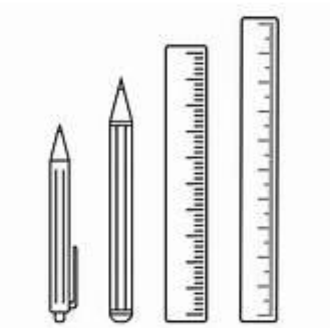
CHAPTER-4

DETERMINANTS

05 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	**Show that** $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$	5
2.	$\begin{vmatrix} \boldsymbol{x} & \boldsymbol{x^2} & \boldsymbol{yz} \\ \boldsymbol{y} & \boldsymbol{y^2} & \boldsymbol{zx} \\ \boldsymbol{z} & \boldsymbol{z^2} & \boldsymbol{xy} \end{vmatrix}$=(x-y) **(**y-z) **(**z-x) **(**xy+yz+zx**).**	5
3.	Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use it to solve the system of equations: $x-y+z=4; x-2y-2z=9 \, and \, 2x+y+3z=1$.	5
4.	Radhika buys 5 pencils, 3 rulers and 1bottle and pays a sum of ₹160. Amit buys 2 pencils, 1 ruler and 3 bottles for ₹190. Also Ankit buys 1 pencil, 2 rulers and 4 bottles for ₹ 250. Express the above in matrix form and find the cost of each article.  	5
5.	Solve the following system of equations , using matrix method; $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4\,;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1\ ;\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$	5
6.	A square matrix A is invertible if A is non singular and $A^{-1}=\frac{1}{\lvert A\rvert}adjA$ If $A=\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $x-y=3\,; 2x+3y+4z=17;\ \ y+2z=7$	5
7.	The management committee of GOKUL DHAM SOCIETY decided to award some of its members (say x) for honesty, some (say y) for helping others and some others(say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times of the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of number of awardees for honesty and supervision is twice the number of awardees for helping	5

Find the number of awardees for each honesty, cooperation and supervision.
Also find the value of 2x + 3y + 5z

8. Solve the system of equations: **(5)**

x+y+z=6000 ;x+3z=11000 ; x-2y+z= 0

9. Solve the following equations by using matrix method **(5)**

$\frac{1}{u}+\frac{1}{v}+\frac{1}{w}=12$, $\frac{3}{v}+\frac{3}{w}+\frac{2}{u}=33$, $\frac{1}{u}+\frac{1}{w}=\frac{2}{v}$

10. Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of Rs. x, y and z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of Rs.29000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of Rs.30500. If the three prizes per person together cost Rs. 9500, then **(5)**

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.
(ii) Solve these equations using matrices.
(iii) Which values are reflected in the questions?

11. A mixture is to be made of three foods A, B, C. The three foods A, B, C contain nutrients P.Q, R as shown below: **(5)**

Food	Ounces per pound of Nutrient		
	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R?

12. If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Hence solve the given equations $2x - 3y + 5z = 11$; **(5)**

$$3x + 2y - 4z = -5;$$

$$x + y - 2z = -3.$$

13.	Given that A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use it to solve the system of equations: x – y + z = 4; x – 2y – 2z = 9; 2x + y +3z = 1	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$C_1 \to C_1+C_2+C_3$ $=\begin{vmatrix}1+x+x^2 & x & x^2\\ 1+x+x^2 & 1 & x\\ 1+x+x^2 & x^2 & 1\end{vmatrix}$ $=\left(1+x+x^2\right)\begin{vmatrix}1 & x & x^2\\ 1 & 1 & x\\ 1 & x^2 & 1\end{vmatrix}$ $R_1 \to R_1-R_3 \quad R_2 \to R_2-R_3$ $=\left(1+x+x^2\right)\begin{vmatrix}0 & x-x^2 & x^2-1\\ 0 & 1-x^2 & x-1\\ 1 & x^2 & 1\end{vmatrix}$ $=\left(1+x+x^2\right)\begin{vmatrix}0 & x(1-x) & -(1-x)(1+x)\\ 0 & (1-x)(1+x) & -(1-x)\\ 1 & x^2 & 1\end{vmatrix}$ Taking (1 – x) common from R_1 and R_2 $=\left(1+x+x^2\right)(1-x)^2\begin{vmatrix}0 & x & -(1+x)\\ 0 & 1+x & -1\\ 1 & x^2 & 1\end{vmatrix}$ Expending along C^1 $=(1+x+x^2)(1-x)^2[-x+(1+x)^2]$ $=(1+x+x^2)(1-x)^2\left(-x+1+x^2+2x\right)$ $=(1-x)(1+x+x^2)(1-x)(1+x+x^2)$ $=\left(1-x^3\right)^2$	5
2.	$R_1 \to R_1-R_3, \quad R_2 \to R_2-R_3$ $=\begin{vmatrix}(x-z) & (x^2-z^2) & yz-xy\\ y-z & y^2-z^2 & zx-xy\\ z & z^2 & xy\end{vmatrix}$ $=(x-z)(y-z)\begin{vmatrix}1 & x+z & -(y)\\ 1 & y+z & -x\\ z & z^2 & xy\end{vmatrix}$ $R_1 \to R_1-R_2$ $=(x-z)(y-z)\begin{vmatrix}0 & x-y & x-y\\ 1 & y+z & -x\\ z & z^2 & xy\end{vmatrix}$	5

	$=(x-z)(y-z)(x-y)\begin{vmatrix} 0 & 1 & 1 \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$ $=(x-z)(y-z)(x-y)\left[-1(xy+zx)+1\left(z^2-y^2-z^2\right)\right]$ $=(x-z)(y-z)(x-y)[-xy-zx-yz]$ $=(x-y)(y-z)(z-x)(xy+yz+zx)$	
3.	$A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ $B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ $AB=8I \Rightarrow B^{-1}=\frac{1}{8}A$ The given system of equation in matrix form is $BX=C$ where $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ C=\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $\Rightarrow B^{-1}=\frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}.\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}=\frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}=\begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x=3; y=-2 \& z=-1$	5
4.	Let the cost of the three articles be ₹ x, ₹ y and ₹z respectively. $5x+3y+z=160; 2x+y+3z=190\ and\ x+2y+4z=250$ In matrix form the equations can be represented as $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix} \Rightarrow x=10, y=20\ and\ z=50$	5
5.	Let $\frac{1}{x}=a$, $\frac{1}{y}=b$, $\frac{1}{z}=c$,then the given system of equation become 2a+3b+10c = 4 4a–6b +5c = 1 6a + 9b – 20c= 2 This system of equation can be written as Ax = B Here, A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ Now, $\lvert A \rvert$ = 2(120-45) -3(-80-30) +10(36+36) = 150+30+720 =1200 ⇨ A^{-1} exist. adj(A) = $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ A^{-1} = 1/1200 $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ Now , X = $A^{-1}B$ Putting the values ,we get , x= 2 ,y = 3 , z = 5	5

6.	$\|A\| = -6 \neq 0,\ \ A^{-1} = \frac{1}{\|A\|} adjA = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ System of equation is as AX = B, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$ $X = A^{-1}B = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\therefore x = 2, y = -1, z = 4$	5
7.	x+ y +z = 12; 2x + 3y + 3z = 33; x – 2y + z = 0 therefore the system of equation can be written as AX = B, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$ $\|A\| = 3, \quad \therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$ $X = A^{-1}B = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ $\therefore x = 3, y = 4, z = 5$ $2x + 3y + 5z = 6 + 12 + 25 = 43$	5
8.	x+y+z=6000 ;x+3z=11000 ; x-2y+z= 0 AX=B A=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$; B=$\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$; X=$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Det A=6 Adj A= $\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$ A^{-1}= $1/6.\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$ $X=A^{-1}B=\begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$ So x=500,y=2000,z=3500	½ ½ ½ 1 ½ ½ +1 ½
9.	Let x=$\frac{1}{u}$, y=$\frac{1}{v}$,z=$\frac{1}{w}$ We have x+y+z=12, 3y+3z+2x=33, x+z=2y i.e. AX=B	½ ½ ½+½

	where $A = \begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 3\\ 1 & -2 & 1\end{bmatrix}$ & $\|A\| = 3$ Now, $\text{Adj}A = \begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}$ So, $X = A^{-1}B = \frac{Adj\ A}{\|A\|}B$ thus $X = \frac{1}{3}\begin{bmatrix}9\\ 12\\ 15\end{bmatrix}$, $\therefore x = 3,\ y = 4,\ z = 5$	½+½ 1 1
10.	According to the question, $2x + 3y + 4z = 29000$ $5x + 2y + 3z = 30500$ $x + y + z = 9500$ The above equations can be written as, $AX = B$ Where $A = \begin{bmatrix}2 & 3 & 4\\ 5 & 2 & 3\\ 1 & 1 & 1\end{bmatrix}, \quad X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}, \quad B = \begin{bmatrix}29000\\ 30500\\ 9500\end{bmatrix}$ Therefore , $X = A^{-1}B$ Solving We get, $x = 2750, \quad y = 3500, \quad z = 3250.$	5
11.	Let x pounds of food A, y pounds of food B and z pounds of food C be needed to form the mixture. According to the question, $x + 3y + 4z = 8$ $2x + y + 2z = 5$ $5x + y + z = 7$ The above equations can be written as, $AX = B$ Where $A = \begin{bmatrix}1 & 3 & 4\\ 2 & 1 & 2\\ 5 & 1 & 1\end{bmatrix}, \quad X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}, \quad B = \begin{bmatrix}8\\ 5\\ 7\end{bmatrix}$ Now, $\|A\| = 11 \neq 0$ So, $A^{-1} = \frac{1}{11}\begin{bmatrix}-1 & 1 & 2\\ 8 & -19 & 6\\ -3 & 14 & -5\end{bmatrix}$, Solving We get,	5

	$x = 1, \quad y = 1, \quad z = 1.$	
12.	.$\vert A\vert$= 2(-4+4) - (-3)(-6+4)+5(3-2) = 2.0+3(-2)+5.1 = 0-6+5 = $-1 \neq 0 \therefore A^{-1}$exists. Then,adj A = $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $\therefore A^{-1} = \frac{\mathbf{1}}{\vert\boldsymbol{A}\vert}$ **adj A** $= \frac{\mathbf{1}}{\mathbf{-1}} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ The given system of equations can be written as a single matrix equation $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ i.e. $AX = B$ $\Longrightarrow X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0.11 + 1.(-5) + (-2).(-3) \\ -2.11 + 9.(-5) + (-23).(-3) \\ -1.11 + 5.(-5) + (-13).(-3) \end{bmatrix}$ $= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$ $\Longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \therefore\ x = 1,\ y = 2,\ z = 3.$	1 1 1 1 1
13.	AB = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ $= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$ $= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8\ I_3$ => $\left(\frac{A}{8}\right)$ B = I => $B^{-1} = \frac{A}{8}$ => $B^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ The given system of equations can be written as a single matrix equation $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ i.e. B X = C=> X = B^{-1} C = $\frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$ => $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ => x =2, y = -2, z= -1	2 1 2