# BGS INTERNATIONAL PUBLIC SCHOOL

**SECTOR-5, DWARKA, NEW DELHI -75**

**PREBOARD - I (2023-24)**

**SUB: APPLIED MATHEMATICS (241)** **SCHOOLCODE: 25279**
**CLASS: XII** **TIME: 3 Hours**
**SET: 2** **M.M:80**

**General instructions:**

(i) This question paper contains 5 sections A, B, C, D and E. Each section is compulsory.
(ii) Section -A carries 20 marks weightage, Section -B carries 10 marks weightage, Section- C carries 18 marks weightage, section- D carries 20marks weightage and section- E carries 3 case based with total weightage of 12 marks.
(iii) Section A : It comprises of **18 MCQ's of 1 mark** each and 2 **Assertion- Reasoning Based** questions
(iv) Section B : It comprises of **5 VSA type questions of 2 marks** each,
(v) Section C: It comprises of **6 SA type of questions of 3 marks** each.
(vi) Section D :It comprises of **4 LA type of questions of 5 marks** each.
(vii) Section E: It has **3 CASE STUDY BASED QUESTION of 4 marks** each.

**SECTION - A**

**(All questions are compulsory. No internal choice is provided in this section)**

Q1. What is the least value of 'x' that satisfies $x \equiv 27 \pmod 4$, when $27 < x \leq 36$ ?

(a) 27 (b) 30 (c) 31 (d) 35

**OR**

In a 2 km race, P can give Q a start of 200 m and R a start of 560 m. Then, in the same race, Q can give R a start of:

(a) 360 m (b) 380 m (c) 400 m (d) 430 m

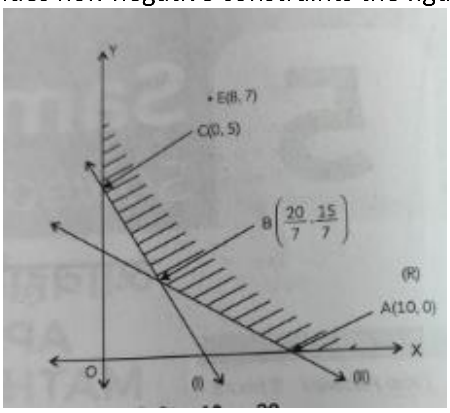
Q 2 A specific characteristics of a sample is called:

(a) Variance (b) statistics (c) parameter (d) population

Q3 Pipe A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the time taken to fill the tank is:

(a) 2 hours (b) $2\frac{3}{4}$ hours (c) 3 hours (d) $3\frac{9}{17}$ hours

Q4. Besides non-negative constraints the figure given below is subject to which of the following constraints:




(a) $x + y \leq 5$; $3x + 10y \geq 30$ (b) $x + y \geq 5$; $3x + 10y \leq 30$ (c) $x + y \leq 5$; $3x + 10y \leq 30$ (d) $x + y \geq 5$; $3x + 10y \geq 30$

**OR**

For the LP problem maximize $Z = 2x + 3y$. The co-ordinates of the corner point Of the bounded feasible region are

A (3, 3), B (20, 3), C (20, 10), D (18, 12), E (12, 12). The Maximum value of Z is

(a) 72 (b) 80 (c) 82 (d) 70

Q5 If $x > y$ and $z < 0$, then :

(a) $xz > yz$ (b) $\frac{x}{z} > \frac{y}{z}$ (c) $xz \geq yz$ (d) $\frac{x}{z} < \frac{y}{z}$

Q6 The values of x which satisfy both the linear inequations simultaneously, are

$4x - 1 < 0,\ 3 - 4x < 0$

(a) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (b) $\left(-\frac{1}{4}, \frac{3}{4}\right)$ (c) $\left(\frac{3}{4}, \frac{1}{4}\right)$ (d) no value

Q7. Fill in the blanks : If A = $\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$, then $A^2$ = ……………… .

**OR**

If A = $[a_{ij}]_{3\times 3}$ = $\begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then cofactor of $a_{12}$ is = ……………. .

Q8. If A = $\begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and AB = $I_3$, then (x + y) equals:

(a) 0 (b) – 1 (c) 2 (d) – 2

Q9. The function $f(x) = a^x$ is increasing on R, if :

(a) $a > 0$ (b) $a < 0$ (c) $0 < a < 1$ (d) $a > 1$

Q10. The normal distribution curve is symmetrical about :

(a) $X = \mu$ (b) $X = \sigma$ (c) $X = \frac{\mu}{\sigma}$ (d) $X = \frac{\sigma}{\mu}$

Q11. $\int_0^{40} \frac{dx}{2x+1}$ = log k, then the value of k is:

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{9}{2}$ (d) 9

Q12. In the equation of trend line y = a + bx, b denotes:

(a) mean of y (b) slope of the trend line (c ) total of x (d) total of xy

Q13. The marks obtained were found normally distributed with mean 75 and variance 100. The percentage of students who scored more than 75 marks is:

(a) 25 % (b) 50 % (c) 75 % (d) 100 %

Q14. A machine costing ₹ 46,000 depreciates at an uniform rate of 12% per annum, then its value after 5 years is :

[use $\left(\frac{22}{25}\right)^5 = 0.5278$]

(a) ₹ 24278.50 (b) ₹ 22730.45 (c) ₹ 31272.20 (d) ₹ 20892.08

**OR**

The amount S of an annuity due A, when the rate of interest is r and n is the number of years, is given by:

(a) $S = \frac{A}{r}(1 + r)^n [(1+ r)^n - 1]$ (b) $S = \frac{A}{r}(1 + r) [(1+ r)^n - 1]$

(c) $S = Ar (1 + r)^n [(1+ r)^n - 1]$ (d) $S = Ar (1 + r) [(1+ r)^n - 1]$

Q15. In one sample t- test, the estimation for the population mean is:

(a) $\frac{\bar{X} - \mu}{S/\sqrt{n}}$ (b) $\frac{\bar{X} - \mu}{S/n}$ (c) $\frac{\bar{X} - \mu}{S^2/\sqrt{n}}$ (d) $\frac{\bar{X} - \mu}{S/n^2}$

Q16. If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$, such that f(2) = 0. Then, f(x) is :

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$ (c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

**OR**

If the function $f(x) = x^3 - 27x + 5$ is strictly increasing when:

(a) $x < -3$ (b) $|x| > 3$ (c) $x > 3$ (d) $|x| < 3$

Q17. If the supply function for a commodity is p = $\sqrt{x+9}$ and the market price is 4, then producer's surplus is :

(a) 3 (b) $\frac{10}{3}$ (c) 10 (d) 15

**OR**

Which of the following differential equations has y = $c_1 e^{x}$ + $c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

Q18. The area under the standard normal curve which lies to the left of z = – 0.56 is:

(a) 0.7123 (b) 0.2877 (c) – 0.2123 (d) 0.2123

**Assertion Reasoning Based Questions**

Q19. **Assertion:** The degree of the given differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 3.

**Reason:** The highest order derivative involved in a differential equation, when it is a polynomial in derivatives, is called its degree.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Assertion (A):**Minor of element 6 in the matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$ is – 3.

**Reason (R):** Minor of an element $a_{ij}$ of a matrix is the determinant obtained by deleting its $i^{th}$ row.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

## SECTION - B

**(All questions are compulsory. In case of internal choice, attempt any one question only)**

Q21. Certain medicine is packed into bags by a machine. A random sample of 10 bags is taken and the mean of their weight (in kg) is found to be 49, their volume is 49.56. Test if the average packing can be taken to be 50 kg.

[Given $t_g$ (0.05) = 2.26]

Q22. A boat goes 48 km downstream in 20 hours. It takes 4 hours more to cover the same distance against the stream. Find the speed of the boat in the still water.

Q23. Using Cramer's rule, solve the system of equations: 3x-y=2 2x+y=11

Q24. If X is a random variable and a, b are real numbers, then prove that E(ax + b) = a E(X) + b

**OR**

A salesman wants to know the average number of units he sells per sales call. He checks his past sales records and comes up with the following probabilities:

| Sales (in Units) | Probability |
|---|---|
| 0 | 0.15 |
| 1 | 0.20 |
| 2 | 0.10 |
| 3 | 0.05 |

| | |
|---|---|
| 4 | 0.30 |
| 5 | 0.20 |

Determine the expected value of the number of units he sells per sales call.

Q25.If the cash equivalent to a perpetuity ₹x payable at the end of every six months is ₹20,000, if money is 5% p.a. compounded half-yearly. Find the value of x.

**OR**

Find the EMI of a loan of 10,00,000 for 15 years at 11% per annum [Given $(1.0092)^{180}$ = 5.19876].

**SECTION – C**

**(All questions are compulsory. In case of internal choice, attempt any one question only)**

Q 26. A man borrows 10,00,000 and agrees to pay interest quarterly at an annual rate of interest of 8%. At the same time, he sets up a sinking fund in order to repay the loan at the end of 5 years. If the sinking fund earns interest at 6% per annum, compounded quarterly, find the quarterly cost of the debt {Given $(1.015)^{20}$ = 1.346852}.

Q27. Surjeet purchased a new house, costing ₹40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is ₹30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? [Use (1.0075)-300=0.1062]

Q 28. Solve the following LPP graphically using corner point method:

Maximise Z=5x+3y

subject to the constraints

$3x + 5y < 15$; $5x + 2y \leq 10$

$x \geq 0, y \geq 0$.

Q29 Three pipes A, B and C can fill a tank together in 8 hours. After working at it together for 2 hours B is closed and A and C can fill the remaining part of the tank in 9 hours. Find the time in which B alone can fill the tank.

Q30. X is a discrete random variable having the following probability distribution:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P (X = $x_i$) | 0 | k | 2k | 2k | $3k$ | $k^2$ | $2k^2$ | $7k^2$ +k |

(A) Determine the constant k

(B) Find P(X < 6)

(C) Find P( X ≥ 6)

**OR**

A box contains 4 white and 6 black balls. If 3 balls are drawn at random, find the mathematical expectation of the number of white balls.

Q31. A sample size of 10 drawn from a normal population has a mean 31 and a variance 2.25. Is it reasonable to assume that the mean of the population is 30? [Use 1% level of significance, given that [($t_9$ (0.1)=3.25]

**OR**

From a sample size of 14 has 52 as mean. The sum of squares of deviations from mean is 117. Can this sample be regarded as taken from the population having 54 as mean? [Given $t_{13}$ (0.05) = 1.77].

**SECTION – D**

**(All questions are compulsory. In case of internal choice, attempt any one question only)**

Q32. If $y = a\left(x + \sqrt{x^2+1}\right)^n + b\left(x - \sqrt{x^2-1}\right)^n$, then show that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$.

**OR**

Assume that the rate at which radioactive substances decay is known to be proportional to the number of such nuclei that are present at the time in a given sample.
In a certain sample, 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. Find what percentage of the original radioactive nuclei will remain after 1000 years.

Q 33 Coded monthly sales figures of a particular brand of T.V. for 18 months commencing January 2020 are as follows:

| Month | January 2020 | February 2020 | March 2020 | April 2020 | May 2020 | June 2020 | July 2020 | August 2020 | September 2020 |
|---|---|---|---|---|---|---|---|---|---|
| Sale of T.V. | 18 | 16 | 23 | 27 | 28 | 19 | 31 | 29 | 35 |
| Month | October 2020 | November 2020 | December 2020 | January 2021 | February 2021 | March 2021 | April 2021 | May 2021 | June 2021 |
| Sale of T.V. | 27 | 28 | 24 | 24 | 28 | 29 | 30 | 29 | 22 |

Calculate 6-monthly moving averages and display these and the original figures on the same graph, using the same axes for both.

**OR**

The revenue from sales tax in a state during 1995-96 to 1999-2000 is shown in the following table. Fit a straight line trend by the method of least squares. Also, tabulate the trend values.

| Year | 1995 -96 | 1996 – 97 | 1997-98 | 1998-99 | 1999-2000 |
|---|---|---|---|---|---|
| Revenue (in ₹ lakhs) | 427 | 612 | 521 | 195 | 490 |

Q34. Aman borrowed a home loan amount of ₹5,000,000 from a bank at an interest rate of 12% per annum for 30 years. Find the monthly installment amount Aman has to pay to the bank.
(Given $(1.01)^{-360} = 0.02781668$)

Q35. A dietician wished to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 unit of vitamin C. The vitamin contents of one kg food is given below:

| Food | Vitamin A | Vitamin B | Vitamin C |
|---|---|---|---|
| X | 1 | 2 | 3 |
| Y | 2 | 2 | 1 |

One kg of food X costs ₹16 and one kg of food Y costs ₹20. Find the least cost of the mixture which will produce the required diet.

## SECTION - E

**(All questions are compulsory. In case of internal choice, attempt any one question only)**

**Case Study - I**

Q36. On her birthday, Radha decided to donate some money to children of an orphanage.

If there are 8 children less, everyone will get ₹10 more. However, if there are 16 children more, everyone will get ₹10 less. Let the number of children be x and amount distributed by Radha to each child be 'y'.

Based on the above information, answer the following questions:

(A) Write the system of equations in matrix form.

(B) Write the inverse of the matrix $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$.

(C) What is the value of x?

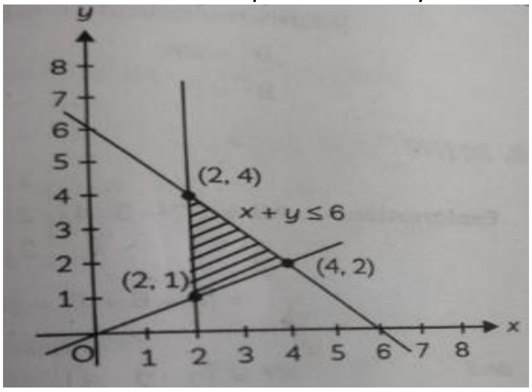
OR

What is the value of y?

Q37. **Case study – II**

One very useful application of linear programming is its a graphical method for solving problems in two variable. Mrs. Meena wanted to use this concept to help students figure out how the area of the 3D-model is composed of 3 straight lines.

On the below diagram (cross section of 3D model), O is the origin. The shaded region R is defined by three inequalities one of the three inequalities is $x + y \le 6$.




Based on the given information, answer the following questions:

(A) Given that the point (x,y) is in the region R, then what is the maximum value of x + 2y ?

(B) Does the point (3, 2.5) lies inside the region R?

(C) What is the area of the region R?

OR

What are the two other inequalities on the graph?

Q38. **Case-Study 3**

Under the pure market competition scenario, the demand function $p_d$ and the supply function $p_s$ for a certion commodity are given as $p_d = \frac{8}{x+1} - 2$ and $p_s = \frac{x+3}{2}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals, find the producer's surplus.

**OR**

Using integration, find the area of the region bounded by the line Y=|x+1|+1 and the line x = – 3 and x = 3 and y = 0.