COMMON PRE-BOARD EXAMINATION 2023-24

Subject: APPLIED MATHEMATICS (241)

Class XII

Time: 3 Hrs.   Max. Marks: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

a) This question paper contains **38** questions. **All** questions are **compulsory**.

b) This question paper is divided into **five** sections – **A, B, C, D** and **E**.

c) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.

d) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

e) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

f) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

g) In **Section E**, Questions no. **36** to **38** are case based questions, carrying **4** marks each.

h) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E. You have to attempt only one alternative in all such questions.

i) Use of calculators is **not** allowed.

SECTION A
(Multiple Choice Questions)
(Each question carries 1 mark)

1   The value of – 60 mod 7 is

    a) 3   b) - 3   c) 4   d) - 4

2   Solution of the inequality $3(2 – x) \geq 2(1 – x)$ for real x is:

    a) $x \geq 8$ b) $x \leq 8$ c) $x \leq 4$ d) $x \geq 4$

3   A man can row upstream at 7 km/h and downstream at 10 km/h. The man's rate in still water is

    a) 17 km/h   b) 3 km/h   c) 8.5 km/h   d) 1.5 km/h

4   In what ratio water should be added to the liquid detergent costing ₹ 480 per litre to get a resulting mixture worth ₹ 300 per litre?

   a) 5:3   b) 3:5   c) 5:8   d) 8:5

5   In a 200 m race, Ram can beat Ramesh by 5 m or 3 seconds. Time taken by Ram to complete the race is

   a) 123 seconds   b) 117 seconds   c) 100 seconds   d) 120 seconds

6   If x=3at, y=at^3, then $\frac{dy}{dx}$ is equal to

   a) 3   b) 3a   c) 3at   d) t^2

7   At 6% converted quarterly, find the present value of a perpetuity of rupees 600 payable at the beginning of each quarter.

   a) ₹ 30, 400   b) ₹ 35, 500   c) ₹ 40, 600   d) ₹ 45, 000

8   Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4x+ 6y be the objective function. The Minimum value of F occurs at

   a) only (0, 2)   b) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

   c) only (3, 0)   d) any point on the line segment joining the points (0, 2) and (3, 0)

9   The data point of a normal variate with mean 15, standard deviation 6 and Z – score 7 is

   a) 57   b) 42   c) 27   d) 75

10 A coin is tossed n times. The probability of getting head atleast once is 0.8, then the least value of n is

   a) 2   b) 3   c) 4   d) 5

11 In a survey question for a sample of 200 individuals, 80 persons gave response 'yes', 100 gave 'NO' response and 20 gave none response. The point estimate of the proportion who respond 'Yes'

   a) 0.4   b) 0.5   c) 0.6   d) 0.8

12 If the calculated value of $|t| < t_v (\propto)$, then the null hypothesis is:

   a) rejected   b) accepted

   c) cannot be determined   d) neither accepted nor rejected

13 A fire in a factory delaying production for some weeks is

   a) Cyclic Trend   b) Secular Trend
   c) Irregular Trend   d) Seasonal Trend

14 For the given five Values 15,24,18,33,42 the three years moving averages are

   a) 19,25,33   b) 19,30,31   c) 19,25,31   d) 19,22,33

15 A machine costing ₹ 40,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 8000. Find the annual depreciation

   a) ₹ 8000   b) ₹ 10000   c) ₹ 5000   d) ₹ 4000

16 Assume an investment's starting value is 2,00,000 and it grows to 12,00,000 in 4 years. Then, CAGR is

   a) 56.5%   b) 50.5%   c) 60.5%   d) 60%

17 Mr. Anil takes a loan of ₹ 2,00,000 with 10% annual interest rate for 5 years. EMI under flat rates system is

   a) ₹ 4000   b) ₹ 5000   c) ₹ 6000   d) ₹ 7000

18 The effective rate of interest which is equivalent to a stated rate of 6% compounded semi-annually is

   a) 0.609%   b) 6.09%   c) 60.9%   d) 60%

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
   a)   Both A and R are true and R is the correct explanation of A
   b)   Both A and R are true but R is not the correct explanation of A
   c)   A is true, but R is false
   d)   A is false, but R is true

19 **Assertion (A):** The maximum profit that a company makes if profit function is given by
 $P(x) = 41 + 24x - 8x^2$; where 'x' is the number of units and P is the profit in rupees is 59.

   Reason (R): The profit is maximum at x = a if $P'(a) = 0$ and $P''(a) > 0$

20 **Assertion (A):** If a die is thrown 6 times and getting even number is success then, probability of getting exactly 2 success is $\frac{15}{64}$.

   Reason (R): In a Binomial distribution the probability is given by
   P (X= r) = ^nC_r $(p)^r(q)^{n-r}$

SECTION B
(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21 At what rate of Interest will the present value of a Perpetuity of ₹ 500 at the end of every 6 months be ₹ 10,000?

22 Using determinant find the value of k, for which points P (3, -2), Q (8, 8) and R (k, 2) are collinear.

OR

If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that $A^2 - 4A + I = O$, where I is an identity matrix and O is a zero matrix.

23 One kind of cake requires 300 g of flour and 15g of fat, another kind of cake requires 150g of flour and 30g of fat. Find the maximum number of cakes which can be made from 7.5kg of flour and 600g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Formulate an L.P.P.

24 Find the value of 3^{50} mod 7

OR

A person can row a boat 5 km an hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

25 Find the solution of the differential equation
$\frac{dx}{x} + \frac{dy}{y} = 0$

SECTION C
(This section comprises of short answer type questions (SA) of 3 marks each)

26 Integrate:
$\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$

OR

Evaluate:
$\int (x^2 + 1) \, logx \, dx$

27 Solve the following system of equations by Cramer's Rule.
3x − 2y + 3z = 8
2x + y − z = 1
4x − 3y + 2z = 4

28 A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that: ($e^{-3.2}$ = 0.041)
a) 2 or less bicycle riders will use the cycle track within an hour?
b) 3 or more bicycle riders will approach the intersection within an hour?

29 Find the intervals in which the function $f(x) = 2x^3 − 9x^2 + 12x + 15$ is strictly increasing or decreasing.

OR

The demand function for a commodity is
$p = 35 − 2x − x^2$
Find the Consumer's Surplus when $p_0 = 20$

30 A machine produces washers of thickness 0.50mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53mm and the standard deviation is 0.03mm. Test the hypothesis at 5% level of significance that the machine is working in proper order. [Given $t_{0.025}$ = 2.262 at 9 degrees of freedom]

31   Amrita bought a car worth ₹ 12, 50, 000 and makes a down payment of ₹ 3, 00, 000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

(Given $1.0125^{-48} = 0.5508565$)

SECTION D
(This section comprises of long answer-type questions (LA) of 5 marks each)

32   A company sets aside a sum of ₹ 10000/- at the end of each year in a sinking fund so that at the end of 20 years it would amount to a balance sufficient to repay the machinery. Assuming that the cost of machinery remains constant at the end of 20 years and that money earns 10% p.a. compound interest, find the cost of the machinery. If the number of years is 10 instead of 20 then what is the cost of the machinery?

($1.1^{20} = 6.73$ and $1.1^{10} = 2.594$)

OR

Examination In-charge of our school buys a copier machine for ₹ 2 lakhs. She estimates that she can use this machine for 6 years and that the machine will only be worth

₹ 10,000/- at the end of its life.

(i)   Determine annual depreciation cost for her responsibility under linear method of depreciation.
(ii)   Prepare a depreciation schedule for the copier machine.

33   A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of rupees 7 and screws B at a profit of rupees 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

34   To manufacture 'x' number of dolls, a company's total cost function C(x) is given by C (x) = 100 + 0.025x^2 and the total revenue function R (x) is described as R (x) = 5x. Given that C (x) and R (x) are in thousand rupees, what number of dolls shall be manufactured to maximize the profit of the company? What is the maximum profit?

OR

An open tank with a square bottom is to contain 4000 cubic cm of liquid is to be constructed. Find the dimension of the tank so that the surface area of the tank is minimum.

35   A factory produces three products every day. Their production on a particular day is 45 tons. It is found that production of third product exceeds the production of first by 8 tons while the production of first and third product is twice the production of second product. Use Matrix method to find the quantity of each product produced every day.

SECTION E
(This section comprises of 3 source-based questions (Case Studies) of 4 mark each)

36 **Case Study I:**
A tank is fitted with 3 taps A, B and C. All the three taps, if opened together, can drain the full tank in $1\frac{1}{2}$ minutes. Taps B and C together take 2 minutes to drain the tank while A and C together take $2\frac{4}{13}$ minutes to drain it. Based on above information answer the following questions.
  (i)   How long will tap A take to drain the tank separately?
  (ii)   How long will tap B take to drain the tank separately?
  (iii) a) How long will taps A and B together take to drain the tank?
                                          OR
  (iii) b) How long will tap C take to drain the tank separately?

37 **Case Study II:**
Let X denote the number of colleges where you will apply after XII and P (X = x) denotes the probability of getting admission in x number of colleges, it is given that

$$P(X = x) = \begin{cases} kx & if\ x = 0\ or\ 1 \\ 3kx & if\ x = 2 \\ 2k(5 - x) & if\ x = 3\ or\ 4 \\ 0 & if\ x > 4 \end{cases}$$

Where k is a positive constant. Based on the above information, answer the following questions.
  (i)   a) What is the value of k
                                          OR
  (i)   b) Find P (X ≤ 3)
  (ii)   Find P (X = 2)
  (iii) Find P (X = 4)

38 **Case Study III:**
When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The following table relates to the tourist arrivals (in millions) during 2004 to 2010 in India:
  a) Fit a straight-line trend by the method of least squares
  b) Estimate the number of tourists that would arrive in the year 2014.

Year	2004	2005	2006	2007	2008	2009	2010
Tourist Arrivals	18	20	23	25	24	28	30

OR

Compute the trends by the method of moving averages, assuming that 4-year cycle is present in the following series.

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Index Number	400	470	450	410	432	475	461	500	480	430