BGS INTERNATIONAL PUBLIC SCHOOL

SECTOR-5, DWARKA, NEW DELHI -75

PREBOARD - I (2023-24)

ANSWER KEY

SUB: APPLIED MATHEMATICS (241) **SCHOOLCODE: 25279**
CLASS: XII **TIME: 3 Hours**
SET: 2 **M.M:80**

SECTION - A

(All questions are compulsory. No internal choice is provided in this section)

Q1. What is the least value of 'x' that satisfies x ≡ 27 (mod 4), when 27 < x ≤ 36 ?

(a) 27 (b) 30 (c) 31 (d) 35

Ans:

OR

In a 2 km race, P can give Q a start of 200 m and R a start of 560 m. Then, in the same race, Q can give R a start of:

(a) 360 m (b) 380 m (c) 400 m (d) 430 m

Ans. (c) 400 m

Q 2 A specific characterstics of a sample is called:

(a) Variance (b) statistics (c) parameter (d) population

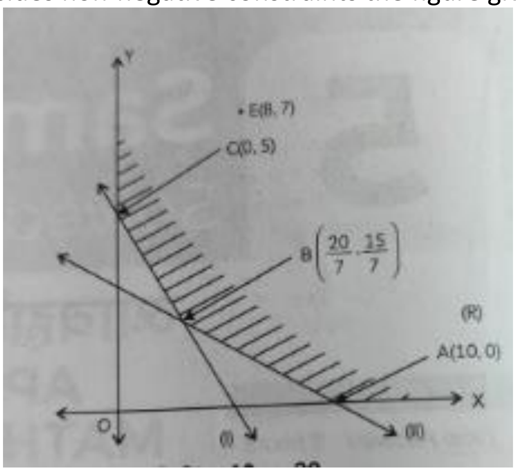
Ans (b) statistics

Q3 Pipe A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the time taken to fill the tank is:

(a) 2 hours (b) $2\frac{3}{4}$ hours (c) 3 hours (d) $3\frac{9}{17}$ hours

Ans: (d) $3\frac{9}{17}$ hours

Q4. Besides non-negative constraints the figure given below is subject to which of the following constraints:



(a) x + y ≤ 5; 3x + 10y ≥ 30 (b) x + y ≥ 5; 3x + 10y ≤ 30 (c) x + y ≤ 5; 3x + 10y ≤ 30 (d) x + y ≥ 5; 3x + 10y ≥ 30

Ans. (d) x + y ≥ 5; 3x + 10y ≥ 30

OR

For the LP problem maximize Z = 2x + 3y. The co-ordinates of the corner point Of the bounded feasible region are A (3, 3), B (20, 3), C (20, 10), D (18, 12),E (12, 12). The Maximum value of Z is

(a) 72 (b) 80 (c)82 (d)70

Ans. (a) 72

Q5 If x > y and z < 0, then :

(a) xz > yz (b) $\frac{x}{z} > \frac{y}{z}$ (c) xz ≥ yz (d) $\frac{x}{z} < \frac{y}{z}$

Ans. (d) $\frac{x}{z} < \frac{y}{z}$

Q6 The values of x which satisfy both the linear inequations simultaneously, are

$4x - 1 < 0\,,\ 3 - 4x < 0$

(a) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (b) $\left(-\frac{1}{4}, \frac{3}{4}\right)$ (c) $\left(\frac{3}{4}, \frac{1}{4}\right)$ (d) no value

Ans. (d) no value

Q7. If A = $\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$, then A^2 = ...

Ans: $\begin{bmatrix} 4 & -3 \\ 0 & 1 \end{bmatrix}$

Or

If A = $[a_{ij}]_{3 \times 3}$ = $\begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then cofactor of a_{12} is = …………….. .

Ans. Co- factor of a_{12} = 4 .

Q8. If A = $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB = I_3, then (x + y) equals:

(a) 0 (b) – 1 (c) 2 (d) – 2

Ans. (a) 0

Q9. The function f(x) = a^x is increasing on R, if :

(a) a > 0 (b) a < 0 (c) 0 < a < 1 (d)a > 1

Ans. (d)a > 1

Q10. The normal distribution curve is symmetrical about :

(a) X = μ (b) X = σ (c) X = $\frac{\mu}{\sigma}$ (d) X = $\frac{\sigma}{\mu}$

Ans. (a) X = μ

Q11. $\int_0^{40} \frac{dx}{2x+1}$ = log k, then the value of k is:

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{9}{2}$ (d) 9

Ans. (d) 9

Q12. In the equation of trend line y = a + bx, b denotes:

(a) mean of y (b) slope of the trend line (c) total of x (d) total of xy

Ans. (b) slope of the trend line

Q13. The marks obtained were found normally distributed with mean 75 and variance 100. The percentage of students who scored more than 75 marks is :

(a) 25 % (b) 50 % (c) 75 % (d) 100 %

Ans : (b) 50 %

Q14. A machine costing ₹ 46,000 depreciates at an uniform rate of 12% per annum, then its value after 5 years is :

[use $\left(\frac{22}{25}\right)^5$ = 0.5278]

(a) ₹ 24278.50 (b) ₹ 22730.45 (c) ₹ 31272.20 (d) ₹ 20892.08

Answer (a) ₹ 24278.50

OR

The amount S of an annuity due A, when the rate of interest is r and n is the number of years, is given by:

(a) S = $\frac{A}{r}$ (1 + r)^n [(1+ r)^n – 1] (b) S = $\frac{A}{r}$ (1 + r) [(1+ r)^n – 1]

(c)S = Ar (1 + r)^n [(1+ r)^n – 1] (d) S = Ar (1 + r) [(1+ r)^n – 1]

Ans. (b) S = $\frac{A}{r}$ (1 + r) [(1+ r)^n – 1]

Q15. In one sample t- test, the estimation for the population mean is:

(a) $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ (b) $\frac{\bar{X}-\mu}{S/n}$ (c) $\frac{\bar{X}-\mu}{S^2/\sqrt{n}}$ (d) $\frac{\bar{X}-\mu}{S/n^2}$

Ans, (a) $\frac{\bar{X}-\mu}{S/\sqrt{n}}$

Q16. If $\frac{d}{dx}$ f (x) = 4x^3 – $\frac{3}{x^4}$, such that f(2) = 0. Then, f(x) is :

(a) x^4 + $\frac{1}{x^3} - \frac{129}{8}$ (b) x^3 + $\frac{1}{x^4} + \frac{129}{8}$ (c) x^4 + $\frac{1}{x^3} + \frac{129}{8}$ (d) x^3 + $\frac{1}{x^4} - \frac{129}{8}$

Ans. (a) x^4 + $\frac{1}{x^3} - \frac{129}{8}$

OR

If the function f(x) = x^3 – 27x + 5 is strictly increasing when :

(a) x < - 3 (b) $|x|$ > 3 (c) x > 3 (d) $|x| < 3$

Ans. (d) $|x| < 3$

Q17. If the supply function for a commodity is p = $\sqrt{x+9}$ and the market price is 4, then producer's surplus is :

(a) 3 (b) $\frac{10}{3}$ (c) 10 (d) 15

Ans. (b) $\frac{10}{3}$

OR

Which of the following differential equations has y = $c_1 e^x + c_2 e^{-x}$ as the general solution ?

(a) $\frac{d^2y}{dx^2}$ + y = 0 (b) $\frac{d^2y}{dx^2}$ – y = 0 (c) $\frac{d^2y}{dx^2}$ + 1= 0 (d) $\frac{d^2y}{dx^2}$ – 1 = 0

Ans. (b) $\frac{d^2y}{dx^2}$ – y = 0

Q18. The area under the standard normal curve which lies to the left of z = – 0.56 is :

(a) 0.7123 (b) 0.2877 (c) – 0.2123 (d) 0.2123

Ans. (b) 0.2877

Assertion Reasoning Based Questions

Q19. Assertion: The degree of the given differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 3.

Reason: The highest order derivative involved in a differential equation, when it is a polynomial in derivatives, is called its degree.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol: (d) Assertion (A) is false but reason (R) is true.

Q20. **Assertion (A):** Minor of element 6 in the matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$ is – 3.

Reason (R): Minor of an element a_{ij} of a matrix is the determinant obtained by deleting its i^{th} row.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol : (c) Assertion (A) is true but reason (R) is false.

SECTION - B

(All questions are compulsory. In case of internal choice, attempt any one question only)

Q21. Certain medicine is packed into bags by a machine. A random sample of 10 bags is taken and the mean of their weight (in kg) is found to be 49, their volume is 49.56. Test if the average packing can be taken to be 50 kg. [Given t_g (0.05) = 2.26]

Ans.

Here, $H_0 : \mu = 50$

$H_1 : \mu \neq 50$

Applying t –test

$$t = \frac{\bar{X} - \mu}{S / \sqrt{n}}$$

$$t = \frac{49 - 50}{\sqrt{49.56 / 10}}$$

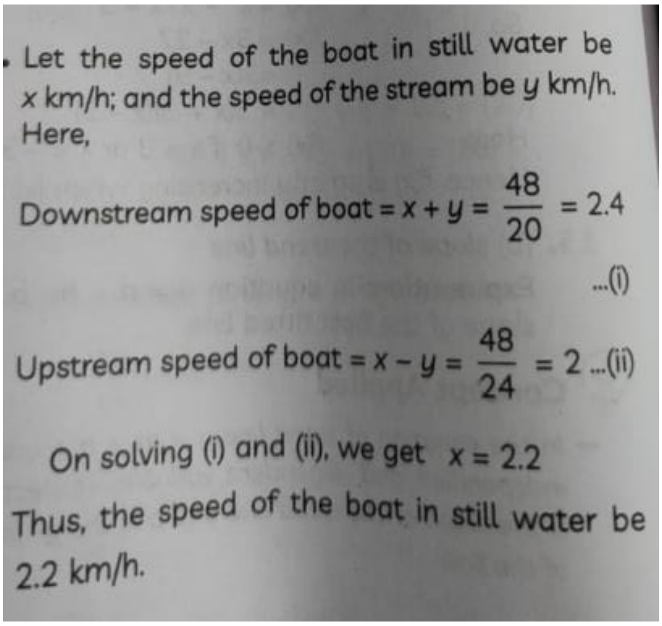
$$t = -0.45$$

$$|t| = 0.45$$

Tabulated $t_{0.05}$ for (10 – 1) i.e. d.f. is 2.26.

Since, $|t| < t_{0.05}$, the null hypothesis is acepted at 5% level of significance and we may conclude that data are consistent with the average packing of 50 kg.

Q22. A boat goes 48 km downstream in 20 hours. It takes 4 hours more to cover the same distance against the stream. Find the speed of the boat in the still water.

Ans. 

Let the speed of the boat in still water be x km/h; and the speed of the stream be y km/h.

Here,

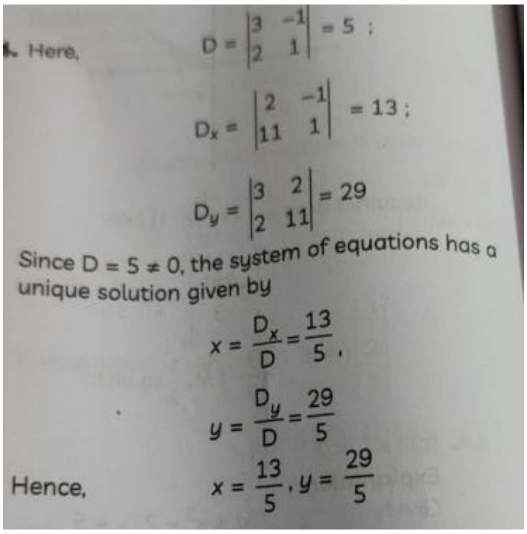
Downstream speed of boat = $x + y = \frac{48}{20} = 2.4$ ...(i)

Upstream speed of boat = $x - y = \frac{48}{24} = 2$ ...(ii)

On solving (i) and (ii), we get $x = 2.2$

Thus, the speed of the boat in still water be 2.2 km/h.

Q23. Using Cramer's rule, solve the system of equations: 3x-y=2 2x+y=11

Ans. 

Here, $D = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 5$;

$D_x = \begin{vmatrix} 2 & -1 \\ 11 & 1 \end{vmatrix} = 13$;

$D_y = \begin{vmatrix} 3 & 2 \\ 2 & 11 \end{vmatrix} = 29$

Since $D = 5 \neq 0$, the system of equations has a unique solution given by

$x = \frac{D_x}{D} = \frac{13}{5}$,

$y = \frac{D_y}{D} = \frac{29}{5}$

Hence, $x = \frac{13}{5}, y = \frac{29}{5}$

Q24 If X is a random variable and a, b are real numbers, then prove that E(ax + b) = a E(X) + b

OR

A salesman wants to know the average number of units he sells per sales call. He checks his past sales records and comes up with the following probabilities:

Sales (in Units)	**Probability**
0	0.15
1	0.20
2	0.10

3	0.05
4	0.30
5	0.20

Determine the expected value of the number of units he sells per sales call.

Ans.

$E(X) = \sum x_i p_i + b\sum p_i$ gives

$E(aX + b) = \sum (ax_i + b)p_i$

$= a \sum x_i p_i + b\sum p_i$

$= a \sum x_i p_i + b \times 1$

$\{\because \sum p_i = 1\}$

$= E(X) + b$

OR

$E(X) = \sum x_i p_i$

$= (0 \times 0.15) + (1 \times 0.20) + (2 \times 0.10) + (3 \times 0.05) + (4 \times 0.30) + (5 \times 0.20)$

$= 0 + 0.20 + 0.20 + 0.15 + 1.2 + 1 = 2.75$

Thus, the expected value of the number of units he would sell per sale call is 2.75.

Q25.If the cash equivalent to a perpetuity ₹x payable at the end of every six months is ₹20,000, if money is 5% p.a. compounded half-yearly. Find the value of x.

OR

Find the EMI of a loan of 10,00,000 for 15 years at 11% per annum [Given $(1.0092)^{180} = 5.19876$]

Ans.

SECTION - B

21. We know that the present value of perpetuity of ₹ R payable at the end of every period forever at a rate of i per period is given by $\frac{R}{i}$.

i.e. $P_\infty = \frac{R}{i}$

Here, R = x, i per period $= \frac{5}{200} = 0.025$,

$P_\infty = 20{,}000$

So, $P_\infty = \frac{R}{i}$

$\Rightarrow 20{,}000 = \frac{x}{0.025}$

$\Rightarrow x = 500$

So the value of x is ₹500.

OR

When P denotes the loan amount, r the interest rate (monthly) and n the number of payments, then EMI $= \frac{P \times r \times (1+r)^n}{(1+r)^n - 1}$

Here, P = ₹10,00,000; r = 0.0092 and n = 15 × 12 = 180 months

So, EMI = ₹10,00,000 × 0.0092 × $\frac{5.19876}{4.19876}$

= 11391.12

Thus, the EMI is ₹11391.12

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

Q 26. A man borrows 10,00,000 and agrees to pay interest quarterly at an annual rate of interest of 8%. At the same time, he sets up a sinking fund in order to repay the loan at the end of 5 years. If the sinking fund earns interest at 6% per annum, compounded quarterly, find the quarterly cost of the debt . { Given $(1.015)^{20} = 1.346852$}

Sol

SECTION - C

26. Here, the amount borrowed = ₹10,00,000;
Rate of interest = 8% = 0.08

So, quarterly interest payment on the loan

$= ₹(10,00,000 \times 0.02) = ₹20,000$

Amount S to be accumulated in the sinking fund $= ₹10,00,000$

Rate of interest = 6% = 0.06 and n = 5 years = 20 quarters

Since interest is compounded quarterly,

$$r = \frac{0.06}{4} = 0.015$$

Let ₹ A be the quarterly instalment payable in the sinking fund. Then,

$$A = \frac{Sr}{(1+r)^n - 1}$$

$$= \frac{10,00,000(0.015)}{(1.015)^{20} - 1}$$

$$= \frac{15,000}{1.346855 - 1}$$

$$= \frac{15,000}{0.346855} = ₹43246 \text{ (approx)}$$

Thus, the quarterly cost of the debt is ₹43,246 + ₹ 20,000, *i.e.*, ₹ 63,246.

Caution

Q27. Surjeet purchased a new house , costing ₹40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly installment is ₹30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use (1.0075)-300=0.1062]

Sol

27. Purchase = ₹ 40,00,000

Down payment = x

Balance = 40,00,000 − x

$$i = \frac{9}{1200} = 0.0075$$

$n = 25 \times 12 = 300$

E = ₹ 30,000

$$\Rightarrow \quad 30000 = \frac{(4000000 - x) \times 0.0075}{1 - (1.0075)^{-300}}$$

$$\Rightarrow \quad 30000 = \frac{(4000000 - x) \times 0.0075}{1 - 0.1062}$$

$\Rightarrow$ x = 424800

Down payment = ₹ 4,24,800

Q 28. Solve the following LPP graphically using corner point method:

Maximise Z=5x+3y

subject to the constraints

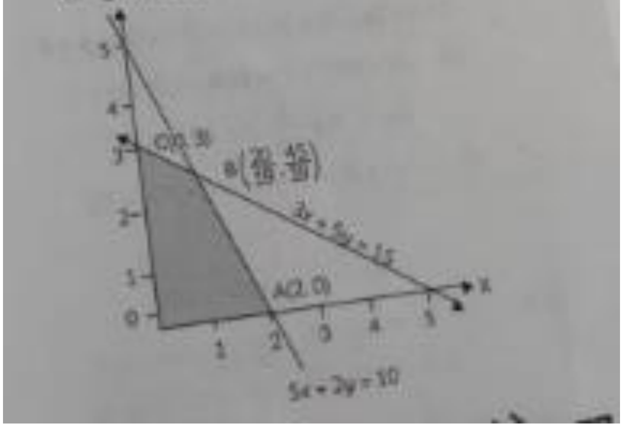
$3x + 5y < 15$; $5x + 2y \leq 10$

$x \geq 0$, $y \geq 0$.

Sol

Down payment = ₹ 4,24,000

28. Here, the graph of the given constraints and the feasible region, so obtained, is shaded in the graph given below



To find intersection point:

Eqⁿ of lines will be $3x + 5y = 15$ ...(i)

$5x + 2y = 10$ ...(ii)

Solving (i) and (ii)

$x = \frac{20}{19}, y = \frac{45}{19}$

Corner points	Corresponding value of Z
O (0, 0)	$5(0) + 3(0) = 0$
A (2, 0)	$5(2) + 3(0) = 10$
$B\left(\frac{20}{19}, \frac{45}{19}\right)$	$5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19} = 12\frac{7}{19}$
C (0, 3)	$5(0) + 3(3) = 9$

Thus, maximum value of Z is $12\frac{7}{19}$ which occurs at $B\left(\frac{20}{19}, \frac{45}{19}\right)$

Q29 Three pipes A, B and C can fill a tank together in 8 hours. After working at it together for 2 hours B is closed and A and C can fill the remaining part of the tank in 9 hours. Find the time in which B alone can fill the tank.

Ans.

Part of the tank that will be filled by three pipes together in 2 hours $= \frac{2}{8} = \frac{1}{4}$

Remaining part of the tank $= 1 - \frac{1}{4} = \frac{3}{4}$

In 9 hours, A and C can fill $\frac{3}{4}$ of the tank.

So, in 1 hour, A and C can fill

$\left(\frac{3}{4} \times \frac{1}{9}\right)$, i.e., $\frac{1}{12}$ of the tank.

Further,

Part of the tank that will be filled by three pipes together in 1 hour $= \frac{1}{8}$

Part of the tank that will be filled by B alone

$= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$

So, B alone can fill the tank in 24 hours.

Q30. X is a discrete random variable having the following probability distribution:

x_i	0	1	2	3	4	5	6	7
P (X = x_i)	0	k	2k	2k	$3k$	k^2	$2k^2$	$7k^2$ +k

(A) Determine the constant k

(B) Find P(X < 6)

(C) Find P(X ≥ 6)

Sol.

30. Since $\sum P(X = x_i) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

(A) $9k + 10k^2 = 1 \Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow k = \frac{1}{10}$ $[\because k \neq -1]$

(B) $P(X < 6) = 1 - [P(X = 6) + P(X = 7)]$

$= 1 - \left\{\frac{2}{100} + \frac{17}{100}\right\}$

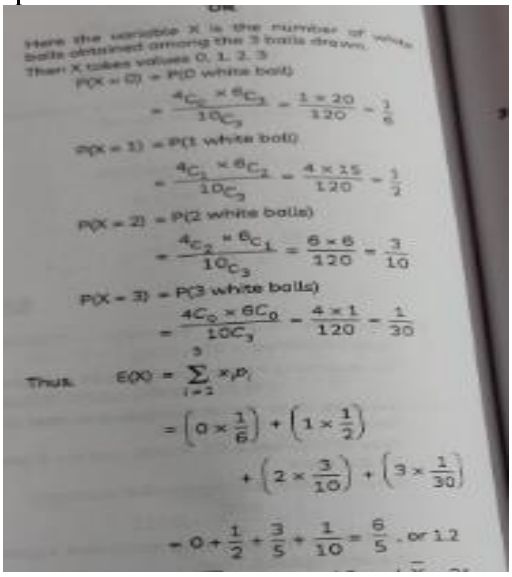
$= \frac{81}{100}$, or 0.81

(C) $P(X \geq 6) = P(X = 6) + P(X = 7)$

$= \frac{2}{100} + \frac{17}{100} = \frac{19}{100}$ or 0.19

OR

A box contains 4 white and 6 black balls. If 3 balls are drawn at random, find the mathematical expectation of the number of white balls.

Sol. 

Here the variable X is the number of white balls obtained among the 3 balls drawn. Then X takes values 0, 1, 2, 3

$P(X = 0) = P(0 \text{ white ball})$

$= \frac{{}^4C_0 \times {}^6C_3}{{}^{10}C_3} = \frac{1 \times 20}{120} = \frac{1}{6}$

$P(X = 1) = P(1 \text{ white ball})$

$= \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{4 \times 15}{120} = \frac{1}{2}$

$P(X = 2) = P(2 \text{ white balls})$

$= \frac{{}^4C_2 \times {}^6C_1}{{}^{10}C_3} = \frac{6 \times 6}{120} = \frac{3}{10}$

$P(X = 3) = P(3 \text{ white balls})$

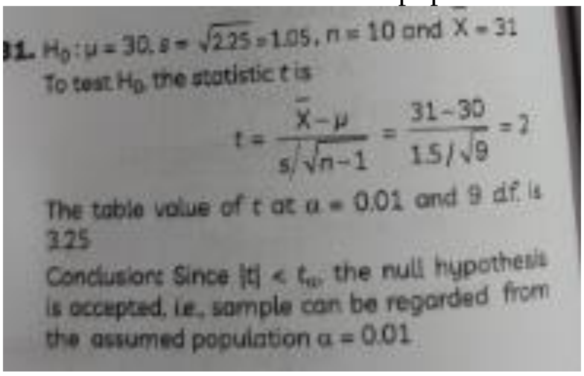
$= \frac{4C_0 \times 6C_0}{10C_3} = \frac{4 \times 1}{120} = \frac{1}{30}$

Thus $E(X) = \sum_{i=1}^{3} x_i p_i$

$= \left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{30}\right)$

$= 0 + \frac{1}{2} + \frac{3}{5} + \frac{1}{10} = \frac{6}{5}$, or 1.2

Q31. A sample size of 10 drawn from a normal population has a mean 31 and a variance 2.25. Is it reasonable to assume that the mean of the population is 30? [Use 1% level of significance, given that [(t_9 (0.1)=3.25]

Sol 

31. $H_0: \mu = 30, s = \sqrt{2.25} = 1.05, n = 10$ and $\bar{X} = 31$

To test H_0, the statistic t is

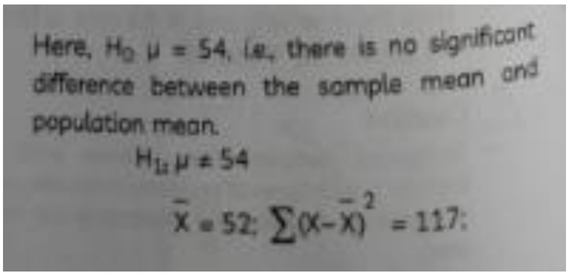
$$t = \frac{\bar{X} - \mu}{s/\sqrt{n-1}} = \frac{31-30}{1.5/\sqrt{9}} = 2$$

The table value of t at $\alpha = 0.01$ and 9 d.f. is 3.25

Conclusion: Since $|t| < t_\alpha$, the null hypothesis is accepted, i.e., sample can be regarded from the assumed population $\alpha = 0.01$

OR

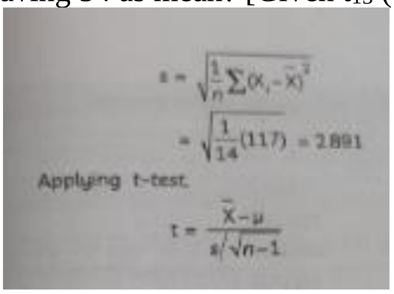
From a sample size of 14 has 52 as mean. The sum of squares of deviations from mean is 117. Can this sample be regarded as taken from the population having 54 as mean? [Given t_{13} (0.05) = 1.77].

Sol. 

Here, H_0: $\mu = 54$, i.e., there is no significant difference between the sample mean and population mean.

$H_1: \mu \neq 54$

$\bar{X} = 52;\ \sum(X - \bar{X})^2 = 117;$



$s = \sqrt{\frac{1}{n}\sum(X_i - \bar{X})^2}$

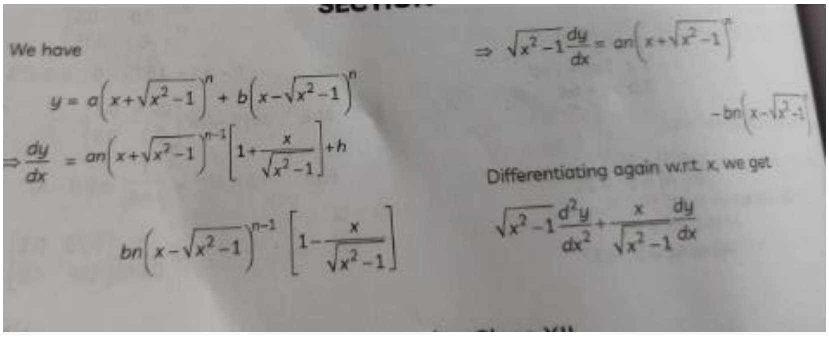
$= \sqrt{\frac{1}{14}(117)} = 2.891$

Applying t-test.

$t = \frac{\bar{X} - \mu}{s/\sqrt{n-1}}$

SECTION – D

Q32. If y = a $\left(x + \sqrt{x^2+1}\right)^n$ + b $\left(x - \sqrt{x^2-1}\right)^n$, then show that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$.



We have

$$y = a\left(x+\sqrt{x^2-1}\right)^n + b\left(x-\sqrt{x^2-1}\right)^n$$

$$\Rightarrow \frac{dy}{dx} = an\left(x+\sqrt{x^2-1}\right)^{n-1}\left[1+\frac{x}{\sqrt{x^2-1}}\right] + h$$

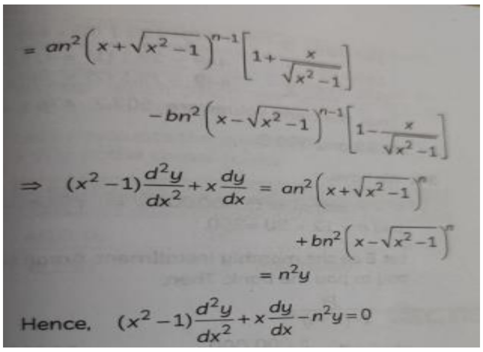
$$bn\left(x-\sqrt{x^2-1}\right)^{n-1}\left[1-\frac{x}{\sqrt{x^2-1}}\right]$$

$$\Rightarrow \sqrt{x^2-1}\frac{dy}{dx} = an\left(x+\sqrt{x^2-1}\right)^n$$

$$-bn\left(x-\sqrt{x^2-1}\right)^n$$

Differentiating again w.r.t. x, we get

$$\sqrt{x^2-1}\frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2-1}}\frac{dy}{dx}$$



$$= an^2\left(x+\sqrt{x^2-1}\right)^{n-1}\left[1+\frac{x}{\sqrt{x^2-1}}\right]$$

$$-bn^2\left(x-\sqrt{x^2-1}\right)^{n-1}\left[1-\frac{x}{\sqrt{x^2-1}}\right]$$

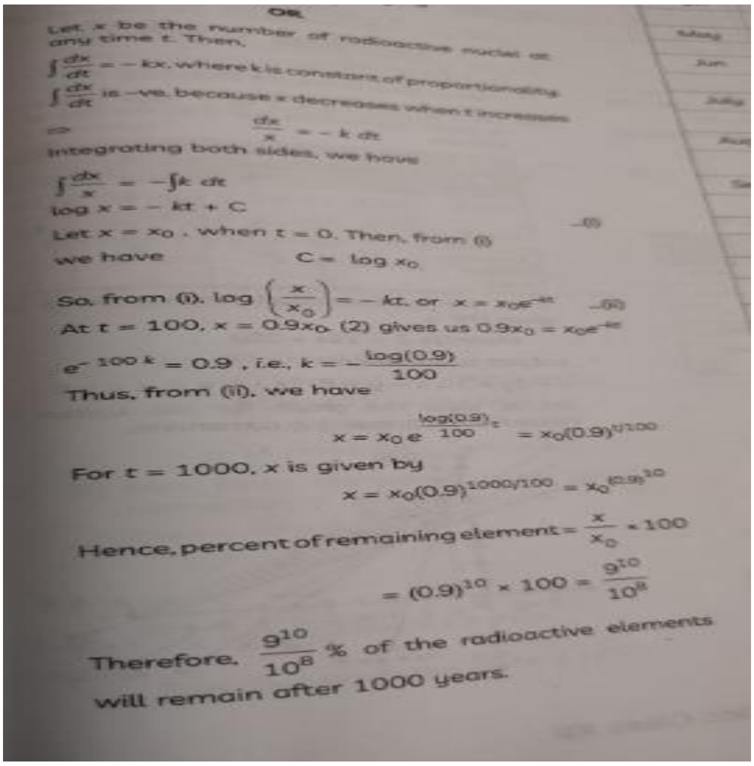
$$\Rightarrow (x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = an^2\left(x+\sqrt{x^2-1}\right)^n + bn^2\left(x-\sqrt{x^2-1}\right)^n = n^2y$$

Hence, $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$

OR

Assume that the rate at which radioactive substances decay is known to be proportional to the number of such nuclei that are present at the time in a given sample.

In a certain sample, 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. Find what percentage of the original radioactive nuclei will remain after 1000 years.

Ans. 

OR

Let x be the number of radioactive nuclei at any time t. Then,

$\frac{dx}{dt} = -kx$, where k is constant of proportionality.

$\frac{dx}{dt}$ is −ve, because x decreases when t increases

$$\Rightarrow \frac{dx}{x} = -k\,dt$$

Integrating both sides, we have

$$\int \frac{dx}{x} = -\int k\,dt$$

$$\log x = -kt + C \quad \text{...(i)}$$

Let $x = x_0$, when $t = 0$. Then, from (i) we have $C = \log x_0$

So, from (i), $\log\left(\frac{x}{x_0}\right) = -kt$, or $x = x_0e^{-kt}$ ...(ii)

At $t = 100$, $x = 0.9x_0$. (2) gives us $0.9x_0 = x_0e^{-kt}$

$e^{-100k} = 0.9$, i.e., $k = -\frac{\log(0.9)}{100}$

Thus, from (ii), we have

$$x = x_0e^{\frac{\log(0.9)}{100}t} = x_0(0.9)^{t/100}$$

For $t = 1000$, x is given by

$$x = x_0(0.9)^{1000/100} = x_0(0.9)^{10}$$

Hence, percent of remaining element $= \frac{x}{x_0} \times 100$

$$= (0.9)^{10} \times 100 = \frac{9^{10}}{10^8}$$

Therefore, $\frac{9^{10}}{10^8}$ % of the radioactive elements will remain after 1000 years.

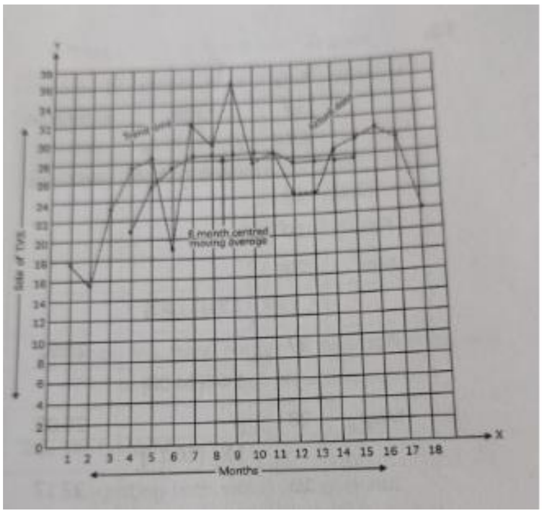
Q 33 Coded monthly sales figures of a particular brand of T.V. for 18 months commencing January 2020 are as follows:

Month	January 2020	February 2020	March 2020	April 2020	May 2020	June 2020	July 2020	August 2020	September 2020
Sale of T.V.	18	16	23	27	28	19	31	29	35
Month	October 2020	November 2020	December 2020	Januar y 2021	February 2021	March 2021	April 2021	May 2021	June 2021
Sale of T.V.	27	28	24	24	28	29	30	29	22

Calculate 6-monthly moving averages and display these and the original figures on the same graph, using the same axes for both.

Months	Sale of T.V.s	6 – monthly total (6-monthly moving average)	6 – monthly centred moving average
Jan (2020)	18		
Feb	16		
Mar	23		
		131 (21.83)	
Apr.	27		22.92
		144 (24.00)	
May	28		25.08
		157 (26.17)	
Jun	19		27.17
		169 (28.17)	
July	31		28.17
		169 (28.17)	
Aug.	29		28.17
		169 (28.17)	
Sept.	35		28.59
		174 (29.00)	
Oct.	27		28.42
		167 (27.83)	
Nov.	28		27.75
		166 (27.67)	
Dec.	24		27.17
		160 (26.67)	
Jan(2021)	24		26.92
		163 (27.17)	
Feb	28		27.25
		164 (27.33)	
Mar	29		27.17
		162 (27.00)	
Apr.	30		
May	29		
Jun	22		

Thus, the 6 – monthly centred moving averages are 22.92, 25.08, 27.17, 28.17, 28.17, 28.59, 28.42, 27.75, 27.17, 26.92, 27.25 and 27.17.



OR

The revenue from sales tax in a state during 1995-96 to 1999-2000 is shown in the following table. Fit a straight line trend by the method of least squares. Also, tabulate the trend values.

Year	1995 -96	1996 – 97	1997-98	1998-99	1999-2000
Revenue (in ₹ lakhs)	427	612	521	195	490

Ans.

OR

Let the equation of the straight line of best fit, with the origin at the middle year 1997-98 and units of x as 1 year, be $y = a + bx$

By the method of least squares, the values of 'a' and 'b' are given by

$$a = \frac{\Sigma y}{n} \text{ and } b = \frac{\Sigma xy}{\Sigma x^2} \quad ...(i)$$

Here, $n = 5$

Calculation for fitting the line of best fit

Year	Revenue (y)	x	x^2	xy
1995-96	427	–2	4	–854
1996-97	612	–1	1	–612
1997-98	521	0	0	0
1998-99	195	1	1	195
1990-2000	490	2	4	980
	$\Sigma y = 2245$	$\Sigma x = 0$	$\Sigma x^2 = 10$	$\Sigma xy = -291$

Using (i), we have

$$a = \frac{\sum y}{n} = \frac{2245}{5} = 449$$

and

$$b = \frac{\sum xy}{\sum x^2} = \frac{-291}{10} = -29.1$$

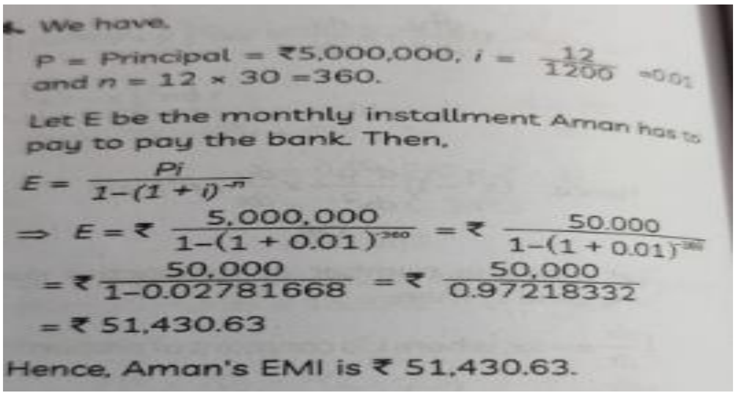
Hence, the required equation of the best fitted straight line is $y = 449 - 29.1x$

Year	x	Trend values ($y = 449 - 29.1x$)
1995-96	–2	449 – 29.1 (–2) = 507.2
1996-97	–1	449 – 29.1 (–1) = 478.1
1997-98	0	449 – 29.1 (0) = 449
1998-99	1	449 – 29.1 (1) = 419.9
1990-2000	2	449 – 29.1 (2) = 390.8

Thus, the trend values are 507.2, 478.1, 449, 419.9 and 390.8.

Q34. Aman borrowed a home loan amount of ₹5,000,000 from a bank at an interest rate of 12% per annum for 30 years. Find the monthly installment amount Aman has to pay to the bank.

(Given $(1.01)^{-360} = 0.02781668$)

Ans. 

We have,

P = Principal = ₹5,000,000, $i = \frac{12}{1200} = 0.01$

and $n = 12 \times 30 = 360$.

Let E be the monthly installment Aman has to pay to pay the bank. Then,

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

$$\Rightarrow E = ₹\frac{5,000,000}{1-(1+0.01)^{-360}} = ₹\frac{50,000}{1-(1+0.01)^{-360}}$$

$$= ₹\frac{50,000}{1-0.02781668} = ₹\frac{50,000}{0.97218332}$$

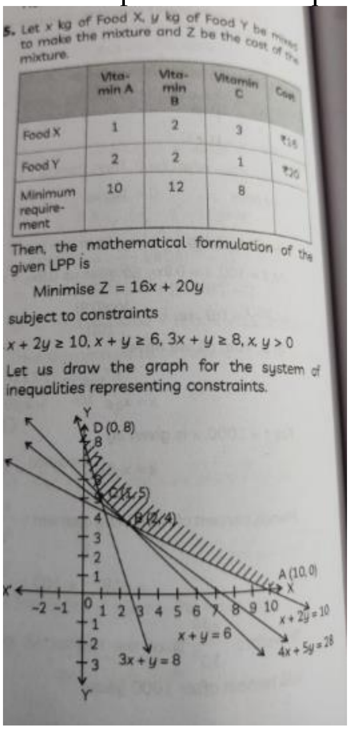
$$= ₹\ 51,430.63$$

Hence, Aman's EMI is ₹ 51,430.63.

Q35. A dietician wished to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 unit of vitamin C. The vitamin contents of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs ₹16 and one kg of food Y costs ₹20. Find the least cost of the mixture which will produce the required diet.

Ans. 

5. Let x kg of Food X, y kg of Food Y be mixed to make the mixture and Z be the cost of the mixture.

	Vitamin A	Vitamin B	Vitamin C	Cost
Food X	1	2	3	₹16
Food Y	2	2	1	₹20
Minimum requirement	10	12	8	

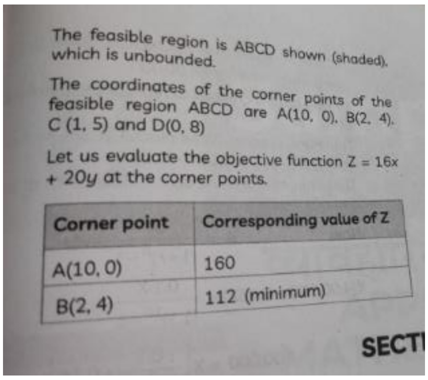
Then, the mathematical formulation of the given LPP is

Minimise $Z = 16x + 20y$

subject to constraints

$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x, y > 0$

Let us draw the graph for the system of inequalities representing constraints.



The feasible region is ABCD shown (shaded), which is unbounded.

The coordinates of the corner points of the feasible region ABCD are A(10, 0), B(2, 4), C (1, 5) and D(0, 8)

Let us evaluate the objective function Z = 16x + 20y at the corner points.

Corner point	Corresponding value of Z
A(10, 0)	160
B(2, 4)	112 (minimum)

SECTI

SECTION - E

(All questions are compulsory. In case of internal choice, attempt any one question only)

Case Study - I

Q36. On her birthday, Radha decided to donate some money to children of an orphanage.

If there are 8 children less, everyone will get ₹10 more. However, if there are 16 children more, everyone will get ₹10 less. Let the number of children be x and amount distributed by Radha to each child be 'y'.

Based on the above information, answer the following questions:

(A) Write the system of equations in matrix form.

(B) Write the inverse of the matrix $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$.

(C) What is the value of x?

OR

What is the value of y?

Ans.

(A) Total amount distributed = xy

A.T.Q.

$(x-8)(y+10) = xy$

$\Rightarrow xy - 8y + 10x - 80 = xy$

$\Rightarrow 5x - 4y = 40$ ...(i)

and $(x+16)(y-10) = xy$

$\Rightarrow xy + 16y - 10x - 160 = xy$

$\Rightarrow 16y - 10x = 160$

$\Rightarrow -5x + 8y = 80$ ...(ii)

Equation (i) and (ii) are the required equations.

Hence, the system of equations in matrix form is

$$\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$$

(B) The equations in matrix form is:

$$\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}^{-1}\begin{bmatrix} 40 \\ 50 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$

$|A| = 40 - 20 = 20$

$\therefore |A| \neq 0$. A is a non-singular matrix.

$$\text{adj } A = \begin{bmatrix} 8 & 5 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

$$\left[\because A^{-1} = \frac{1}{|A|}(\text{adj}A)\right]$$

(C) Since the system of equation is matrix form:

$$\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{matrix} 40 \\ 80 \end{matrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ 80 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 320+320 \\ 200+400 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$x = 32$

OR

Since the system of equation is matrix form:

$$\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{matrix} 40 \\ 80 \end{matrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ 80 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} 320+320 \\ 200+400 \end{bmatrix}$$

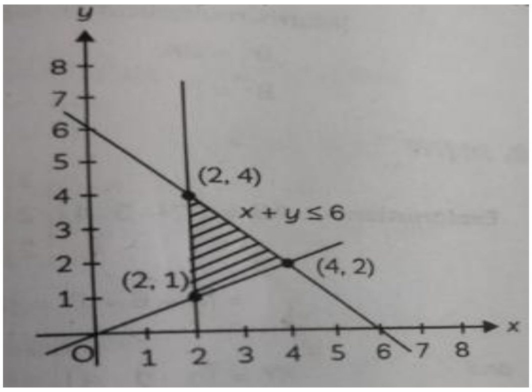
$$= \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$y = 30$

Q37. **Case study – II**

One very useful application of linear programming is its a graphical method for solving problems in two variable. Mrs. Meena wanted to use this concept to help students figure out how the area of the 3D-model is composed of 3 staight lines.

On the below diagram (cross section of 3D model), O is the origin. The shaded region R is defined by three inequalities one of the three inequalities is $x + y \leq 6$.



Based on the given information, answer the following questions:

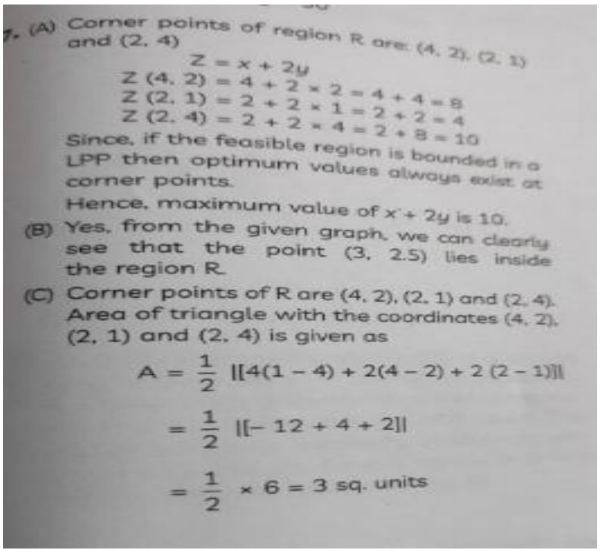
(A) Given that the point (x,y) is in the region R, then what is the maximum value of x + 2y ?

(B) Does the point (3, 2.5) lies inside the region R?

(C) What is the area of the region R?

OR

What are the two other inequalities on the graph?

Ans. 

7. (A) Corner points of region R are: (4, 2), (2, 1) and (2, 4)

$Z = x + 2y$

$Z(4, 2) = 4 + 2 \times 2 = 4 + 4 = 8$

$Z(2, 1) = 2 + 2 \times 1 = 2 + 2 = 4$

$Z(2, 4) = 2 + 2 \times 4 = 2 + 8 = 10$

Since, if the feasible region is bounded in a LPP then optimum values always exist at corner points.

Hence, maximum value of x + 2y is 10.

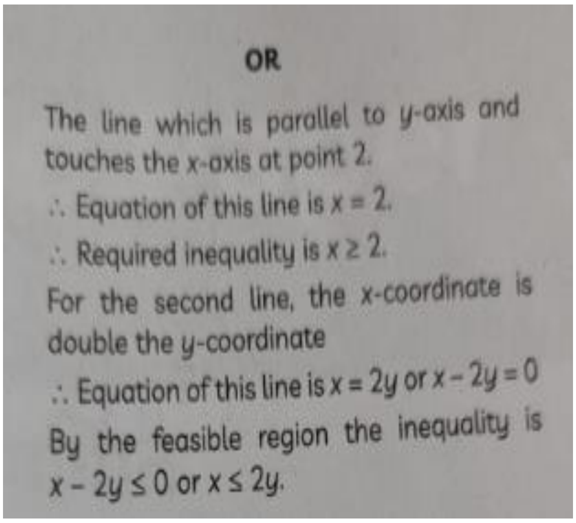
(B) Yes, from the given graph, we can clearly see that the point (3, 2.5) lies inside the region R.

(C) Corner points of R are (4, 2), (2, 1) and (2, 4). Area of triangle with the coordinates (4, 2), (2, 1) and (2, 4) is given as

$A = \frac{1}{2} |[4(1-4) + 2(4-2) + 2(2-1)]|$

$= \frac{1}{2} |[-12 + 4 + 2]|$

$= \frac{1}{2} \times 6 = 3$ sq. units



OR

The line which is parallel to y-axis and touches the x-axis at point 2.

∴ Equation of this line is $x = 2$.

∴ Required inequality is $x \geq 2$.

For the second line, the x-coordinate is double the y-coordinate

∴ Equation of this line is $x = 2y$ or $x - 2y = 0$

By the feasible region the inequality is $x - 2y \leq 0$ or $x \leq 2y$.

Q38. **Case-Study 3**

Under the pure market competition scenario, the demand function p_d and the supply function p_s for a certion commodity are given as $p_d = \frac{8}{x+1} - 2$ and $p_s = \frac{x+3}{2}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals, find the producer's surplus.

OR

Using integration, find the area of the region bounded by the line Y=|x+1|+1 and the line x = – 3 and x = 3 and y = 0.

Ans.

38. Under pure competition, $p_d = p_s$

$$\Rightarrow \quad \frac{8}{x+1} - 2 = \frac{x+3}{2}$$

$$\Rightarrow \quad x^2 + 8x - 9 = 0$$

$$\Rightarrow \quad x = -9, 1$$

$$\therefore \quad x = 1$$

When $x_0 = 1$ $\quad (\because x \neq -9)$

$$\Rightarrow \quad p_0 = 2$$

$\therefore$ Producer's surplus $= 2 - \int_0^1 \frac{x+3}{2} dx$

$$= 2 - \left[\frac{x^2}{4} + \frac{3x}{2}\right]_0^1 = \frac{1}{4}$$

OR

Given equations of the curves are:

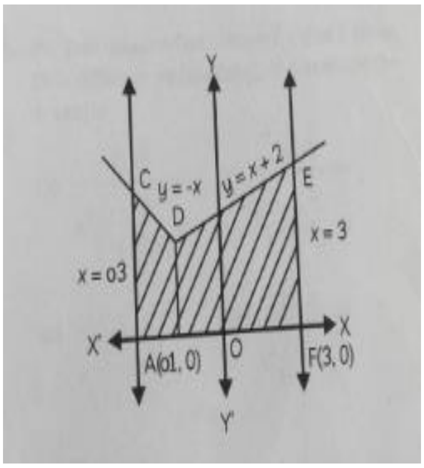
$$y = |x + 1| + 1$$

$$= \begin{cases} (x+1)+1, \text{if } x+1 \geq 0 \\ -1-x+1, \text{if } x+1 < 0 \end{cases}$$

$$= \begin{cases} x+2, \text{if } x \geq -1 \\ -x, \text{if } x < -1 \end{cases}$$

$$x = -3, x = 3 \text{ and } y = 0$$

Sketch of the region bounded by the given curves is shown below:



Required area $= \int_{-3}^{-1} y dx + \int_{-1}^{3} y dx$

$$= \int_{-3}^{-1} (-x) dx + \int_{-1}^{3} (x+2) dx$$

$$= \left[-\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1-9)$$

$$+ \left[\left(\frac{9}{2}+6\right) - \left(\frac{1}{2}-2\right)\right]$$

$= 16$ sq. units.