BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION (BSSCA)
PRE-BOARD EXAMINATION (2023-2024)
SET-A

Date: 18-12-2023   **Max. Marks: 80**

Subject: Applied Mathematics (Code: 241)   **Time: 3hours**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there is internal choice in some questions.

2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA) questions of 2 marks each.

4. Section C has 6 Short Answer (SA) questions of 3 marks each.

5. Section D has 4 Long Answer (LA) questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.

7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

(All questions are compulsory. No internal choice is provided in this section)

1. Find the non-negative remainder when 7^{30} is divided by 5.   1

    (a) -1   (b) 1   (c) 4   (d) 3

2. The best-fitted trend line is one for which the sum of squares of residuals or error is   1

    (a) Positive   (b) Minimum   (c) Negative   (d) Maximum

3. A sample of 100 wall clocks is taken at random. Out of 100, it was found that 25 wall clocks are digital and 75 are analogue. The point estimate of population proportion of analogue clocks is   1

    (a) 0.25   (b) 0.50   (c) 0.75   (d) None of these

4. If p > q and r < 0, then which of the following is true?   1

    (a) pr < qr   (b) p$-r$ < q $- r$ (c) p + r < q + r   (d) None of these

5. Write the number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ 1

    (a) 3   (b) 4   (c) 2   (d) 0

6. A six faced die is thrown until 1 appears, the probability that 1 appears in even   1
number of trials, is

    (a) $\frac{5}{11}$ (b) $\frac{5}{6}$ (c) $\frac{6}{11}$ (d) $\frac{1}{6}$

7. If f(a+b-x) = f(x), then $\int_a^b xf(x)dx$ is equal to   1

    (a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b)$\frac{a+b}{2}\int_a^b f(b+x)dx$ (c)$\frac{b-a}{2}\int_a^b f(x)dx$ (d))$\frac{a+b}{2}\int_a^b f(x)dx$

8. At what rate of interest will the present value of a perpetuity of Rs 500 payable   1
at the end of every 6 months be Rs10,000?

    (a) 5%   (b) 8%   (c) 10%   (d) 4%

9. Evaluate: $\int_{-1}^2 \frac{|x|}{x} dx$ 1

    (a) 0   (b) -1   (c) 2   (d) 1

10. A fire in a factory delaying production for some weeks is   1

    (a) secular trend   (b) irregular trend   (c) cyclical trend   (d) seasonal trend

11. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{x^3+y}{x}$ is   1

    (a) xe^x (b) $\frac{x}{e^x}$ (c) $\frac{e^x}{x}$ (d) e^x

12. If the mean of a binomial distribution is 81, then the standard deviation lies   1
between

    (a) 0 and 81   (b) 0 and 9   (c) 0 and 3   (d) None of these

13. The objective function of an LPP is   1

    (a) a constant   (b) a function to   (c) a relation   (d) None of these

                      be optimized   between the

                                    variables

14. The sum of the degree and the order of the below differential equation is   1

$$\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^3\right] = 0$$

    (a) 4   (b) 3   (c) 2   (d) None of these

15. A machine makes a car wheel and in a random sample of 26 wheels, the test statistic is found to be 3.07. As per the t-distribution test (of 5% level of significance), what can you say about the quality of wheels produced by the machine? (Use $t_{25}(0.05) = 2.06$)   1

(a) Superior quality   (b) Inferior quality   (c) same quality   (d) Cannot say

16. If 'm' is the mean of the Poisson distribution, then its standard deviation is given by   1

(a) $\sqrt{m}$ (b) m^2 (c) m   (d) $\frac{m}{2}$

17. A company buys a machine at a cost of Rs 5000. If the company decides on a salvage value of Rs 1000 and a useful life of 5 years, then annual depreciation cost is   1

(a) Rs 500   (b) Rs 600   (c) Rs 700   (d) Rs 800

18. At what points, the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero?   1

(a) (3,0), (-1,0)   (b) (3,0), (1,2)   (c) (1,2), (-1,0)   (d) (1,2), (1,-2)

ASSERTION REASON BASED QUESTIONS In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): If a is any positive real number, then $a + \frac{1}{a} \geq 1$.   1

Reason (R): Let a and b be two distinct positive real numbers. Then

$\frac{a+b}{2} \geq \sqrt{ab}$.

20. Assertion (A): For a binomial distribution, if mean = 5 and variance = 4, then the number of trials is 25.   1

Reason (R): The variance of a binomial distribution = $\sqrt{npq}$.

Section B

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

21. What sum of money is needed to invest now, so as to get Rs 6000 at the beginning of every month forever, if the money is worth 9% per annum compounded monthly? 2

22. Suppose 2x3 matrix A =[a_{ij}], whose elements are given by $a_{ij} = (\frac{i+2j}{2})$. Construct the matrix A. 2

OR

If A – 2B = $\begin{pmatrix} 1 & 5 \\ 3 & 7 \end{pmatrix}$ and 2A – 3B = $\begin{pmatrix} -2 & 5 \\ 0 & 7 \end{pmatrix}$, then find matrix B.

23. A dealer wishes to purchase toys A and B. He has Rs. 580 and has space to store 40 items. A costs Rs. 75 and B costs Rs. 90. He can make profit of Rs. 10 and Rs.15 by selling A and B respectively. Assuming that he can sell all the items that he can buy, express this problem as an LPP to maximise profit. (No need to solve) 2

24. The mean weekly sales of chocolate bars in candy stores was 146.3 bars per store. After an advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertisement campaign successful? [Given $t_{21}(0.05) = 1.721$] 2

25. In what ratio must a grocer mix two varieties of tea worth Rs 60 per kg and Rs 65 per kg so that by selling the mixture at Rs 68.20 per kg, he may gain 10%? 2

OR

A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 hours and 10 hours respectively. If all the three pipes are opened simultaneously, how long will it take to fill the tank?

Section C

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

26. Evaluate: $\int \frac{dx}{x(x^5+3)}$ dx　　　　　　　　　　　3

OR

Evaluate: $\int \frac{x^2+1}{(x+1)^2} e^x$ dx

27. Mrs Sumitra invested Rs 10,000 in a company's fund. Her yearly investment values are shown in the table given below. Calculate CAGR (compound annual growth rate) of her investment.　　　3

Year	0	1	2	3
Amount (in Rupees)	10,000	12,800	11,500	9,400

28. Determine for what values of x, the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is strictly increasing and for what values of x, it is strictly decreasing.　　　3

29. Given that x= −9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$. Find the other two roots.　　　3

OR

There are 2 families A and B. There are 4 men, 6 women and 2 children in family A and 2 men, 2 women and 4 children in family B. The recommended daily allowance for calories is man: 2400, woman: 1900, child: 1800 and for proteins, man: 55 gm, woman: 45 gm and child: 33 gm. Represent the above information by matrices. Using matrix operation, calculate the total requirement of calories and proteins for each of the two families.

30. The marginal revenue of selling x units of a commodity is given by　　　3

MR= $50e^{\frac{x}{10}}(1 + \frac{x}{10})$. If the revenue obtained on selling 10 units of commodity is Rs 500e, find the revenue function.

31. A company has set-up a sinking fund so that it can accumulate Rs 6,00,000 in 10 years. How much amount should the company deposit at the end of every six months if interest is 5% p.a. compounded semi-annually?　　　3

[Given that $(1.025)^{20} = 1.637$]

Section D

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

32. An unbiased coin is tossed n times. Let the random variable X denote the number of times the head occurs. If P(X=1), P(X=2) and P(X=3) are in AP, find the value of n.   5

OR

A manufacturer of glass plates knows that 2% of his product is defective. If he sells the glass plates in box of 100 and guarantees that not more than 4 plates will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? [Given e^{-2} = 0.135]

33. A machine initially costs Rs 6400 with no scrap value. The cost of operating is Rs 500 in the first year and increases by Rs 800 in each successive year. Determine (i) the number of years it has to be operated for minimizing total cost per year, and (ii) corresponding cost per year.   5

OR

Find the dimensions of a rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also, find its maximum volume.

34. Find the minimum value of Z for the problem minimise Z = 400x + 200y, subject to constraints 5x+2y ≥ 30, 2x+y ≤ 15, x≤ y and x, y≥ 0.   5

35. A total amount of Rs 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}$% respectively. The total annual interest from these three accounts is Rs 550. Equal amounts have been deposited in 5% and 8% savings accounts. Using matrix method, find the amount deposited in each of these three accounts.   5

Section E

(This section comprises of 3 source based questions (Case Studies) of 4 mark each)

36. **Case Study 1:**

A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream.

Based on the above information, answer the below questions.

(a)	What is the ratio between the speed of the boat and speed of the water current respectively?	2
(b) (i)	If the speed of the stream is 9 kmph, find the speed of the boat.	1
(b) (ii)	If the same boat goes to a place 30 km upstream and returns to the same place, find the time taken to travel this distance.	1

37. **Case Study 2:**

The 119 km Bengaluru-Mysore expressway project on NH-275 is a 6-10 lane access-controlled highway developed by the Ministry of Road Transport and Highway. The aim of this project is to decongest the traffic movement between Bengaluru and Mysore and eventually reduce the travel time. But due to break-neck speed, motorists often fail to react quickly enough when faced with sudden stops, leading to collisions. To reduce the number of accidents, police have instructed NHAI to install radar units to measure the speed of vehicles.

The speeds are normally distributed with a mean speed of 80 kmph and a standard deviation of 10 kmph. Answer the below questions, assuming that a vehicle is chosen at random. [Given $F(1) = 0.8413$, $F(2) = 0.9772$ and $F(3) = 0.9986$]

(a)	Find the probability that the vehicle is running at less than 60 kmph.	1

(b) Find the probability that the vehicle is running at more than 100 kmph $1\frac{1}{2}$

(c) Find the probability that the vehicle is running between 90 kmph and 110 kmph. $1\frac{1}{2}$

38. **Case Study 3:** Rameshwaram Café is a quick service restaurant. The café is trending in social media because it has caught the fancy of many start-ups, knowing that this 10 by 10 square feet store does business in crores in a month and clocks around more than Rs 50 crores in a year. Let us analyse their revenue in 8 months:

Month	1	2	3	4	5	6	7	8
Revenue (in Crore)	5	7	9	6	4	5	5	8

Based on the above information, answer the following questions. Show steps to support your answer.

Use the method of least-squares to find the best-fit trend line equation for Rameshwaram Café's business. What is the estimated sale for the 10th month? 3+1

OR

Plot the given data on a graph sheet. Calculate the three monthly moving average and plot this on the same graph sheet.

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