BGS INTERNATIONAL PUBLIC SCHOOL

SECTOR-5, DWARKA, NEW DELHI -75

PREBOARD - I (2023-24)

ANSWER KEY

SUB: APPLIED MATHEMATICS (241) **SCHOOLCODE: 25279**
CLASS: XII **TIME: 3 Hours**
SET: 1 **M.M:80**

SECTION - A

(All questions are compulsory. No internal choice is provided in this section)

Q1. [(3×7) + 5] $mod 4$ is

(a) 3 (b) 2 (c) 4 (d) 5

Ans: b.2

Solution: [(3×7)+5]$mod4$=26$mod4$

26=6×4+2⇒26≡2 ($mod4$) = 2

OR

A person can row a boat 5 km an hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

(a) 1.5 km/hr (b) 2.5 km/hr (c) 2km/hr (d) 3 km/hr

Ans : b. 2.5 km/hr

Q 2 In what ratio must rice at Rs 69 per kg be mixed with rice at Rs 100 per kg so that the mixture be worth Rs 80 per kg?

(a) 11:20 (b) 11: 10 (c) 20:11 (d). 10:11

Ans: c. 20:11

Q3 pipes A and B together can fill a pipe in 4 hours ,pipe B take 6 hours more than A to fill the tank ,if they opened separately .The time taken by A to fill the tank alone is

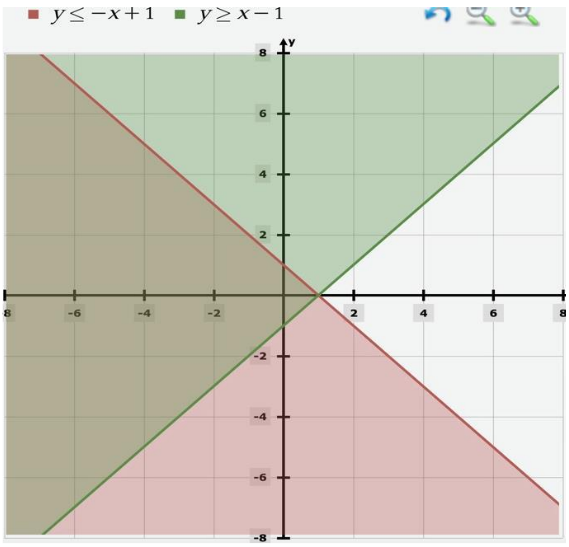
(a) 2 hours (b) 4 hours (c) 6hours (d) 8 hours

Ans: c. 6hours

Q4. The feasible region of the inequality x +y ≤ 1 and x – y ≤ 1 lies in quadrants.

(a) Only I and II (b) Only I and III (c) Only II and III (d) All the four

ANSWER : d



OR

For the LP problem maximize Z = 2x + 3y. The co-ordinates of the corner point Of the bounded feasible region are A (3, 3), B (20, 3), C (20, 10), D (18, 12),E (12, 12). The Maximum value of Z is

(a) 72 (b) 80 (c)82 (d)70

Ans. (a) 72

Q5. A # B, means A is greater than B ,
A * B, means A is smaller than B ,
A % B, means A is equal to B
A @ B, means A is greater than equal to B,
A © B, means A is smaller than equal to B

Statement: S © P @ Q # R **Conclusion I:** S @ R **Conclusion II:** R * P

(a) Only conclusion I is true
(b) Only conclusion II is true
(c) Both conclusion I and II are true
(d) Neither conclusion I nor II is true
(e) Either conclusion I or II is true

Q6. If matrix A= $\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \propto A$ then write the value of $\propto$

(a) 9 (b) 6 (c) 18 (d)12

Ans. (b) 6

Q7. A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B 'AB is a:

(a) symmetric matrix
(b) skew-symmetric matrix
(c) Diagonal matrix
(d) upper triangular symmetric

Ans: (b) skew-symmetric matrix A is a skew-symmetric matrix ⇒ A' = -A Consider (B'AB)' = (AB)'(B')' = B'A'(B')' = B'A'B = B'(-A)B = -B'AB As (B'AB) = -B'AB Hence, B'AB is a skew-symmetric matrix

Q8. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ write A^{-1} in terms of A

(a) $\begin{bmatrix} 2 & -3 \\ -5 & -2 \end{bmatrix}$ (b) $\frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 3 \\ 5 & -2 \end{bmatrix}$ (d) $\frac{1}{19}\begin{bmatrix} 2 & -3 \\ 5 & -2 \end{bmatrix}$

Solution:- $A^{-1} = \frac{Adj(A)}{|A|}$

$= \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$= \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$= \frac{1}{19} A$

Q9. Find the intervals in which the functions f(x) = x2 – 4x + 6 is strictly increasing

(a) $(-\infty,\mathbf{2})\cup(\mathbf{2},\infty)$ (b) $(\mathbf{2},\infty)$ (c) $(-\infty,\mathbf{2})$ (d) $(-\infty,\mathbf{2}]\cup[\mathbf{2},\infty)$

Q10. Suppose that demand is given by the equation x_d =500 – 50P, where xd is quantity demanded, and P is the price of the good. Supply is described by the equation x_s= 50 + 25P where x_s is quantity supplied. What is the equilibrium price

(a) 100 (b) 200 (c) 250 (d) 300

Q11. $\int_{-2}^{2} x^5\, dx$

(a) $2\int_{-2}^{2} x^5\, dx$ (b) $\frac{32}{5}$ (c) $\frac{64}{5}$ (d) 0

Ans. (d) 0

Q12. The highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation is called____ of the differential equation.

(a) homogeneous (b) power (c) degree (d) order

Ans. (c) degree

Q13. In a Poisson Distribution, if 'n' is the number of trials and 'p' is the probability of success, then the mean value is given by?

(a) m = np (b) $m = (np)^2$ (c) m = np(1-p) (d) m = p

Answer: (a)

Explanation:
(For a discrete probability function, the mean value or the expected value is given byMean(μ)=∑xp(x)

For Poisson Distribution P(x) = $\frac{\lambda^x e^{-\lambda}}{X!}$, substitute in above equation and solve to get μ = m = np).

Q14. The shape of the Normal Curve is
(a) Bell Shaped (b) Flat (c) Circular (d) Spiked

Answer: (a) **Explanation:** Due to the nature of the Probability Mass function, a bell-shaped curve is obtained.

OR

Which one is not a requirement of a binomial distribution.
(a) There are 2 outcomes for each trial. (b) The probability of success must be same for all trails
(c) There is a finite number of trails (d) The outcomes must be dependent on each other

Answer: option (d)

Q15. Given below are the consumer price index numbers (CPI) of the industrial workers.

Year	**2014**	**2016**	**2017**
Index Number	**145**	**150**	**190**

Find the best fitted trend line by the method of least squares
(a) y = 180+25x (b) y = 180+50x (c) y = 100+25x (d) None of these

9. D
Explanation::-

Year	Index number	X = xi – A = xi – 2017	X^2	XY
2014	145	-3	9	-435
2016	150	-1	1	-150
2017	190	0	0	0

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n=3, $\sum X = -4$,$\sum XY = -585$, ,$\sum X^2 = 10$, $\sum X$=485

a=$\frac{\sum Y}{n}$ = $\frac{485}{3}$ = 161.66

b=$\frac{\sum XY}{\sum X^2}$ =$\frac{-585}{10}$ = -58.5 ,

Therefore, the required equation of the straight-line trend is given by

y = a + bx , y = 161.66-58.5x,

OR

Increase in the number of patients in the hospital due to heat stroke is
(a) Secular trend (b) Irregular variation (c) Seasonal variation (d) cyclical variation

Ans. (c) Seasonal variation

Q16. $\int \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}$ dx is equals to
(a) $\frac{e^{2x}}{2}$ + c (b) $\frac{e^{2x}}{2x}$ + c (c) $\frac{e^{x}}{2}$ + c (d) $\frac{e^{2x}}{2x}$ + c

Ans. (b) $\frac{e^{2x}}{2x}$ + c

OR

The normal to the curve x^2 = 4y passing through (1, 2) is
(a) x + y = 3 (b) x – y = 3 (c) x + y = 1 (d) x – y = 1

Ans. (a) x + y = 3

Q17 At 6% converted quarterly, find the present value of a perpetuity of Rs 600 payable at the beginning of each quarter.

(a) Rs 30,400 (b) Rs 35,500 (c) Rs 40,600 (d) Rs 45,000

Answer: a) Rs 40,600

$$R = 600, \qquad i = \frac{0.06}{4} = 0.015$$

$$\because P = R + \frac{R}{i} = 600 + \frac{600}{0.015} = 600 + 40{,}000 = 40{,}600$$

OR

CAGR stands for

(a) Compound Aggregate Growth Rate (b) Compound Annual Growth Rate

(c) Computed Annual Growth Rate (d) Computed Aggregate Growth Rate

Answer: b) Compound Annual Growth Rate

Q18. It is given that at x = 1, the function $f(x) = x^3 - 12x^2 + kx + 7$ attains maximum value, then the value of 'k'

(a) 10 (b) 12 (c) 21 (d) 13

Ans. (c) 21

Assertion Reasoning Based Questions

Q19. Assertion: For two matrices A and B of order 3, $|A|=3, |B|=-4$ then $|2AB|=-96$

Reason: For a matrix A of order n and a scalar $k \det(kA)=k$ raised to the power n. (det A)

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Q20. **Assertion (A):** An annuity in which the periodic payment begins on a fixed date and continue forever is called perpetuity.

Reason (R): The amount or future value of perpetuity is defined.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol : (c) Assertion (A) is true but reason (R) is false.

SECTION - B

(All questions are compulsory. In case of internal choice, attempt any one question only)

Q21 The marginal cost of production of x units of a commodity is 30+2x. It is known that fixed costs are Rs.120. Find the total cost of producing 100units.

Ans. $C(x) = 120 + 30x + x^2$

Or

If the demand function for a commodity is $p = 25 - x^2$, find the consumers' surplus for $p_0 = 9$.

Ans. $\frac{128}{3}$

Q22. Verify that $y + x + 1 = 0$ is a solution of differential equation $(y – x)\, dy – (y^2 – x^2)\, dx = 0$.

$(y - x)[dy - (y + x)\,dx] = 0 \Rightarrow \frac{dy}{dx} = y + x$ ...(*i*)

Given $y + x + 1 = 0 \Rightarrow \frac{dy}{dx} + 1 = 0 \Rightarrow \frac{dy}{dx} - (y + x) = 0 \Rightarrow \frac{dy}{dx} = x + y$ ...(*ii*)

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Ans. From (*i*) and (*ii*) get the result.

Q23. How much money is needed to endure a series of lectures costing 2500 at the beginning of each year indefinitely, if money is worth 3% compounded annually?

Solution: We have R= 2500, i = 0.03 Money needed to endure a series of lectures costing R.S 2500 at the beginning of each year means the present value of a perpetuity of R.S 2500 payable at the beginning of each year

$P = R + \frac{R}{i} = 2500 + \frac{2500}{0.03}$

= R.S 85833.33

Q24. Experiance shows that 1.4 % of telephone calls received are wrong numbers. Determine the probability that among 150 calls received 2 are wrong numbers.

Solution:

Given : n =150 and p = 1.4 % = 14 /1000

$\therefore \lambda = np = 150 \times \frac{14}{1000} = 2.1$

$P(X=2) = \frac{(\lambda^2 e^{-\lambda})}{2!} = \frac{4.41 \times 0.122}{2} = 4.41 \times 0.061 = 0.269$

OR

For a certain type of laptops the charging time of batteries is normally distributed with mean 50 hours and standard deviation 15 hours. Arun has one of these laptops, Find the probability that the charging time of battery will be between 50 to 70 hours .

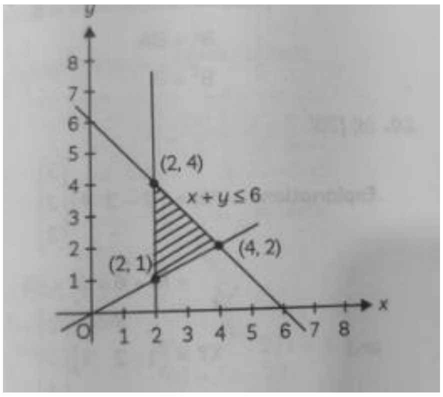
Solution:

Given : $\mu = 50$ and $\sigma = 15$

$P(50 < x < 70) = P\left(\frac{50-50}{15} < Z < \frac{70-50}{15}\right)$

$P(0 < Z < 1.33) = F(1.33) - F(0) = 0.9082 - 0.5 = 0.4082$

Q25. One very useful application of linear programming is its a graphical method for solving problems in two variable. Mrs. Meena wanted to use this concept to help students figure out how the area of the 3D-model is composed of 3 straight lines.On the below diagram (cross section of 3D model), O is the origin. The shaded region R is defined by three inequalities one of the three inequalities is $x + y \leq 6$.



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(i) Given that the point (x, y) is in the region R, then what is the maximum value of x+ 2y ?

(ii) Does the point (3, 2.5) lies inside the region R?

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

Q 26. A firm has the cost function $\boldsymbol{C}=\frac{x^3}{3}-\boldsymbol{7x^2}+\boldsymbol{111x}+\boldsymbol{50}$ and demand function $\boldsymbol{x}=\boldsymbol{100}-\boldsymbol{p}$

(i) Write the total revenue function in terms of $\boldsymbol{x}$

(ii) Formulate the total profit function P in terms of $\boldsymbol{x}$

(iii) Find the profit maximizing level of output $\boldsymbol{x}$. What is the maximum profit ?

Sol. (i) Revenue function = $100x-x^2$

(ii) Profit function = $-\frac{x^3}{3}+6x^2-111x-50$

(iii) $x=11$

(iv) Maximum Profit = 111.3

OR

The marginal revenue function for a firm is given by $\frac{5x^2+30x+51}{(x+3)^2}$

Show that the revenue function is given by $\frac{2x}{x+3}$ + 5x

Ans. $\frac{2x}{x+3}$ + 5x

Q27. Find the Probability distribution of the number of Successes of two tosses of a die. Where a Success is defined as "the number greater than 4". Also find the Mean. Variance and Standard deviation of the distribution.

Solution:

When a die is tossed S={1.2.3.4.5.6}

Let E be the event ''the number greater than 4''

E={5.6}

n (E)=2

p=P(E) $=\frac{2}{6}=\frac{1}{3}$

q=1- $\frac{1}{3}=\frac{2}{3}$

As the die is tossed twice there are 2 bernoullian trials.

Let X denote the number of successes the X can take values 0,1,2

$P(0) = 2C_0\, p^0 q^2 = (\frac{2}{3})^2 = \frac{4}{9}$

$P(1) = 2C_1\, p^1 q^1 = 2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

$P(2) = 2C\, p^2 q^0 = (\frac{1}{3})^2 = \frac{1}{9}$

Probability distribution of Number of successes is

x	0	1	2
P(x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean $=\Sigma x_i p_i = (0)(\frac{4}{9}) + (1)\left(\frac{4}{9}\right) + 2\left(\frac{1}{9}\right) = \frac{6}{9}$

$\Sigma p_i x^2 = (0)(\frac{4}{9}) + (1)\left(\frac{4}{9}\right) + 4\left(\frac{1}{9}\right) = \frac{8}{9}$

Variance= $\Sigma p_i x^2 - (\Sigma x_i p_i)^2 = \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$

Standard Deviation= $\sqrt{Var} = \frac{2}{3}$

Q 28. Find the effective rate of interest equivalent to a nominal rate of 6% compounded
(i) Semi-annually (ii) Quarterly (iii) Continuously

Solution:

(i) When compounded semi-annually

We have r = 0.06 and m = 2

$$r_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.06}{2}\right)^2 - 1 = 0.0609 = 6.09\%$$

(ii) When compounded quarterly

We have r = 0.06 and m = 4

$$r_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.06}{4}\right)^2 - 1 = 0.0613 = 6.13\%$$

(iii) When compounded continuously

$$r_{eff} = e^r - 1 = e^{0.06} - 1 = 0.0618 = 6.18\%$$

Q29 Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ = 1 + a² + b² + c²

Ans.

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} (a^2+1) & a^2 & a^2 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2+1 & a^2+b^2+c^2+1 & a^2+b^2+c^2+1 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

$$= (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

$$= (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & -1 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$= (a^2+b^2+c^2+1)$$

OR

Show that the matrix B^TAB is symmetric or skew symmetric accordingly when A is symmetric or skew symmetric

Q30. If A = $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11A = 0$

Ans.

Q31. A steamer can go 24 km in still water in 50 minutes. One day, it went 22.5 km upstream and returned the same distance in downstream. If the difference between the time taken to travel upstream and downstream was 25 minutes, then what was the speed of stream in km per hour?

Solution:

Speed of steamer in still water=24000/50=480m/minute=8m/sec

Let speed of stream=v m/sec

In upstream ,the speed of steamer=(8-v) m/ses

In downstream the speed of steamer=(8+v) m/sec

22500/(8-v) - 22500/(8+v) = 25 × 60

22500/(8-v) - 22500/(8+v) = 1500

v =2m/sec

$=2 \times \frac{18}{5} \frac{km}{hr}$

=7.2km/hr

SECTION – D

Q32. A random sample of size 16 has 53 as mean. The sum of squares of deviations from mean is 150. Can this sample be regard as taken from the population having 56 as mean? Level of significance is 5% (right tail t-test). diet?

Ans: $H_o : \mu = 56$

$H_1 : \mu > 56$

$n = 16,\ \bar{x} = 53,\ \mu = 56,\ \sum(x_i - \bar{x})^2 = 150$

$\therefore\ S^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{150}{15} = 10$

$\Rightarrow S = \sqrt{10} = 3.162$

$t = \frac{\bar{x} - \mu}{(S/\sqrt{n})} = \frac{53-56}{(3.162\ /\ \sqrt{16})} = \frac{4 \times (-3)}{3.162} = -3.795.$

Tabulated value of $t = 1.753$ $(df = 15,\ 0.05)$

Since the calculated value of $|t| = 3.795$ is greater than the tabulated value 1.753.

So, H_o can be rejected.

(Note: Area is always positive numerically on right hand side)

Ans.

OR

Q 33 Consider the following data:

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Production	137	140	34	137	151	121	124	159	157	169	172	150

Calculate a suitable moving average and show on a graph against the original data.

SOLUTION:

In order to find which moving average will be appropriate, we will have to

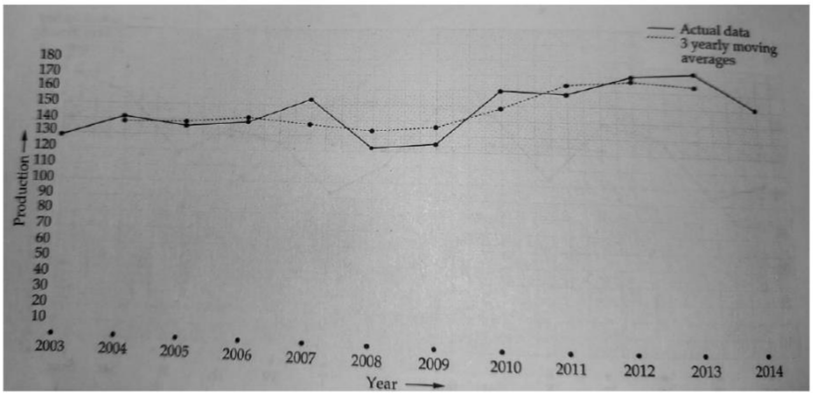
estimate the length of the cycle of the above data. We observe that the data has the pattern

(137, 140, 134), (137, 151, 121), (124, 159, 157), (169, 172, 150). Thus, we have cycle length of 3.

So, we will calculate 3 yearly moving averages as shown in the following table:

Year	Production	3-year moving total	3-year moving average
2003	137	-	-
2004	140	311	103.66
2005	34	311	103.66
2006	137	322	107.33
2007	151	409	136.33
2008	121	396	132
2009	124	404	134.66
2010	159	440	146.66

2011	157	485	161.66
2012	169	498	166
2013	172	491	163.66
2014	150	-	-



Q34 Suppose Mr. X invested Rs.1,00,000/- in a mutual fund and the value of the investment at the time of redemption was Rs.1,50,000/-. If CAGR for the investment is 8%, calculate the number of years for which he has invested the amount. If CAGR is 4 % what is the number of years of investment?

Solution:

$$\text{CAGR\%}=\left[\left(\frac{FV}{PV}\right)^{1/n}-1\right]\times 100$$

$$8=\left[\left(\frac{1,50,000}{1,00,000}\right)^{1/n}-1\right]\times 100$$

$$\frac{8}{100}=\left[\left(\frac{3}{2}\right)^{1/n}-1\right]$$

$$\left(\frac{3}{2}\right)^{1/n}=1.08$$

Taking log

$$\frac{1}{n}[log3-log2]=log1.08$$

$$n=\frac{log3-log2}{log1.08}=\frac{1761}{334}$$

logn =log1761-log334

logn =0.7221

n= antilog(0.7221)=5.273

n(approx.)=5 years

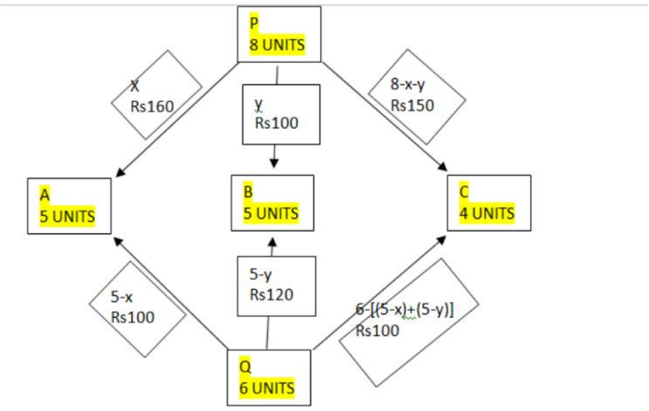
ii)If CAGR is 4% , then $0.04=\left(\frac{3}{2}\right)^{1/n}-1$

By taking log and antilog in $\left(\frac{3}{2}\right)^{1/n}=1.04$, we get n=1.036

n(approx.)=1 year

Q35. (Transportation problem) There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/TO	COST (IN Rs)		
	A	B	C
	160	100	150
	100	120	100



Ans. Total transportation cost Z is given by

$Z = 160\,x + 100\,y + 150\,(8 - x - y) + 100\,(5 - x) + 120\,(5 - y) + 100\,(x + y - 4)$

$= 10\,(x - 7\,y + 190)$

Minimize $Z = 10\,(x - 7y + 190)$

subject to the constraints:

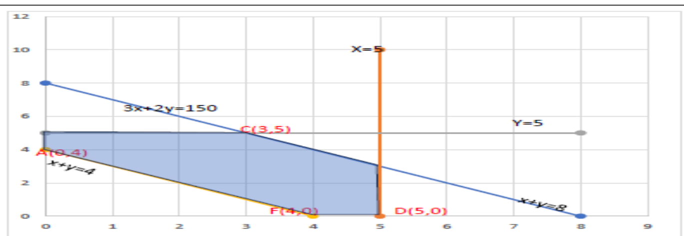
$x \geq 0,\ y \geq 0$ ... (1)

$x + y \leq 8$ ... (2)

$x \leq 5$ ... (3)

$y \leq 5$ ... (4)

and $x + y \geq 4$ (5)



Corner point	Z=10(x-7y+190)
(0,4)	1620
(0,5)	1550
(3,5)	1580
(5,3)	1740
(5,0)	1950
(4,0)	1940

As Feasible region bounded (so by corner point theorem)

Hence, minimum value of Z is 1550 at (0,5)

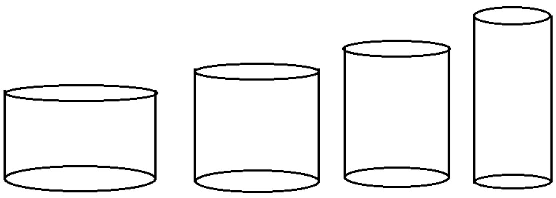
OR

SECTION - E

(All questions are compulsory. In case of internal choice, attempt any one question only)

Case Study - I

Q36. A company is planning to launch a new product and decides to pack the new product in closed right circular cylindrical cans of volume 432π cm^3 . The cans are to be made from tin sheet. The company tried different options.



Based on the above information, answer the following questions:

(a) If r cm is the radius of the base of the cylinder and h cm is height, then find a relation between r and h.

(b) If S cm^2 is the surface area of the closed cylindrical can, then find S in terms of r.

(c) Find the minimum surface area of cylindrical can

Q37. **Case study – II**

A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.
Based on the above information, answer the following questions

(i) Find the probability that in a sample of 100 bulbs selected at random none of the bulbs are defective (Use $e^{-6}=0.0024$)

(ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs.

(iii) Find the probability that the sample of 100 bulbs will include not more than one defective bulb.

OR

Find the Mean and Variance of the distribution of number of defective bulbs in a sample of 100 bulbs

Solutions

N=100, p=6/100, λ=np =100x(6/100) =6

$P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$

(i) $P(0) = \frac{e^{-6}6^0}{0!} = e^{-6} = 0.0024$

(ii) $P(2) = \frac{e^{-6}6^2}{2!} = 0.0024 \times \left(\frac{36}{2}\right) = 0.0432$

(iii) $P(0)+P(1) = e^{-6} + 6e^{-6} = 7e^{-6} = 0.0168$

OR

Mean = λ =6

Variance=λ=6

Q38. **Case-Study 3**

In mathematics modular arithmetic is a system of arithmetic for integers where numbers ”wrap around” when reaching a certain value called modulus . A familiar use of modular arithmetic is in the 12 hour clock in which the day is divided into two 12 hour periods. If the time is 7:00 now, then 8 hours later it will be 3:00. Simple addition would result in 7+8=15, but clocks “wrap around” every 12 hours. Because the hour number starts over after it reaches 12, this is arithmetic modulo 12. In terms of the definition,15 is congruent to 3 modulo 12. So 15:00 on a 24 hour clock is displayed 3:00 on a 12 hour clock. Based on the above information answer the following questions

(a) Evaluate 3^6 (mod4)

(b) What is the least positive of x for which $100 \equiv x \ (mod 7)$

(c) Evaluate (137+995) mod 12

OR

Find the last digit of 12^{12}

Solution:

(a) $3^2 \equiv 1(mod4)$

$(3^2)^3 \equiv 1^3$ (mod4)

$3^6 \equiv 1(mod\ 4)$

$3^6 mod\ 4 = 1$

(b) $100 \equiv x(mod7)$

100-x is divisible by 7 when x=2,100-2=98 which is divisible by 7

Therefore the least positive value of x=2

(c)(137+995)mod 12

= (5+11)mod 12=16mod 12=4

OR

$12^{12} mod 10$

12 mod 10=2

$12 \equiv 2\ mod 10 12^6 \equiv 2^6\ (mod 10)$

=64(mod10)=4

$(12^6)^2 = 4^2 = 16\ mod\ 10 = 6$

Therefore last digit =6