Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
APPLIED MATHEMATICS PAPER CODE 465

General Instructions: -

1 You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2 **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**

3 Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**

4 The Marking scheme carries only suggested value points for the answers
These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.

5 The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

6 Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

7 If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

8 If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	● Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

APPLIED MATHEMATICS (Subject Code–241)

(PAPER CODE: 465)

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	The last (unit) digit of $(22)^{12}$ is : (a) 2 (b) 4 (c) 6 (d) 8	
Sol.	(c) 6	**(1)**
2.	The least non-negative remainder, when 3^{15} is divided by 7 is : (a) 1 (b) 5 (c) 6 (d) 7	
Sol.	(c) 6	**(1)**

3.	If $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix}$, then AB is : (a) $\begin{bmatrix} -5 & 10 \\ 0 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5 \\ 25 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 10 & -25 \\ -5 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$	
Sol.	(d) $\begin{bmatrix} \mathbf{-5} & \mathbf{10} \\ \mathbf{0} & \mathbf{-25} \end{bmatrix}$	**(1)**
4.	If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then the values of x and y are : (a) $x = 3, y = 5$ (b) $x = 5, y = 3$ (c) $x = 2, y = 7$ (d) $x = 7, y = 2$	
Sol.	(a) $\boldsymbol{x = 3, y = 5}$	**(1)**
5.	The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is : (a) 7 : 8 (b) 8 : 7 (c) 5 : 7 (d) 7 : 5	
Sol.	(b) 8 : 7	**(1)**

6.	If $\frac{\lvert x+1 \rvert}{x+1} > 0, x \in \mathbb{R}$, then : (a) $x \in [-1, \infty)$ (b) $x \in (-1, \infty)$ (c) $x \in (-\infty, -1)$ (d) $x \in (-\infty, -1]$	
Sol.	(b) $\boldsymbol{x \in (-1, \infty)}$	**(1)**
7.	A and B are square matrices each of order 3 such that $\lvert A \rvert = -1$ and $\lvert B \rvert = 3$. What is the value of $\lvert 3AB \rvert$? (a) -9 (b) -18 (c) -27 (d) -81	
Sol.	(d) $\boldsymbol{-81}$	**(1)**
8.	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is : (a) -1 (b) 0 (c) 1 (d) 3	
Sol.	(a) $\boldsymbol{-1}$	**(1)**
9.	The relation between 'Marginal cost' and 'Average cost' of producing 'x' units of a product is : (a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$ (c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$	
Sol.	(d) $\boldsymbol{\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)}$	**(1)**

10.	$\int (x-1)\,e^{-x}\,dx$ is equal to : (a) $(x-2)\,e^{-x}+C$ (b) $xe^{-x}+C$ (c) $-xe^{-x}+C$ (d) $(x+1)\,e^{-x}+C$	
Sol.	(c) $\mathbf{-xe^{-x}+C}$	**(1)**
11.	The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is : (a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $xy=C$ (c) $\log x \log y = C$ (d) $x+y=C$	
Sol.	(b) $\boldsymbol{xy=C}$	**(1)**
12.	If X is a Poisson variable such that $P(X = 1) = 2P(X = 2)$, then $P(X = 0)$ is : (a) e (b) $\frac{1}{e}$ (c) 1 (d) e^2	
Sol.	(b) $\boldsymbol{\frac{1}{e}}$	**(1)**
13.	If the calculated value of $\lvert t\rvert < t_v(\alpha)$, then the null hypothesis is : (a) rejected (b) accepted (c) cannot be determined (d) neither accepted nor rejected	
Sol.	(b) accepted	**(1)**

14.	For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as : (a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$ (c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 - 1$	
Sol.	(c) $\boldsymbol{n_1 + n_2 - 2}$	**(1)**
15.	The straight line trend is represented by the equation : (a) $y_c = a + bx$ (b) $y_c = a - bx$ (c) $y_c = na + b\Sigma x$ (d) $y_c = na - b\Sigma x$	
Sol.	(a) $\boldsymbol{y_c = a + bx}$	**(1)**
16.	The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by : (a) Ri (b) $R + \frac{R}{i}$ (c) $\frac{R}{i}$ (d) R – Ri	
Sol.	(c) $\boldsymbol{\frac{R}{i}}$	**(1)**
17.	The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is : (a) 10·25% (b) 10·38% (c) 10·47% (d) 10·53%	
Sol.	(b) 10.38%	**(1)**
18.	Region represented by $x \geq 0, y \geq 0$ lies in (a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant	
Sol.	(a) I quadrant	**(1)**

	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.	
19.	*Assertion (A) :* The function $f(x) = (x + 2)\, e^{-x}$ is increasing in the interval $(-1, \infty)$. *Reason (R) :* A function f(x) is increasing, if $f'(x) > 0$.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**
20.	*Assertion (A) :* The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$. *Reason (R) :* If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**
	SECTION B This section comprises very short answer (VSA) type questions of **2 marks each.**	
21(a).	Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?	
Sol.	Let B be closed after n minutes. Then, pipe A runs for 18 minutes and B runs for n minutes to fill the tank. $\therefore \frac{18}{24} + \frac{n}{32} = 1$	**(1)**

	$\Rightarrow \frac{3}{4} + \frac{n}{32} = 1 \Rightarrow n = 8.$ Hence, pipe B must be closed after 8 min	**(1)**
	OR	
21(b).	In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.	
Sol.	Suppose A takes ‘t’ seconds to run 1 km race. Then, B takes (t + 30) seconds and C takes (t + 30 + 15) seconds, i.e. (t + 45) seconds. We find A beats C by (30 + 15) seconds = 45 seconds and it is given that A beats C by 180 metres. $\therefore$ C runs 180 m in 45 seconds $\Rightarrow$ C runs 1000 m in $\left(\frac{45}{180} \times 1000\right)$ seconds = 250 seconds. $\therefore\ t + 45 = 250 \Rightarrow t = 205$ Hence, A takes 205 seconds to run 1 km	**$(\frac{1}{2})$** **(1)** **$(\frac{1}{2})$**
22.	Solve for x : $\frac{x+3}{x-2} \le 2.$	
Sol.	$\frac{\mathbf{x+3}}{\mathbf{x-2}} - \mathbf{2} \le \mathbf{0} \quad \Rightarrow \frac{\mathbf{-x+7}}{\mathbf{x-2}} \le 0$ or $\frac{\mathbf{x-7}}{\mathbf{x-2}} \ge 0$ Thus, the solution set is $(-\infty, 2) \cup [7, \infty)$	**(1)** **(1)**
23(a).	Solve the following system of equations by Cramer’s rule : $2x - y = 17,\ 3x + 5y = 6$	
Sol.	Here, D = $\begin{vmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{3} & \mathbf{5} \end{vmatrix} = 13$ $D_1 = \begin{vmatrix} \mathbf{17} & \mathbf{-1} \\ \mathbf{6} & \mathbf{5} \end{vmatrix} = 91$ $D_2 = \begin{vmatrix} \mathbf{2} & \mathbf{17} \\ \mathbf{3} & \mathbf{6} \end{vmatrix} = -39$ Thus, $x = \frac{\mathbf{D_1}}{\mathbf{D}} = 7;\ y = \frac{\mathbf{D_2}}{\mathbf{D}} = -3$	**$(\frac{1}{2})$** **$(\frac{1}{2})$** **$(\frac{1}{2})$**

		$(\frac{1}{2})$
	OR	
23(b).	Determine the integral value(s) of x for which the matrix A is singular : $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$	
Sol.	A is singular gives $\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$ i.e. $(x + 1) [(x + 2) (x – 6) – 2] + 3[– 5x + 30 – 8] + 4[– 5 – 4x – 8] = 0$ i.e. $(x + 1)\ (x^2 – 4x – 14) – 15x + 66 – 52 – 16x = 0$ i.e. $x^3 – 3x^2 – 49x = 0$ $x = 0,\ \frac{3\pm\sqrt{205}}{2}$ Hence, x = 0 is the only integral value.	$(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
24.	A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.	
Sol.	Here, $6y = x^3 + 2$ $\Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ As $\frac{dy}{dt} = 8\frac{dx}{dt}$, we have $48\cancel{\frac{dx}{dt}} = 3x^2\cancel{\frac{dx}{dt}} \Rightarrow x = 4, – 4$ when x = 4, y = 11; when x = – 4, $y = \frac{-31}{3}$.	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$

	∴ Points on the curve are (4, 11), $\left(-\ \mathbf{4},\ \frac{-\mathbf{31}}{\mathbf{3}}\right)$	
25.	Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0\cdot135$)	
Sol.	Let p be the probability that an item is defective so, $p = \frac{\mathbf{2}}{\mathbf{100}} = 0\cdot02$. Here n = 100 ∴ m = np = 2 $P(X = r) = \frac{m^r}{r!}e^{-m} = \frac{2^r e^{-2}}{r!}$ $\Rightarrow \mathbf{P(X = 3)} = \frac{\mathbf{2^3 e^{-2}}}{\mathbf{3!}} = \frac{\mathbf{4}}{\mathbf{3}} \times \mathbf{0\cdot135 = 0\cdot18}$	$\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$
	SECTION C This section comprises short answer (SA) type questions of **3 marks each.**	
26(a).	A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.	
Sol.	Let the original quantity of dettol be x litres and the quantity of Dettol replaced by water be y litres. So, $y = \frac{\mathbf{x}}{\mathbf{3}}$. After 3 operations the quantity of dettol left $= x\left(\mathbf{1} - \frac{\mathbf{y}}{\mathbf{x}}\right)^{\mathbf{3}}$. After 3 operations the quantity of water in the bottle $= x - x\left(\mathbf{1} - \frac{\mathbf{x}}{\mathbf{3x}}\right)^{\mathbf{3}}$ Hence, the required ratio is $x\left(1 - \frac{x}{3x}\right)^3 : \left[x - x\left(1 - \frac{x}{3x}\right)^3\right]$ $= \left(1 - \frac{1}{3}\right)^3 : \left[1 - \left(1 - \frac{1}{3}\right)^3\right]$ $= \frac{8}{27} : \frac{19}{27}$ $= 8 : 19$	**(1)** **(1)** **(1)**

26(b).	A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?	
Sol.	Here, $n_A = 3$, $n_B = 7$ and $n_C = 10$. $\frac{1}{n} = \frac{1}{n_A} - \frac{1}{n_B} - \frac{1}{n_C}$ $\Rightarrow \frac{1}{n} = \frac{1}{3} - \frac{1}{7} - \frac{1}{10}$ $\Rightarrow \frac{1}{n} = \frac{19}{210} \Rightarrow \text{n} = 11\frac{1}{19}$ Hence, the tank is filled in $11\frac{1}{19}$ hours.	**(2)** **(1)**
27.	Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.	
Sol.	$y = x^3 - 6x^2 + 9x - 8$ $\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9$ $\Rightarrow \frac{dy}{dx} = 3(x-1)(x-3)$ Critical points are 1, 3 Showing, x=1 is a point of local maxima. Showing, x=3 is a point of local minima.	**(1)** **(1)** $(\frac{1}{2})$ $(\frac{1}{2})$
28.	An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.	
Sol.	Let A be the event of obtaining two sixes in the first five throws of a die. Let B be the event of obtaining a six in the sixth throw of a die.	

	Then required probability = P(AB) = P(A) P(B) Here, $P(B) = \frac{1}{6}$ and $P(A) = 5_{C_2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$ Thus, Required probability = $\frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$	**(2)** **(1)**
29.	The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. (Use $t_{0\cdot005} = 1\cdot729$ for 19 d.f.)	
Sol.	We are given $\mu = 50,\ \bar{x} = 55, SD = 10, n = 20$ $H_0: \mu = 50$ $H_1: \mu > 50$ $t = \frac{\bar{x}-\mu}{\frac{SD}{\sqrt{n}}} = \frac{55-50}{\frac{10}{\sqrt{20}}} = 2\cdot236$ $t_{\text{cal value}} > t_{\text{tab value}}$ Hence H_0 is rejected. So, Advertising Campaign was successful.	**(1)** **(2)**
30(a).	An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.	
Sol.	Here C = ₹ 4,50,000	

S = ₹ 1,00,000

and n = 5 years.

Annual depreciation D = $\frac{C - S}{n}$ = ₹ 70,000 **(2)**

Thus, yearly depreciation schedule is as follows:

Years	Book value at the beginning of the year (in ₹)	Depreciation (in ₹)	Book value at the end of the year (in ₹)
1	4,50,000	70,000	3,80,000
2	3,80,000	70,000	3,10,000
3	3,10,000	70,000	2,40,000
4	2,40,000	70,000	1,70,000
5	1,70,000	70,000	1,00,000

(1 for correct table)

30(b). Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

{Given $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}

Sol. Here P = ₹ 9,50,000, i = $\frac{15}{1200}$ = $0{\cdot}0125$ **($\frac{1}{2}$)**

n = 48 months **($\frac{1}{2}$)**

Using the reducing balancing method,

$$E = \frac{Pi}{1 - (1 + i)^{-n}} = \frac{9{,}5{,}0000 \times 0{\cdot}0125}{1 - (1 + 0{\cdot}0125)^{-48}}$$ **(1)**

$$=\frac{11875}{1-(1.0125)^{-48}} = \frac{11875}{1-0.5508565}$$ $(\frac{1}{2})$

$= ₹\ 26,439{\cdot}21$ $(\frac{1}{2})$

31. Maximise z = 300x + 190y

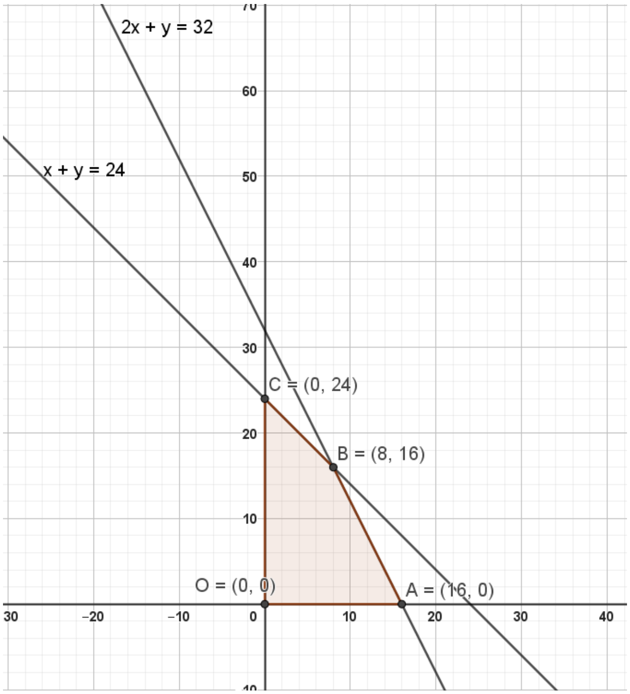
subject to constraints :

$x + y \le 24,$

$2x + y \le 32,$

$x \ge 0, y \ge 0.$

Sol.



(2)

Corner Points	Value of Z
O (0,0)	0
A (16,0)	4800

(1)

B (8,16)	5440 → Max Value
C (0,24)	4560

So Z is maximum at B (8,16)

Max Value of Z = 5440

SECTION D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32(a). Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = I$.

Sol. Here, $|A| = -(-4-3) - (12+1) + 2(9-1)$

$= 7 - 13 + 16 = 10 \neq 0$ **(1)**

$$\Rightarrow \text{adj}(A) = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$ **$(2\frac{1}{2})$**

$$\text{Hence } A^{-1} = \frac{1}{10}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$ **$(\frac{1}{2})$**

$$AA^{-1} = \frac{1}{10}\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(1)**

OR

32(b).	Using matrix method, solve the following system of equations for x, y and z : $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 2$	
Sol.	The matrix equation AX = B is $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $\lvert A \rvert = 10$ $\text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Here $A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ Thus, $x = 2$, $y = -1$, $z = 1$	$(\frac{1}{2})$ **(1)** **(2)** $(\frac{1}{2})$ **(1)**
33(a).	Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.	
Sol.	Let the two parts be x and 15 – x. Then, let $y = x^2(15 - x)^3$ $\Rightarrow \frac{dy}{dx} = x(15 - x)^2 (-5x + 30)$	**(1)** **(1)**

	$\frac{dy}{dx}$ = 0 gives x = 0, 15, 6 Rejecting x = 0, 15. Hence x = 6 Showing, $x = 6$ is a point of maxima So, y is maximum when x = 6. Hence two parts are 6 and 9	**$(1\frac{1}{2})$** **(1)** **$(\frac{1}{2})$**
	OR	
33(b).	Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).	
Sol.	Let P (x, y) be the required point which is nearest to Q (1, 4). Then distance PQ should be minimum and hence $(PQ)^2$ should be minimum. Now, $(PQ)^2 = (x-1)^2 + (y-4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$ $= \frac{y^4 - 32y + 68}{4}$ Let D = $\frac{y^4 - 32y + 68}{4}$ $\frac{dD}{dy} = y^3 - 8$ $\frac{dD}{dy} = 0 \Rightarrow y = 2$ Showing, y = 2 is a point of minima Thus, the point is (2, 2)	**$(\frac{1}{2})$** **(1)** **(1)** **$(\frac{1}{2})$** **(1)** **$(\frac{1}{2})$** **$(\frac{1}{2})$**

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year :	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹) :	65	68	70	72	75	67	73

Sol. Consider year 2014 as the year of origin. Calculation of trend values by method of least squares.

Year	Sales (in lakh ₹) y	Deviations from 2014 (x)	Squares of Deviations (x^2)	Sales deviation (xy)
2010	65	– 4	16	– 260
2012	68	– 2	4	– 136
2013	70	– 1	1	– 70
2014	72	0	0	0
2015	75	1	1	75
2016	67	2	4	134
2019	73	5	25	365
n = 7	$\sum y = 490$	$\sum x = 1$	$\sum x^2 = 51$	$\sum xy = 108$

(2 for correct table)

The equation of the straight-line trend is

$$y_c = a + bx$$

Two normal equations are

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$\Rightarrow 490 = 7a + b$ and $108 = a + 51b$

$\Rightarrow$ a = 69·9 and b = 0·75 **(1)**

$y_c = 69.9 + 0.75x$ **(1)**

Thus, trend values are

$$y_{2010} = 69{\cdot}9 + 0{\cdot}75(-4) = 66{\cdot}90$$

$$y_{2012} = 69{\cdot}9 + 0{\cdot}75(-2) = 68{\cdot}40$$

$$y_{2013} = 69{\cdot}9 + 0{\cdot}75(-1) = 69{\cdot}15$$

$$y_{2014} = 69{\cdot}9 + 0{\cdot}75(0) = 69{\cdot}90$$

$$y_{2015} = 69{\cdot}9 + 0{\cdot}75(1) = 70{\cdot}65$$

$$y_{2016} = 69{\cdot}9 + 0{\cdot}75(2) = 71{\cdot}40$$

$$y_{2019} = 69{\cdot}9 + 0{\cdot}75\ (5) = 73{\cdot}65$$

(1 for correct trend values)

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1{\cdot}4)^{1/6} = 1{\cdot}058$]

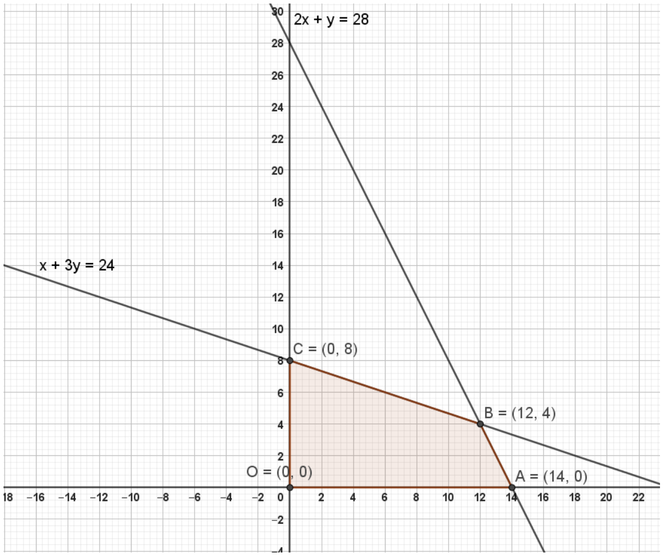
Sol. CAGR is the mean annual growth rate of an investment over a specified period of time longer than one year. **(1)**

$$\text{CAGR} = \left[\frac{\textbf{Ending investment amount}}{\textbf{Start amount}}\right]^{\frac{1}{\text{no.of years}}} - \mathbf{1}$$ **(1)**

P.V. = ₹ 10,000

	F.V. = ₹ 14,000	**(1)**
	n = 6 years	
	So, CAGR = $\left(\frac{\mathbf{14000}}{\mathbf{10000}}\right)^{\mathbf{1/6}}$ – 1 = $(1{\cdot}4)^{1/6}$ – 1	**$(\frac{1}{2})$**
	= 1·058 – 1	**$(\frac{1}{2})$**
	= 0·058	**$(\frac{1}{2})$**
	Hence, CAGR = 5·8%	**$(\frac{1}{2})$**
	SECTION E This section comprises of 3 case-study based questions of **4 marks each.**	
36.	A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs. Based on the above information, answer the following questions : (i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0{\cdot}0024$) (ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. (iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. **OR** (iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.	
Sol.	n= 100, p = $\frac{\mathbf{6}}{\mathbf{100}}$, m= np	
	Here m = 100 × $\frac{\mathbf{6}}{\mathbf{100}}$ = 6.	
	$P(r) = e^{-m}\frac{m^r}{r!}$	
	(i) $P(0) = e^{-m}\frac{m^0}{0!} = e^{-6} = 0{\cdot}0024$	**(1)**
	(ii) $P(2) = e^{-m}\frac{m^2}{2!} = e^{-6} \times \frac{36}{2} = 0.0432$	

	(iii)(a) $P(0) + P(1) = e^{-6} + e^{-6}\frac{m^1}{1!} = e^{-6} + 6e^{-6} = 7e^{-6} = 0{\cdot}0168$ **OR** (iii)(b) Mean = Variance = m = np = 6	**(1)** **(1+1)** **(1+1)**
37.	A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively. Based on the above information, answer the following questions : (i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. (ii) Write the constraint that relates the number of craftsmanship hours. (iii) (a) Determine the maximum profit (in ₹) earned by the factory. **OR** (iii) (b) How many bats and rackets respectively, are to be manufactured to earn maximum profit ?	
Sol.	(i) Z = 10x + 20y (ii) $x + 3y \leq 24$ (iii) (a) other constraints are $2x + y \leq 28$ $x \geq 0$ $y \geq 0$	**(1)** **(1)**



(1)

(1)

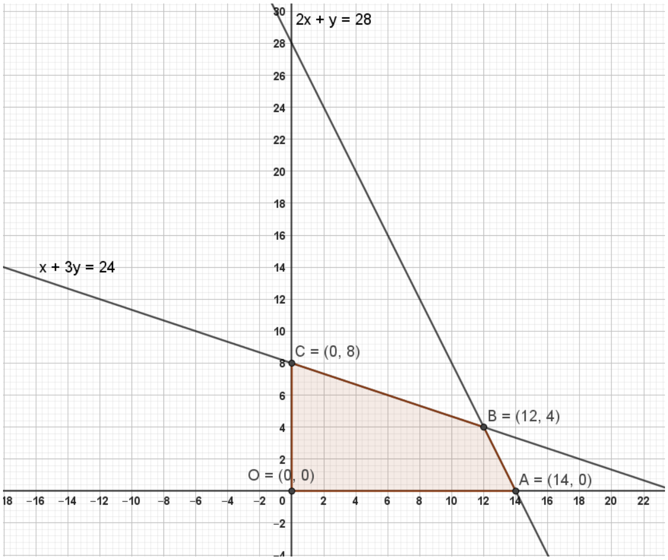
Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

∴ P is maximum at B (12,4); which is ₹ 200

OR

(iii) (b)

(1)



(1)

Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

12 bats and 4 rackets

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI.

(ii) Find the principal paid by Mr. Aggarwal in the 150th instalment.

(iii) (a) Find the total interest paid by Mr. Aggarwal.

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ?

[Use $(1{\cdot}00625)^{240} = 4{\cdot}4608$; $(1{\cdot}00625)^{91} = 1{\cdot}7629$]

Sol. Given P = ₹ 30,00,000, $i = \frac{7{\cdot}5}{1200} = 0{\cdot}00625$

and n = 12 × 20 = 240 months

(i) EMI $= \frac{P\,i}{1-(1+i)^{-n}}$

$= \frac{30{,}00{,}000 \times 0{\cdot}00625}{1-(1{\cdot}00625)^{-240} - 1}$ $\left(\frac{1}{2}\right)$

$= \frac{30{,}00{,}000 \times 0{\cdot}00625 \times 4{\cdot}4608}{3{\cdot}4608}$ $\left(\frac{1}{2}\right)$

= ₹ 24167.82

(ii) Interest paid on 150th instalment

$= \frac{\text{EMI} \times [(1+i)^{240-150+1} - 1]}{(1+i)^{240-150+1}}$

$= \frac{24167 \times [1{\cdot}7629 - 1]}{1{\cdot}7629}$ $\left(\frac{1}{2}\right)$

= ₹ 10458.70

⇒ Principal paid in 150th instalment = EMI – interest

=₹ (24167.82 – 10458.70) $\left(\frac{1}{2}\right)$

	= ₹ 13709.12	
	(iii) (a) Total Interest paid = n × EMI – P	
	= ₹ (240 × 24167.82 – 30,00,000)	**(1)**
	= ₹ 28,00,276.80	**(1)**
	OR	
	(iii) (b) Total amount paid = n x EMI	
	= 240 x 2416.81	**(1)**
	= ₹ 5800276.8	**(1)**