MARKING SCHEME CLASS XII MATHEMATICS PRE-BOARD-2 (2023-24)

SECTION-A

A1. $2A+B=0 \Rightarrow B=-2A \Rightarrow B=-2\begin{bmatrix}3 & 4\\5 & 2\end{bmatrix}=\begin{bmatrix}-6 & -8\\-10 & -4\end{bmatrix}$ $\therefore$ option (b)

A2. $|\text{adj}A|=64 \Rightarrow |A|^{3-1}=64 \Rightarrow |A|=\pm 8$ $\therefore$ option (d)

A3. here, $x=-3,\ y=-1,\ z=4$ $\therefore x+y+z=0$ $\therefore$ option (d)

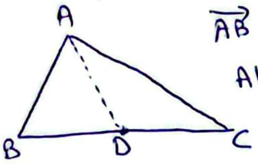
A4. $f(x)=2x^3+9x^2+12x-1 \Rightarrow f'(x)=6x^2+18x+12=6(x+1)(x+2)$

$f'(x)=0 \Rightarrow x=-1,-2$ $\quad -\infty \overset{+}{\text{——}} -2 \overset{-}{\text{——}} -1 \overset{+}{\text{——}} \infty$ $\therefore x\in(-2,-1)$ $\therefore$ option (b)

A5. $n(A)=5,\ n(B)=6$ Since $n(A)<n(B)$ $\therefore$ option (d)

A6. order $=3$, degree $=2$ $\therefore$ sum $=3+2=5$ $\therefore$ option (a)

A7. clearly bounded region will be formed $\therefore$ option (d)

A8. 

$\vec{AB}+\vec{BC}=\vec{AC} \Rightarrow \vec{BC}=2\hat{i}-2\hat{j}+2\hat{k}$ $\therefore \vec{BD}=\hat{i}-\hat{j}+\hat{k}$

Also, $\vec{AB}+\vec{BD}=\vec{AD} \Rightarrow \vec{AD}=2\hat{i}+3\hat{k}$ $\therefore$ option (d)

A9. $I=\int_0^{\pi/6}\sec^2\left(\frac{x}{6}-\frac{\pi}{6}\right)dx=\left[\tan\left(x-\frac{\pi}{6}\right)\right]_0^{\pi/6}=\tan 0-\tan\left(-\frac{\pi}{6}\right)=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

$\therefore$ option (a)

A10. $A^2-3A+I=0$ (given). $\Rightarrow I=-A^2+3A$ —— ①

Now, $A^{-1}=xA+yI \Rightarrow AA^{-1}=xA^2+yIA \Rightarrow I=xA^2+yA$ —— ②

from ① + ②,, $x=-1,\ y=3$ $\therefore x+y=2$ $\therefore$ option (b)

A11. $(\hat{i}\times\hat{j})\cdot\hat{j}+(\hat{j}\times\hat{i})\cdot\hat{k}=\hat{k}\cdot\hat{j}-\hat{k}\cdot\hat{k}=-1$. $\therefore$ option (d)

A12. option (c)

A13. $|4A^{-1}|=4^2\times|A^{-1}|=16\times\frac{1}{|A|}=16\times\frac{1}{2}=8$ $\therefore$ option (c)

A14. $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{8}}{1-\frac{3}{4}}=\frac{1}{2}$ $\therefore$ option (a)

A15. $\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1 \Rightarrow \cos^2\theta=0 \Rightarrow \theta=\frac{\pi}{2}$ $\therefore$ option (c)

A16. $x\,dy-(1+x^2)dx=dx \Rightarrow x\,dy=(2+x^2)dx \Rightarrow dy=\left(\frac{2}{x}+x\right)dx$

$\Rightarrow y=2\log x+\frac{x^2}{2}+C$ $\therefore$ option (d)

A17. div. by $\cos x$, $y=\frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)$

$\therefore \frac{dy}{dx}=-\sec^2\left(\frac{\pi}{4}-x\right)$ $\therefore$ option (a)

A18. line is $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-1/2}{6}$; d.r. $=\langle 2,-3,6\rangle$

d.c. $=\langle\frac{2}{7},-\frac{3}{7},\frac{6}{7}\rangle$ $\therefore$ option (d)

A19. $y = \sin^{-1}x + 2\cos^{-1}x = \sin^{-1}x + \cos^{-1}x + \cos^{-1}x = \frac{\pi}{2} + \cos^{-1}x$

Now, $\cos^{-1}x \in [0, \pi]$ $\therefore$ $\frac{\pi}{2} + \cos^{-1}x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ $\therefore$ A is false.

but, R is true. $\therefore$ option (d).

A20. d.r. of line I $= \langle -2, -4, -4 \rangle = \langle 1, 2, 2 \rangle$

d.r. of line II $= \langle 2, 4, 4 \rangle = \langle 1, 2, 2 \rangle$

as d.r's are proportional, lines are parallel. $\therefore$ A is true.

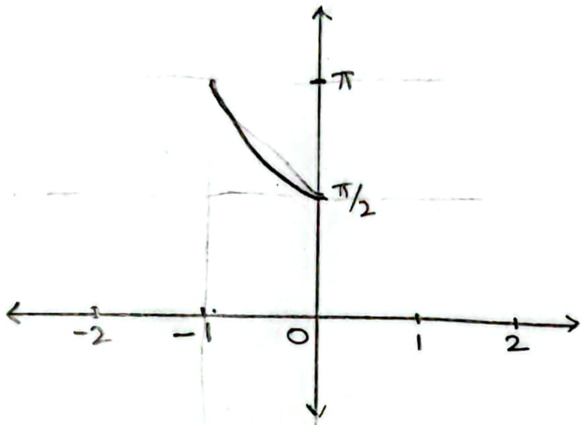
but R is false. $\therefore$ option (c)

SECTION-B

A21. $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}(\cos\pi) + \tan^{-1}1$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{4}\right)\right) + \pi + \frac{\pi}{4} = \frac{\pi}{4} + \pi + \frac{\pi}{4} = \frac{3\pi}{2}$

OR



Range $= \left[\frac{\pi}{2}, \pi\right]$

A22. $\cancel{p = (7\hat{i} - \hat{j} + 8\hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}$

$\frac{(\hat{i} + \hat{j} + \hat{k}).(p\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{p^2+1+4}} = \frac{1}{3} \Rightarrow 8p^2 - 18p + 4 = 0 \Rightarrow p = 2$ or $\frac{1}{4}$

A23. Differentiating $3y = ax^3 + 1$ gives $3.\frac{dy}{dx} = 3ax^2$

when $x = 1$, $\frac{dy}{dx} = 2$ $\therefore$ $3(2) = 3a(1)^2 \Rightarrow a = 2$

OR

$f(x) = \frac{16\sin x}{4 + \cos x} - x$

$\Rightarrow f'(x) = \frac{16[4 + \cos x].\cos x + 16\sin^2 x}{(4 + \cos x)^2} - 1$

$= \frac{\cos x(56 - \cos x)}{(4 + \cos x)^2}$

when $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$; $56 - \cos x > 0$ and $(4 + \cos x)^2 > 0$

$\therefore$ $f'(x) < 0 \Rightarrow f$ is st. dec. in $\left(\frac{\pi}{2}, \pi\right)$

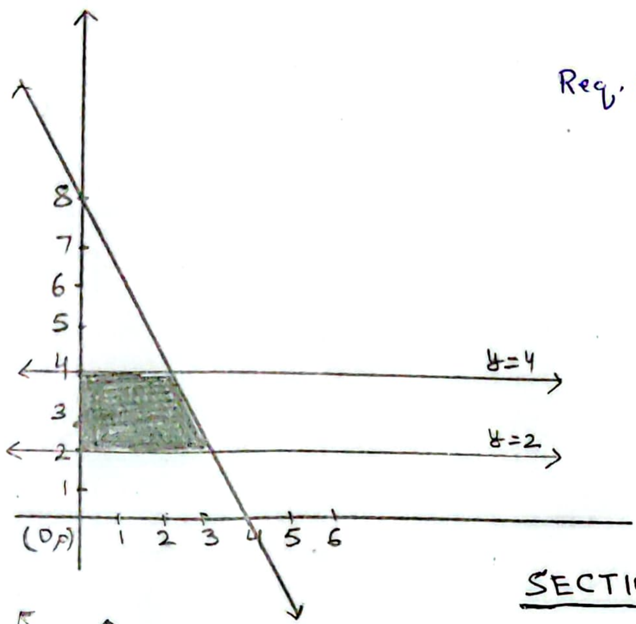
A24. General point on line is $P(\lambda, 2\lambda+1, 2\lambda-1)$.

Now, $OP = \sqrt{11} \Rightarrow OP^2 = 11$

$\Rightarrow \lambda^2 + (2\lambda+1)^2 + (2\lambda-1)^2 = 11 \Rightarrow k = \pm 1$

$\therefore$ co-ordinates of points are $(1,3,1)$ and $(-1,-1,-3)$

A25.



$$\text{Req. Area} = \int_2^4 x \cdot dy$$

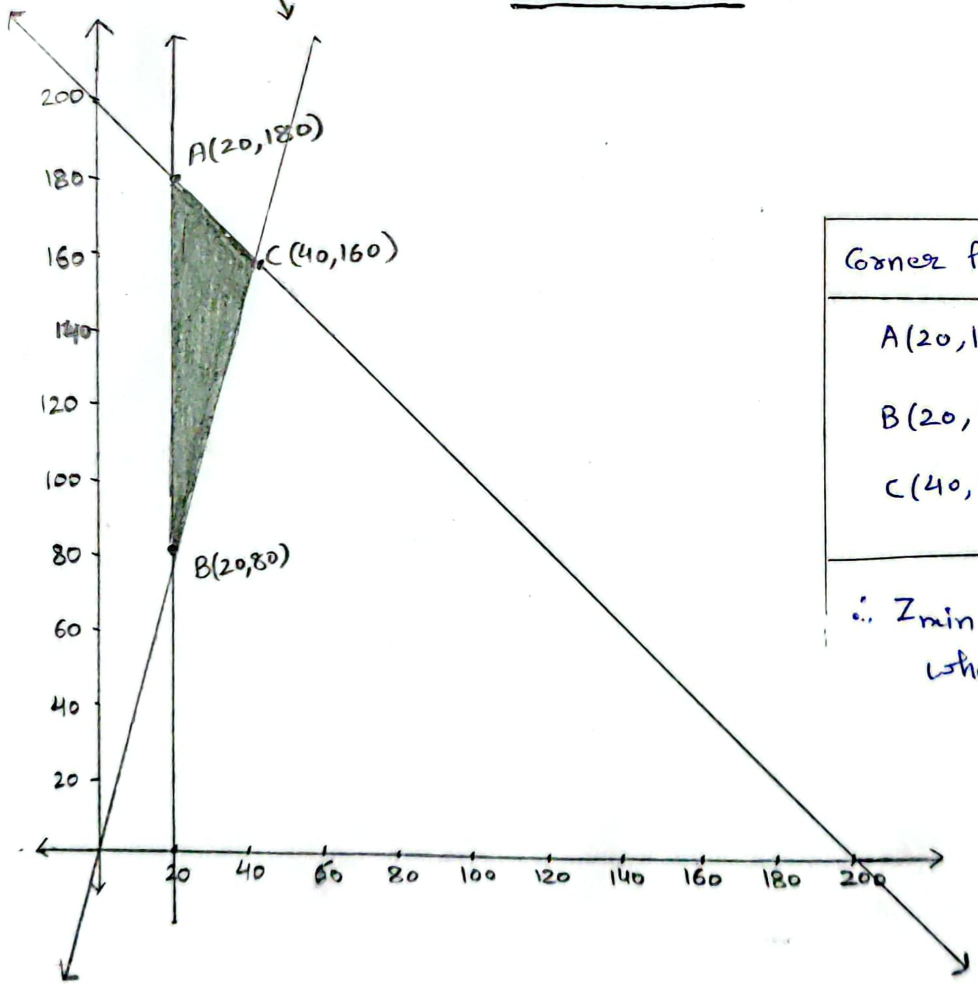
$$= \int_2^4 \frac{1}{2}(8-y)\,dy$$

$$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4$$

$= 5$ sq. units

SECTION-C

A26.



Corner Point	$Z = 500x + 400y$
A(20,180)	82000
B(20,80)	42000
C(40,160)	84000

$\therefore Z_{min} = 42000$

when $x = 20$, $y = 80$

A27. X : No. of red balls out of the two balls drawn.
then $X = 0, 1, 2$

Probability Distribution table is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\text{Mean} = 0\times\frac{1}{4} + 1\times\frac{1}{2} + 2\times\frac{1}{4} = 1$ Ans.

OR

Let $P(A) = x$, $P(B) = y$

$P(A)\cdot P(\bar{B}) = \frac{1}{4} \Rightarrow x(1-y) = \frac{1}{4}$ —— ①

$P(\bar{A})\cdot P(B) = \frac{1}{6} \Rightarrow (1-x)\cdot y = \frac{1}{6}$ —— ②

solving, $x - y = \frac{1}{12}$

eleminating y, we get $12x^2 - 13x + 3 = 0 \Rightarrow x = \frac{1}{3}, \frac{3}{4}$

$\therefore P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ or $P(A) = \frac{3}{4}, P(B) = \frac{2}{3}$

A28. $I = \int \frac{x^4}{(x-1)(x^2+1)} dx$

$= \int \left[x+1 + \frac{1}{(x-1)(x^2+1)}\right] dx$

$= \int \left(x+1 + \frac{1}{2(x-1)} - \frac{1}{2}\cdot\frac{x+1}{x^2+1}\right) dx$

$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$

OR

$I = \int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}} d\theta$

Put $\sin\theta = t$, $\cos\theta\, d\theta = dt$

$\therefore I = \int \frac{\cos\theta}{\sqrt{\sin^2\theta - 3\sin\theta + 2}} d\theta$

$= \int \frac{dt}{\sqrt{t^2 - 3t + 2}} = \int \frac{dt}{\sqrt{\left(t-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$= \log\left|\left(t - \frac{3}{2}\right) + \sqrt{t^2 - 3t + 2}\right| + C$

$= \log\left|\left(\sin\theta - \frac{3}{2}\right) + \sqrt{\sin^2\theta - 3\sin\theta + 2}\right| + C$

A29. Given diff. eqn is : $\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\sqrt{x^2+4}}{1+x^2}$

$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$

soln is given by : $y\cdot(1+x^2) = \int \frac{\sqrt{x^2+4}}{1+x^2}\cdot(1+x^2)\cdot dx = \int \sqrt{x^2+4}\, dx$

$\therefore y\cdot(1+x^2) = \frac{x\sqrt{x^2+4}}{2} + 2\log\left|x + \sqrt{x^2+4}\right| + C$

OR

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2} \quad \text{—— (1)}$$

Put $y = vx$ —— (2)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (3)}$$

using (2) & (3) from (1).

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2} \qquad \Rightarrow \int \frac{1+v^2}{v^3}\,dv = -\int \frac{dx}{x}$$

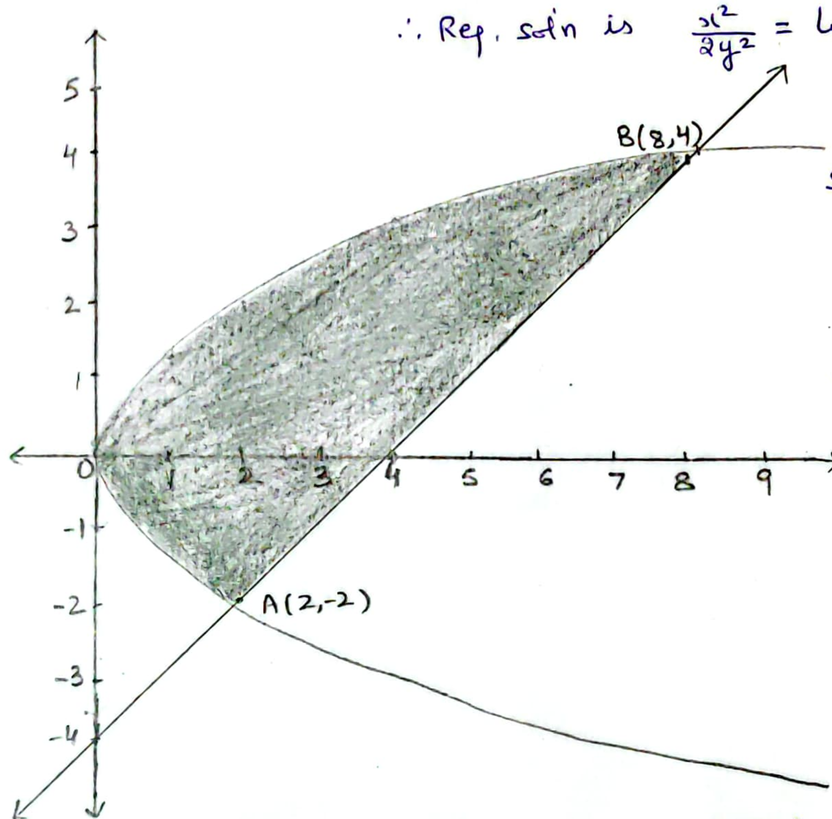
$$\Rightarrow -\frac{1}{2v^2} + \log|v| = -\log|x| + \log C$$

$$\Rightarrow -\frac{x^2}{2y^2} = \log\left|\frac{c}{y}\right|$$

when $x=0$, $y=1$ gives $c=1$

$\therefore$ Req. sol^n is $\frac{x^2}{2y^2} = \log|y|$

A30.



Solving $y^2 = 2x$ and $y = x-4$ we get $A(2,-2)$, $B(8,4)$

Req. Area, A

$$= \int_{-2}^{4} (y \text{ of Line})\,dx - \int_{-2}^{4} (y \text{ of Par.})\,dx$$

$$= \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy$$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$$

$= 18$ sq. units.

A31. Let $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ and $v = \sin^{-1}(2x\sqrt{1-x^2})$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\therefore u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right) = \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x \qquad \therefore \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

Again, $v = \sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta.\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$

$\therefore \frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{1}{2}$$

Note: If the substitution made is $x = \cos\theta$, answer will be $-\frac{1}{2}$.

SECTION-D

A32. $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow |A| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1}$ exists

here, $A_{11} = -2$, $A_{12} = -1$, $A_{13} = 3$

$A_{21} = -1$, $A_{22} = 2$, $A_{23} = -1$

$A_{31} = 5$, $A_{32} = -5$, $A_{33} = -5$

$\text{adj} A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

Given system of equations can be written as $AX = B$; $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$

Now, $X = A^{-1} \cdot B$

$$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$$

$\therefore x = 400,$
$y = 300,$
$z = 200$

A33. Vector eqn of required line through $(1,2,-4)$ is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and cartesian eqn is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda$

Equation of line through $A(3,3,-5)$ and $B(1,0,-11)$ is :

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Distance between parallel lines is given by $d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

Here, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$

$\therefore (\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\Rightarrow d = \frac{\sqrt{293}}{7}$

OR

Let d.r. of required line be a, b, c.

Since it is $\perp$ to two given lines;

$\therefore a + 2b + 3c = 0$ and $-3a + 2b + 5c = 0$

solving, $a = 4\lambda$, $b = -14\lambda$, $c = 8\lambda$

$\therefore$ Eqn of line is $\frac{x+1}{4\lambda} = \frac{y-3}{-14\lambda} = \frac{z+2}{8\lambda} = \mu$

or $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4} = \lambda'$

vector Eqn of line is $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda'(2\hat{i} - 7\hat{j} + 4\hat{k})$

Distance from origin $= \frac{|(-\hat{i} + 3\hat{j} - 2\hat{k}) \times (2\hat{i} - 7\hat{j} + 4\hat{k})|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|}$

$= \frac{|-2\hat{i} + \hat{k}|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|} = \frac{\sqrt{5}}{\sqrt{69}}$

A34. $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ——— ①

$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right) . \sin\left(\frac{\pi}{2}-x\right) \cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right) + \cos^4\left(\frac{\pi}{2}-x\right)} dx$

$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right) . \cos x . \sin x}{\cos^4 x + \sin^4 x} dx$ ——— ②

adding ① + ②,

$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \pi . \int_0^{\pi/4} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$\Rightarrow I = \frac{\pi}{2} . \int_0^{\pi/4} \frac{\tan x . \sec^2 x}{(\tan^2 x)^2 + 1} dx$ $(\because$ div. by $\cos^4 x)$

Putting $\tan^2 x = t \Rightarrow 2 \tan x . \sec^2 x \, dx = dt$

$I = \frac{\pi^2}{16}$

OR

$I = \int_0^{\pi/2} 2 \sin x \cos x . \tan^{-1}(\sin x) \, dx$

Let $\sin x = t \Rightarrow \cos x \, dx = dt$

$\therefore I = \int_0^1 2t . \tan^{-1} t \, dt$

$= t^2 . \tan^{-1} t \Big]_0^1 - \int_0^1 \frac{t^2}{1+t^2} dt$

$= \left[t^2 . \tan^{-1} t - t + \tan^{-1} t\right]_0^1 = \frac{\pi}{2} - 1$

A35. $f : [-4,4] \to [0,4]$

$f(x) = \sqrt{16-x^2}$

for onto. Let $y = f(x) \Rightarrow y = \sqrt{16-x^2} \Rightarrow y^2 = 16 - x^2 \Rightarrow x^2 = 16 - y^2 \Rightarrow x = \pm\sqrt{16-y^2}$

Now, $16 - y^2 \geq 0 \Rightarrow y^2 \leq 16 \Rightarrow -4 \leq y \leq 4 \Rightarrow y \in [-4,4]$;

but $y \geq 0 \Rightarrow y \in [0,4]$ ie. R_f = co-domain

$\Rightarrow f$ is onto.

for one-one

$f(-1) = f(1) = \sqrt{15}$, but $-1 \neq 1 \Rightarrow f$ is not one-one.

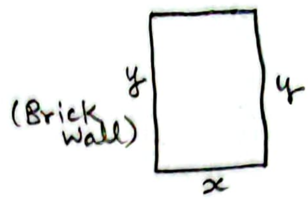
Now, $f(a) = \sqrt{7}$

$\Rightarrow \sqrt{16-a^2} = \sqrt{7} \Rightarrow a = \pm 3$

SECTION-E

A36. (i) (a) $2x + y = 200$

(b) $A(x) = xy = x(200-2x)$



(ii) from (a) & (b) of (i)

$A(x) = x(200-2x)$

$= 200x - 2x^2$

for max./min, $\frac{dA}{dx} = 0 \Rightarrow 200 - 4x = 0 \Rightarrow x = 50$

Now, $\frac{d^2A}{dx^2} = -4 < 0 \quad \therefore A$ is maximum at $x = 50$.

Thus, Max. $A(x) = 200(50) - 2(50)^2 = 5000$ sq.m.

A37. (i) $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

(ii) $P(E) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2)$

$= 0.65 \times 0.35 + 0.35 \times 0.8$

$= 0.35 \times 1.45 = 0.51$

(iii) (a) $P(E_1|E) = \dfrac{P(E_1).P(E|E_1)}{P(E_1).P(E|E_1) + P(E_2).P(E|E_2)} = \dfrac{0.65 \times 0.35}{0.51} = 0.45$

(b) $P(E_2|E) = \dfrac{P(E_2).P(E|E_2)}{P(E_1).P(E|E_1) + P(E_2).P(E|E_2)} = \dfrac{0.35 \times 0.8}{0.51} = 0.55$

A38. (i) for the year 2000, $t = 0$ and $V(0) = -2$, and the number of vehicles can't be negative $\therefore$ the given fn. $V(t)$ can't be used.

(ii) $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$\Rightarrow V'(t) = \frac{3}{5}t^2 - 5t + 25$

$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right] > 0$ always

$\therefore V(t)$ is an increasing fn.