SOLUTION KEY- Class XII Mathematics PB-1 (2023-24)

SECTION-A

A1. $n(A)=2$ $\therefore$ No. of reflexive relations $= 2^{n(n-1)} = 2^{2\times 1} = 4$ $\therefore$ option (b)

A2. A, B, C are collinear $\Rightarrow ar(\Delta ABC) = 0$

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0 \Rightarrow 3(2-8) + 2(k-8) + 1(8k-16) = 0 \Rightarrow k = 5$$

$\therefore$ option (c)

A3. $\begin{vmatrix} x & 2 \\ 3 & x-2 \end{vmatrix} = 0 \Rightarrow x^2 - 2x - 6 = 0$

$\therefore$ Product of all possible values of x is $\frac{c}{a}$ ie. $-\frac{6}{1} = -6$

$\therefore$ option (b)

A4. $|A| = |kA| \Rightarrow |A| = k^n . |A| \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

$\therefore$ sum $= -1 + 1 = 0$ $\therefore$ option (d)

A5. $A.(adj A) = 3I \Rightarrow |A|.I = 3I \Rightarrow |A| = 3$

also, $|adj A| = |A|^{n-1} = 3^2 = 9$

$\therefore |A| + |adj A| = 3 + 9 = 12$ $\therefore$ option (a)

A6. As f is cts. at $x = 0$

$\Rightarrow \lim_{x\to 0} f(x) = f(0) \Rightarrow 2k = \lim_{x\to 0} \left(\frac{\sin x}{x} + \cos x\right) \Rightarrow 2k = 1 + 1 \Rightarrow k = 1$

$\therefore$ option (a)

A7. $x = a\cos^2\theta \Rightarrow \frac{dx}{d\theta} = 2a\cos\theta(-\sin\theta)$

$y = b\sin^2\theta \Rightarrow \frac{dy}{d\theta} = 2b\sin\theta(\cos\theta)$

$\therefore \frac{dy}{dx} = \frac{2b\sin\theta\cos\theta}{-2a\sin\theta\cos\theta} = -\frac{b}{a}$ $\therefore$ option (c)

A8. option (a)

A9. $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ —(1)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ —(2)

add, $2I = \int_0^{\pi/2} 1.dx \Rightarrow 2I = [x]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$ $\therefore$ option (c)

A10. order $= 2$, degree $= 2$ $\therefore$ Product $= 2\times 2 = 4$ $\therefore$ option (a)

A11. $\frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{2\log x}{x^2 \log x}$; $P = \frac{1}{x\log x}$ $\therefore IF = e^{\int \frac{1}{x\log x} dx} = e^{\int \frac{dt}{t}} = e^{\log t}$

$(\because \log x = t \Rightarrow \frac{1}{x}dx = dt)$

$= e^{\log(\log x)}$

$= \log x$

$\therefore$ option (c)

A12. Req. Projection $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{3(1)+(-1)(2)+(-2)(-3)}{\sqrt{1+4+9}} = \frac{\sqrt{14}}{2}$ $\therefore$ option (a)

A13. $\vec{a}+\vec{b}=-\vec{c} \Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = (-\vec{c})\cdot(-\vec{c}) \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|.|\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow (3)^2+(5)^2+2(3)(5)\cos\theta = (7)^2$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\therefore$ option (d)

A14. $\vec{b} = \hat{i}-\vec{a} = \hat{i}-2\hat{i}+2\hat{j}-2\hat{k} \Rightarrow \vec{b} = -\hat{i}+2\hat{j}-2\hat{k} \Rightarrow |\vec{b}| = \sqrt{1+4+4}$

$\Rightarrow |\vec{b}| = 3$ $\therefore$ option (b)

A15. d.r. of I line $= \langle 7,-5,1\rangle$

d.r. of II line $= \langle 1, \frac{\lambda}{2}, 3\rangle$

here, $7(1)+(-5)(\frac{\lambda}{2})+1(3) = 0 \Rightarrow -\frac{5\lambda}{2} = -10 \Rightarrow \lambda = 4$ $\therefore$ option (b)

A16. req. line refers to y-axis, whose d.c. are 0, 1, 0 $\therefore$ option (d)

A17. $I = \int e^{-x}\left(\frac{x+1}{x^2}\right)dx$; Put $-x = t \Rightarrow dx = -dt$ also, $x = -t$

$\therefore I = \int e^{t}\left(\frac{-t+1}{t^2}\right)dt = \int\left(\frac{1}{t}-\frac{1}{t^2}\right)\cdot e^{t}\,dt = \frac{e^t}{t}+C$

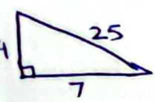
i.e. $I = \frac{e^{-x}}{-x}+C = -\frac{e^{-x}}{x}+C$ $\therefore$ option (d)

A18. $Z = 11x+7y$

$\therefore Z_{A(3,2)} = 11(3)+7(2) = 47$

$Z_{B(0,3)} = 11(0)+7(3) = 21$

$Z_{C(0,5)} = 11(0)+7(5) = 35$ $\therefore Z_{min} = 21$ $\therefore$ option (a)

A19. $\cot\left(\cos^{-1}\frac{7}{25}\right) = \cot\left(\cot^{-1}\frac{7}{24}\right) = \frac{7}{24}$ 

$\therefore$ A is true.

R is also true but not used in A. $\therefore$ option (b)

A20. $P(A)\times P(B) = \frac{3}{5}\times\frac{1}{5} = \frac{3}{25} \neq P(A\cap B)$ $\therefore$ A is not true.

but R is true. $\therefore$ option (d)

SECTION-B

A21. $\sin\left(\frac{\pi}{6}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \sin\left(\frac{\pi}{6}+\sin^{-1}\frac{\sqrt{3}}{2}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right) = \sin\frac{\pi}{2} = 1$ Ans

OR

$y = \sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\therefore y = \sin^{-1}\left(\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$

$\Rightarrow y = \frac{\pi}{4} + \theta \Rightarrow y = \frac{\pi}{4} + \sin^{-1}x$

A22. $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x+1)(x+2)$

for inc/dec, $f'(x) = 0 \Rightarrow x = -1, -2$

$-\infty \quad - \quad -2 \quad + \quad -1 \quad - \quad \infty$

$\therefore$ f is dec. when $x \in (-\infty, -2) \cup (-1, \infty)$ & inc. when $x \in (-2, -1)$

A23. $\vec{r} = \frac{4(2\hat{i}+3\hat{j}+4\hat{k}) - 1(-\hat{i}+\hat{j}+\hat{k})}{4-1} \Rightarrow \vec{r} = \frac{8\hat{i}+12\hat{j}+16\hat{k}+\hat{i}-\hat{j}-\hat{k}}{3}$

$\therefore \vec{r} = \frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$ or $3\hat{i} + \frac{11}{3}\hat{j} + 5\hat{k}$

OR

Given: $|\vec{a}| = |\vec{b}| = 1$; also $|\vec{a}+\vec{b}| = 1$

Now, $|\vec{a}+\vec{b}| = 1 \Rightarrow |\vec{a}+\vec{b}|^2 = 1 \Rightarrow (\vec{a}+\vec{b}).(\vec{a}.\vec{b}) = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 1$

$\Rightarrow 1+1+2\vec{a}.\vec{b} = 1 \Rightarrow \vec{a}.\vec{b} = -\frac{1}{2}$

Now, $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$= 1+1-2\left(-\frac{1}{2}\right) = 3 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{3}$, Hence shown.

A24. $\vec{d_1} = 2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{d_2} = 3\hat{i}-4\hat{j}-\hat{k}$

ar(||gm) $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Now, $\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -6 \\ 3 & -4 & -1 \end{vmatrix} = \hat{i}(-3-24) - \hat{j}(-2+18) + \hat{k}(-8-9)$

$= -27\hat{i} - 16\hat{j} - 17\hat{k}$

$\therefore |\vec{d_1} \times \vec{d_2}| = \sqrt{(-27)^2 + (-16)^2 + (-17)^2} = \sqrt{1274}$

$\therefore$ Req. area $= \frac{1}{2}.\sqrt{1274}$ sq. units

A25. $I = \int \frac{\sin 3x}{\sin x}dx$

$= \int \frac{3\sin x - 4\sin^3 x}{\sin x}dx$

$= \int (3 - 4\sin^2 x)dx = \int \left[3 - 4\left(\frac{1-\cos 2x}{2}\right)\right]dx = \int (1 + 2\cos 2x)dx$

$= x + 2 \cdot \frac{\sin 2x}{2} + C$

$= x + \sin 2x + C$

SECTION - C

A26. $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let $x = \sin A$, $y = \sin B$ $\Rightarrow$ $A = \sin^{-1}x$ and $B = \sin^{-1}y$

$\therefore \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow \cancel{2}\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) = a\left[\cancel{2}\,\cancel{\cos}\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow$ $A - B = 2\cot^{-1}a$

$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx} = 0$ $\Rightarrow$ $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$, H.P.

OR

$y = e^{\sin x} + (\tan x)^x$

$\Rightarrow y = u + v$, where, $u = e^{\sin x}$, $v = (\tan x)^x$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ——— ①

Now, $u = e^{\sin x}$ $\Rightarrow$ $\frac{du}{dx} = \cos x \cdot e^{\sin x}$ ——— ②

Now, $v = (\tan x)^x$ $\Rightarrow$ $\log v = x\cdot\log(\tan x)$

$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{\tan x}\times\sec^2 x + \log(\tan x)\cdot 1$

$\Rightarrow \frac{dv}{dx} = (\tan x)^x\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$ ——— ③

from ①, ② and ③,

$\frac{dy}{dx} = \cos x\cdot e^{\sin x} + (\tan x)^x\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$ Ans

A27. Let $I = \int\frac{e^x}{(1+e^x)(2+e^x)}dx$

Put $e^x = t$ $\Rightarrow$ $e^x dx = dt$

$\therefore I = \int\frac{dt}{(t+1)(t+2)} = \int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$

$= \log|t+1| - \log|t+2| + C$

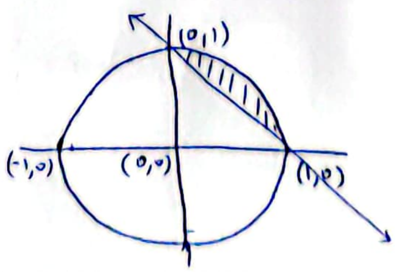
$= \log\left|\frac{t+1}{t+2}\right| + C = \log\left|\frac{e^x+1}{e^x-1}\right| + C$

A28. $R = \{(x,y) : x^2+y^2 \le 1 \le x+y\}$

$x^2+y^2 = 1$; $x+y = 1$

Point of intersection :

$(0,1)$ and $(1,0)$



Required area $= \int_0^1 (y \text{ of circle})\, dx - \int_0^1 (y \text{ of line})\, dx$

$$= \int_0^1 \sqrt{1-x^2}\, dx - \int_0^1 (1-x)\, dx$$

$$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ sq. units}$$

A29. Given diff. eq'n is :

$$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

eq'n becomes, $v + x\frac{dv}{dx} = \frac{2x\sin v - vx\cos v}{vx - x\cos v}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v} \Rightarrow \int \frac{v - \cos v}{2\sin v - v^2}\, dv = \int \frac{dx}{x}$$

Put $2\sin v - v^2 = t \Rightarrow (2\cos v - 2v)\, dv = dt \Rightarrow (v - \cos v)\, dv = -\frac{dt}{2}$

$$\therefore -\frac{1}{2}\int \frac{dt}{t} = \int \frac{dx}{x} \Rightarrow -\frac{1}{2}\log|t| = \log|x| + \log C$$

$$\Rightarrow \log\left(\frac{1}{2\sin v - v^2}\right) = \log(cx)^2$$

$$\Rightarrow 2\sin v - v^2 = \frac{1}{c^2x^2}$$

$$\Rightarrow 2\sin\left(\frac{y}{x}\right) - \frac{y^2}{x^2} = \frac{c'}{x^2}$$

OR

$(x^2+1)\cdot\frac{dy}{dx} - 2xy = (x^2+1)^2\cdot\cos x \quad ; \quad y(0) = 0$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2+1}\cdot y = (x^2+1)\cdot\cos x$$

$$\text{I.F.} = e^{\int \frac{-2x}{x^2+1}dx} = \frac{1}{x^2+1}$$

sol'n is given by,

$$y\cdot\frac{1}{x^2+1} = \int \frac{(x^2+1)\cdot\cos x}{x^2+1}\, dx \Rightarrow \frac{y}{x^2+1} = \int \cos x\, dx$$

$$\Rightarrow \frac{y}{x^2+1} = \sin x + C$$

$$\Rightarrow y = (x^2+1)\cdot\sin x + C(x^2+1)$$

when $x = 0, y = 0 \Rightarrow 0 = 0 + C \Rightarrow C = 0$

$\therefore y = (x^2+1)\sin x$ Ans.

A30. $P(\text{success}) = P(S) = \frac{1}{2}$, $P(\text{failure}) = P(F) = \frac{1}{2}$

$$P(A) = P(S) + P(FFS) + P(FFFFS) + \dots \infty$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} + \dots \infty$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$$

$a = \frac{1}{2}$, $r = \frac{1}{4}$

$S_\infty = \frac{a}{1-r}$

$\therefore$ $P(B) = 1 - \frac{2}{3} = \frac{1}{3}$

A31. Min. $Z = 5x + 10y$

$x + 2y = 120$

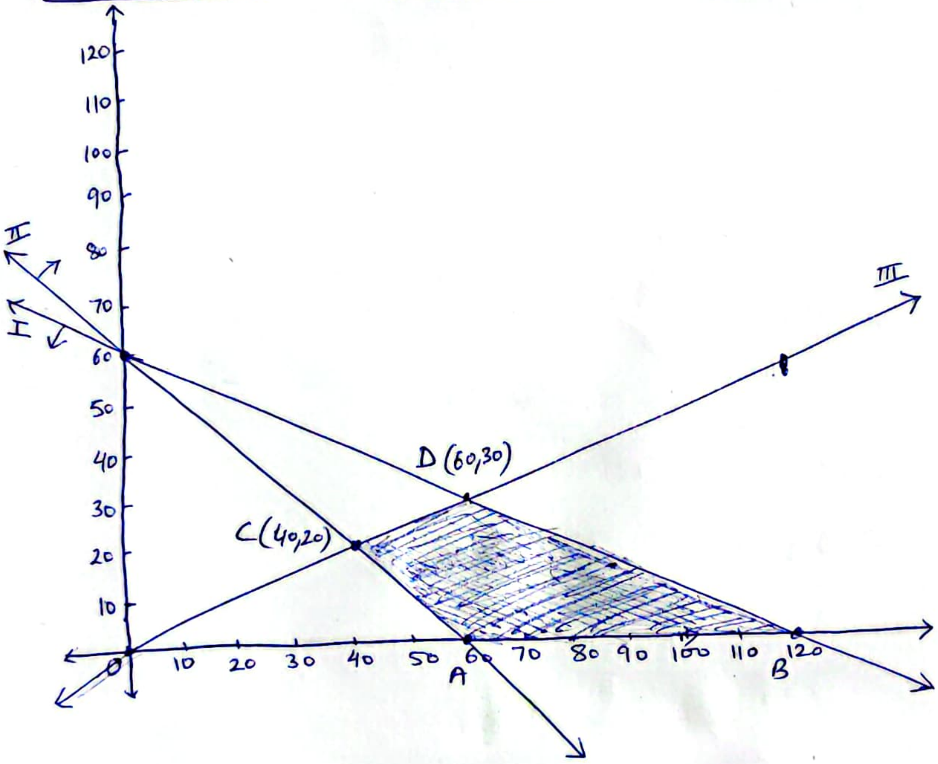
x	0	120
y	60	0

$x + y = 60$

x	0	60
y	60	0

$x = 2y$

x	0	120
y	0	60



Corner Pt.	$Z = 5x + 10y$
A(60,0)	300 → min.
B(120,0)	600
C(40,20)	400
D(60,30)	600

$Z_{min} = 300$ when $x = 60$, $y = 0$

SECTION-D

A32. $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 2 \\ 4 & 2 & -3 \end{bmatrix} \Rightarrow |A| = 2(6-4) - 3(-9-8) + 4(6+8) = 111 \neq 0$

$\Rightarrow A^{-1}$ exists.

Now, $A_{11} = 2$, $A_{12} = 17$, $A_{13} = 14$

$A_{21} = 17$, $A_{22} = -22$, $A_{23} = 8$

$A_{31} = 14$, $A_{32} = 8$, $A_{33} = -13$

$\text{adj}A = \begin{bmatrix} 2 & 17 & 14 \\ 17 & -22 & 8 \\ 14 & 8 & -13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{111}\begin{bmatrix} 2 & 17 & 14 \\ 17 & -22 & 8 \\ 14 & 8 & -13 \end{bmatrix}$

Given eqn's can be written as $AX = B$

where, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 17 \\ 11 \\ 8 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}.B$

$$= \frac{1}{111}\begin{bmatrix} 2 & 17 & 14 \\ 17 & -22 & 8 \\ 14 & 8 & -13 \end{bmatrix}\begin{bmatrix} 17 \\ 11 \\ 8 \end{bmatrix}$$

$$= \frac{1}{111}\begin{bmatrix} 333 \\ 111 \\ 222 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ $\therefore$ $x = 3, y = 1, z = 2$

A33. $(a,b) \, R \, (c,d) \Leftrightarrow a+d = b+c$

for reflexive: $(a,b) \, R \, (a,b) \Rightarrow a+b = b+a$, always true

$\therefore R$ is reflexive

for symmetric: If $(a,b) \, R \, (c,d) \Rightarrow a+d = b+c$

$\Rightarrow d+a = c+b$

$\Rightarrow c+b = d+a \Rightarrow (c,d) \, R \, (a,b)$

$\Rightarrow R$ is symm.

for transitive: let $(a,b), (c,d), (e,f) \in A \times A$

If $(a,b) \, R \, (c,d) \Rightarrow a+d = b+c$

and $(c,d) \, R \, (e,f) \Rightarrow c+f = d+e$

adding, $a + \cancel{d} + \cancel{c} + f = b + \cancel{c} + \cancel{d} + e$

$\Rightarrow a+f = b+e \Rightarrow (a,b) \, R \, (e,f) \Rightarrow R$ is transitive

Since R is reflexive, symmetric & transitive $\Rightarrow R$ is equivalence.

for equivalence class:

$(2,5) \, R \, (a,b) \Rightarrow 2+b = 5+a \Rightarrow b-a = 3$

$\therefore$ $[(2,5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

OR

$f: R-\{3\} \to R-\{1\}$

$f(x) = \dfrac{x-2}{x-3}$.

For <u>one-one</u>. Let $x_1, x_2 \in R-\{3\}$.

Let $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one

for <u>onto</u>

Let $y = f(x)$

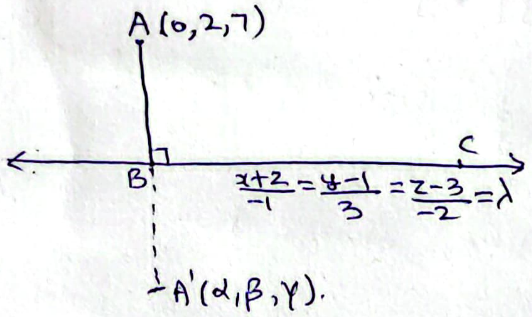
$\Rightarrow y = \dfrac{x-2}{x-3} \Rightarrow (x-3)\cdot y = x-2 \Rightarrow xy - 3y = x-2$

$\Rightarrow xy - x = 3y - 2 \Rightarrow x = \dfrac{3y-2}{y-1}$

clearly, $y-1 \neq 0 \Rightarrow y \neq 1$ ie. $y \in R-\{1\}$ ie. $R_f =$ co-domain

$\Rightarrow f$ is onto

A34.



Any point on BC is $(-\lambda-2, 3\lambda+1, -2\lambda+3)$

Let $B(-\lambda-2, 3\lambda+1, -2\lambda+3)$

Now,

d.r. of AB $= \langle -\lambda-2, 3\lambda-1, -2\lambda-4 \rangle$

d.r. of BC $= \langle -1, 3, -2 \rangle$

as, $AB \perp BC \Rightarrow -1(-\lambda-2) + 3(3\lambda-1) - 2(-2\lambda-4) = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

$\therefore$ Point $B\left(\frac{-3}{2}, \frac{-1}{2}, 4\right)$

Let image of A is $A'(\alpha, \beta, \gamma)$

Now, $AB = \sqrt{\left(\frac{-3}{2}-0\right)^2 + \left(\frac{-1}{2}-2\right)^2 + (4-7)^2} = \dfrac{\sqrt{70}}{2}$ units

(iii) B is mid-point on AA'.

Using mid-point formula,

$\left(\dfrac{0+\alpha}{2}, \dfrac{2+\beta}{2}, \dfrac{7+\gamma}{2}\right) = \left(\dfrac{-3}{2}, \dfrac{-1}{2}, 4\right) \Rightarrow \alpha = -3, \beta = -3, \gamma = 1$

$\therefore$ image $A'(-3, -3, 1)$ Ans.

OR

Given lines in vector form are:

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ or $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu'(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here, $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$S.D. = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

here, $\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$\therefore |(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{81 + 196 + 16} = \sqrt{293}$

also, $|\vec{b}| = \sqrt{4 + 9 + 36} = 7$

$\therefore$ Req. distance $= \dfrac{\sqrt{293}}{7}$ units.

A35. $I = \displaystyle\int_{-5}^{0} (|x| + |x+2| + |x+5|)\,dx$

critical points are $x = 0, -2, -5$

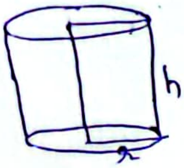
$$\therefore I = \int_{-5}^{-2} [-x - (x+2) + (x+5)]\,dx + \int_{-2}^{0} [-x + (x+2) + (x+5)]\,dx$$

$$= \int_{-5}^{-2} (-x+3)\,dx + \int_{-2}^{0} (x+7)\,dx$$

$$= \left[-\frac{x^2}{2} + 3x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 7x\right]_{-2}^{0} = \left(-8 + \frac{55}{2}\right) + (12) = \frac{63}{2}$$

SECTION-E

A36.



$$75\pi = 2\pi r h + \pi r^2 \Rightarrow h = \frac{75\pi - \pi r^2}{2\pi r}$$

(i) $V = \pi r^2 h \Rightarrow V = \pi r^2 \left(\dfrac{75\pi - \pi r^2}{2\pi r}\right)$

$$\Rightarrow V = \frac{r}{2}(75\pi - \pi r^2) \text{ or } \frac{75}{2}\pi r - \frac{\pi}{2} r^3$$

(ii) $\dfrac{dV}{dr} = \dfrac{75\pi}{2} - \dfrac{3}{2}\pi r^2$

(iii) for critical point, $\dfrac{dV}{dr} = 0 \Rightarrow 75\pi = 3\pi r^2 \Rightarrow r = 5$ cm

Now, $\dfrac{d^2V}{dr^2} = -3\pi r \Rightarrow \left.\dfrac{d^2V}{dr^2}\right]_{r=5cm} = -15\pi < 0 \quad \therefore$ V is max. when $r = 5$cm

Also, $h = \dfrac{75\pi - \pi r^2}{2\pi r} = \dfrac{75\pi - 25\pi}{10\pi} = 5 \quad \therefore h = r$

$\therefore h > r$ is false.

A37. Given: $\frac{dr}{dt} = 1$ cm/s

(i) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\Rightarrow \left.\frac{dV}{dt}\right]_{r=6} = 4\pi (6)^2 \times 1 = 144\pi$ cm³/s

(ii) $\frac{dV}{dt} = 400\pi$ cm³/s (given)

$\Rightarrow 4\pi r^2 \cdot \frac{dr}{dt} = 400\pi \Rightarrow \frac{dr}{dt} = \frac{100}{r^2}$ $\therefore \frac{100}{r^2} = 1 \Rightarrow r^2 = 100$

$\Rightarrow r = 10$ cm

Now, $A = 4\pi r^2$

$\Rightarrow \frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$= 8\pi (10)(1) = 80\pi$ cm²/s

A38. Let E_1: seed is of type A_1

E_2: seed is of type A_2

E_3: seed is of type A_3

A: seed will germinate

(a) $P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{49}{100}$

(b) $P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{49}{100}} = \frac{24}{49}$