians of ΔBCD. Based on the above information, answer the following questions. **(i)** The coordinates of point G are (a) (2, 3, 3) (b) (3, 3, 2) (c) (3, 2, 3) (d) (0, 2, 3) **(ii)** The length of vector $\overrightarrow{AG}$ is (a) $\sqrt{17}$ units (b) $\sqrt{11}$ units (c) $\sqrt{13}$ units (d) $\sqrt{19}$ units **(iii)** Area of ΔABC (in sq. units) is (a) $\sqrt{10}$ (b) $2\sqrt{10}$ (c) $3\sqrt{10}$ (d) $5\sqrt{10}$ **(iv)** The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is (a) 5 units (b) 9.32 units (c) 10 units (d) 11 units **(v)** The length of the perpendicular from the vertex D on the opposite face is (a) $\frac{6}{\sqrt{10}}$ units (b) $\frac{2}{\sqrt{10}}$ units (c) $\frac{3}{\sqrt{10}}$ units (d) $8\sqrt{10}$ units	
7.	Two vectors $\hat{j}+\hat{k}$ and $3\hat{\imath}-\hat{j}+4\hat{k}$, represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a triangle ABC. Find the length of the median through A.	5
8.	Let $\vec{a}=4\hat{\imath}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{\imath}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{j}-\hat{k}$. find the vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a}=21$.	5
9.	Read the following passage and answer the following questions A person purchased an air plant , plant holder which is in shape of tetrahedron .Let A,B,C, D be the co-ordinates of the air plant holder where A(1,2,3),B(3,2,1),C(2,1,2),D(3,4,3). (i)Find the vector $\overrightarrow{AB}$.  (ii)Find the vector $\overrightarrow{CD}$. (iii)Find the unit vector along $\overrightarrow{BC}$ vector. (iv)Find the area Δ BCD.	5(1+1 +1+2)
10.	Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\|\vec{a}\times\vec{b}\|/2$. (i)Also find the area of parallelogram, whose diagonals are $2\hat{\imath}-\hat{j}+\hat{k}$ and $\hat{\imath}+3\hat{j}-\hat{k}$.	5(3+ 2)

ANSWERS:

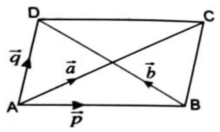
Q. NO	ANSWER	MARKS
1.	$\vec{d}$ is $\perp$ to $\vec{a}$ and $\vec{b}$ So, we take the cross product of $\vec{a}$ and $\vec{b}$ i.e. $(\hat{i} + 4\hat{j} + 2\hat{k}) \times (3\hat{i} - 2\hat{j} + 7\hat{k})$ $= 0 - 2\hat{i} \times \hat{j} + 7\hat{i} \times \hat{k} + 12\hat{j} \times \hat{i} - 0 + 28\hat{j} \times \hat{k} +$ $6\hat{k} \times \hat{i} - 4\hat{k} \times \hat{j} + 0$ $= -2\hat{k} - 7\hat{j} - 12\hat{k} + 28\hat{i} + 6\hat{j} + 4\hat{i}$ $= 32\hat{i} - \hat{j} - 14\hat{k}$ $\vec{d}$ would be a multiple of the obtained cross product, such that $\vec{c}.\vec{d} = 15$ $\Rightarrow (2\hat{i} - \hat{j} + 4\hat{k}).(32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}) = 15$ $\Rightarrow 64\lambda + \lambda - 56\lambda = 15$ $\therefore 9\lambda = 15$ $\therefore \lambda = \frac{5}{3}$ $\therefore \vec{d} = \frac{160\hat{i} - 5\hat{j} - 70\hat{k}}{3}$	**5**
2.	To prove: $\overline{a} + \overline{b} + \overline{c}$ is equally inclined to $\overline{a}, \overline{b}$ and $\overline{c}$. Given: $\overline{a}.\overline{b} = \overline{b}.\overline{c} = \overline{a}.\overline{c} = 0$ Angle between: $\overline{a} + \overline{b} + \overline{c}$ and $\overline{a}$: $\cos\theta_1 = \frac{(\overline{a} + \overline{b} + \overline{c}).\overline{a}}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{a}\rvert}$ $\Longrightarrow \cos\theta_1 = \frac{\lvert\overline{a}\rvert^2 + \overline{a}.\overline{b} + \overline{a}.\overline{c}}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{a}\rvert}$ $\Longrightarrow \cos\theta_1 = \frac{\lvert\overline{a}\rvert^2 + 0 + 0}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{a}\rvert}$ $= \frac{\lvert\overline{a}\rvert}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert}$ Angle between $\overline{a} + \overline{b} + \overline{c}$ and $\overline{b}$ $\Longrightarrow \cos\theta_2 = \frac{(\overline{a} + \overline{b} + \overline{c}).\overline{b}}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{b}\rvert}$ $= \frac{\overline{a}.\overline{b} + \lvert\overline{b}\rvert^2 + \overline{b}.\overline{c}}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{b}\rvert}$ $= \frac{0 + \lvert\overline{b}\rvert^2}{\lvert\overline{a} + \overline{b} + \overline{c}\rvert . \lvert\overline{b}\rvert}$	**5**

	$= \dfrac{\lvert\overline{b}\rvert}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert}$ Angle between $\overline{a}+\overline{b}+\overline{c}$ and $\overline{c}$: $\Longrightarrow \quad \cos\theta_3 = \dfrac{(\overline{a}+\overline{b}+\overline{c}).\overline{c}}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert\,.\,\lvert\overline{c}\rvert}$ $= \dfrac{\overline{a}.\overline{c}+\overline{b}.\overline{c}+\lvert\overline{c}\rvert^2}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert\,.\,\lvert\overline{c}\rvert}$ $= \dfrac{0+0+\lvert\overline{c}\rvert^2}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert\,.\,\lvert\overline{c}\rvert}$ $= \dfrac{\lvert\overline{c}\rvert}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert}$ $\because \lvert\overline{a}\rvert = \lvert\overline{b}\rvert = \lvert\overline{c}\rvert = p(\text{let})$ $\therefore \cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \dfrac{p}{\lvert\overline{a}+\overline{b}+\overline{c}\rvert}$ Hence proved.	
3.	$\vec{A} = 2\vec{a}-3\vec{b}$ $\vec{B} = 3\vec{a}-2\vec{b}$ The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{m\vec{b}+n\vec{a}}{m+n}$ and $\frac{m\vec{b}-n\vec{a}}{m-n}$ respectively. ∴ The position vector of the point dividing the line internally in the ratio 2:3 is $\frac{2\times(3\vec{a}-2\vec{b})+3\times(2\vec{a}-3\vec{b})}{2+3}$ $= \frac{12}{5}\vec{a} - \frac{13}{5}\vec{b}$ And the position vector of the point dividing the line internally in the ratio 2:3 is $\frac{2\times(3\vec{a}-2\vec{b})-3\times(2\vec{a}-3\vec{b})}{2-3}$ $= -5\vec{b}$	5
4.	Diagonals are represented by the vectors $\vec{d}_1 = 3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\vec{d}_2 = \hat{\imath}-3\hat{\jmath}+4\hat{k}$. Let $\vec{a}$ and $\vec{b}$ be two adjacent sides of the parallelogram. Thus, $\vec{a}+\vec{b} = 3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\vec{a}-\vec{b} = \hat{\imath}-3\hat{\jmath}+4\hat{k}$ Which gives $\vec{a} = 2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{b} = \hat{\imath}+2\hat{\jmath}-3\hat{k}$ The area of the parallelogram $\lvert\vec{a}\times\vec{b}\rvert = (a_2b_3-b_2a_3)\hat{\imath} + (a_3b_1-a_1b_3)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k}$ Here $a_1=2$, $a_2=-1$, $a_3=1$, $b_1=1$, $b_2=2$, $b_3=-3$ $\therefore \vec{a}\times\vec{b} = (3-2)\,\hat{\imath} + (1+6)\,\hat{\jmath} + (4+1)\hat{k} = \hat{\imath} + 7\hat{\jmath} + 5\hat{k}$ $\Rightarrow \lvert\vec{a}\times\vec{b}\rvert = \sqrt{1+49+25} = \sqrt{75} = 5\sqrt{3}$ Hence area = $5\sqrt{3}$ sq. units	5

5.	**b** **d** **b** **b** **a**	5
6.	**c** **b** **c** **b** **a**	5
7.	Using triangle law of addition $$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{CA}\overrightarrow{bd}=\frac{1}{2}\overrightarrow{BC}\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}$$ And solving $\left\|\overrightarrow{AD}\right\|=\frac{1}{2}\sqrt{34}$	5
8.	$\frac{1}{3}(-i+16j+13k).$	5
9.	As coordinates of A,B,C are : A(1,2,3), B(3,2,1), C(2,1,2), D(3,4,3). (i) $\overrightarrow{AB}=\overrightarrow{OB}$ - $\overrightarrow{OA}$ =(3-1)) $\hat{\imath}$+(2-2)) $\hat{\jmath}$ +(1-3) $\hat{k}$ = 2 $\hat{\imath}$ - 2 $\hat{k}$ Similarly find $\overrightarrow{CD}$ (ii) $\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}$ =$\hat{\imath}+3\hat{\jmath}+\hat{k}$ (a) $\because \overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(2-3)\hat{i}+(1-2)\hat{j}+(2-1)\hat{k}=-\hat{i}-\hat{j}+\hat{k}$ $\because \widehat{BC}=\frac{\overrightarrow{BC}}{\|\overrightarrow{BC}\|}=\frac{-\hat{i}-\hat{j}+\hat{k}}{\sqrt{(-1)^2+(-1)^2+1^2}}=\frac{-\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ $=-\frac{1}{\sqrt{3}}\hat{i}-\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$ Which is a unit vector along $\overrightarrow{BC}$.) (b) $\because \overrightarrow{BC}=-\hat{i}-\hat{j}+\hat{k}$ $\overrightarrow{BD}=\overrightarrow{OD}-\overrightarrow{OB}=(3-3)\hat{i}+(4-2)\hat{j}+(3-1)\hat{k}$ $=2\hat{j}+2\hat{k}$ $\because \overrightarrow{BC}$ and $\overrightarrow{BD}$ are adjacent sides of ΔBCD. $=\overrightarrow{BC}\times\overrightarrow{BD}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-1&1\\0&2&2\end{vmatrix}=\hat{i}(-2-2)-\hat{j}(-2-0)+k(-2+0)=-4\hat{i}+2\hat{j}-2\hat{k}$ $\therefore$ Area of $\Delta BCD=\frac{1}{2}\|\overrightarrow{BC}\times\overrightarrow{BD}\|$ $=\frac{1}{2}\sqrt{(-4)^2+2^2+(-2)^2}=\frac{1}{2}\times\sqrt{16+4+4}$ $=\frac{1}{2}\sqrt{24}=\frac{1}{2}\times 2\sqrt{6}=\sqrt{6}$ sq. units	5

10. Let $ABCD$ be a parallelogram such that

$$\overrightarrow{AB} = \vec{p}, \overrightarrow{AD} = \vec{q} \Rightarrow \overrightarrow{BC} = \vec{q}$$



By triangle law of addition, we get

$$\overrightarrow{AC} = \vec{p} + \vec{q} = \vec{a} \quad \text{[say]} \quad ...(i)$$

Similarly, $\overrightarrow{BD} = -\vec{p} + \vec{q} = \vec{b}$ [say] ...(*ii*)

On adding equation (*i*) and (*ii*), we get

$$\vec{a} + \vec{b} = 2\vec{q} \Rightarrow \vec{q} = \frac{1}{2}(\vec{a} + \vec{b})$$

On subtracting equation (*ii*) from equation (*i*), we get

$$\vec{a} - \vec{b} = 2\vec{p} \Rightarrow \vec{p} = \frac{1}{2}(\vec{a} - \vec{b})$$

Now, $\vec{p} \times \vec{q} = \frac{1}{4}(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$$= \frac{1}{4}(\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b})$$

$$= \frac{1}{4}[\vec{a} \times \vec{b} + \vec{a} \times \vec{b}] \quad [\because \vec{a} \times \vec{a} = 0 = \vec{b} \times \vec{b}]$$

$$= \frac{1}{2}(\vec{a} \times \vec{b})$$

So, area of a parallelogram $ABCD = |\vec{p} \times \vec{q}| = \frac{1}{2}|\vec{a} \times \vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

$$= \frac{1}{2}|(2\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} + 3\hat{j} - \hat{k})|$$

$$= \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}\right| = \frac{1}{2}|[\hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)]|$$

$$= \frac{1}{2}|-2\hat{i} + 3\hat{j} + 7\hat{k}| = \frac{1}{2}\sqrt{4+9+49}$$

$$= \frac{1}{2}\sqrt{62} \text{ sq. units}$$

5