Name _____ **Section**_____ **Roll No.**_____

CRPF PUBLIC SCHOOL, ROHINI
PRE-BOARD -2 EXAMINATION (2023-24)
CLASS XII
MATHEMATICS (SET-B)

Time Allowed: 3 hrs **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has **18 MCQ's and 02 Assertion Reasoning based questions** of 1 mark each.

3. Section B has **5 Very Short Answer (VSA)-type questions** of 2 marks each.

4. Section C has **6 Short Answer (SA)-type questions** of 3 marks each.

5. Section D has **4 Long Answer (LA)-type questions** of 5 marks each.

6. Section E has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A (MCQ) 1 Mark Questions	
Q1	If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to : (a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$
Q2	Let A be a 3×3 matrix such that $\lvert \text{adj } A \rvert = 64$. Then $\lvert A \rvert$ is equal to : (a) 8 only (b) -8 only (c) 64 (d) 8 or -8
Q3	If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $x + y + z$ is : (a) 10 (b) 6 (c) 8 (d) 0
Q4	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$

Q5 If the set A contains 5 elements and the set B contains 6 elements, then the number of both one-one and onto mapping from A to B is

(a) 720 (b) 120

(c) 30 (d) 0

Q6 The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$, is :

(a) 5 (b) 6

(c) 7 (d) 4

Q7 The number of feasible solutions of the linear programming problem given as

Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \le 12,\ x + 2y \le 10,\ x \ge 0, y \ge 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

Q8 In Δ ABC, $\vec{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\vec{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

Q9 $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

Q10 If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2

(c) 3 (d) -3

Q11 The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is :

(a) 2 (b) 0

(c) 1 (d) -1

Q12 The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

Q13 If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals :

(a) 4 (b) 2

(c) 8 (d) $\frac{1}{32}$

Q14 If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

Q15 If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is :

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

Q16 The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2\log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2\log x + \frac{x^2}{2} + C$

Q17 If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

Q18

Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are :

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$

(d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

Assertion Reasoning Based Questions

Q19

Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion (A) : Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

a. Both **A** and **R** are correct and **R** is the correct explanation of **A**
b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c. **A** is correct but **R** is not correct
d. **A** is not correct but **R** is correct

Q20

Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion (A) : A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

a. Both **A** and **R** are correct and **R** is the correct explanation of **A**
b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c. **A** is correct but **R** is not correct
d. **A** is not correct but **R** is correct

SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each

Q21

Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$.

OR

Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

Q22	If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.
Q23	A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a. **OR** Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
Q24	Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.
Q25	Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.
	SECTION – C (Short Answer (SA)-type questions) 3 Marks Each
Q26	Determine graphically the minimum value of the following objective function : $z = 500x + 400y$ subject to constraints $x + y \leq 200$, $x \geq 20$, $y \geq 4x$, $y \geq 0$.
Q27	Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **OR** A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).
Q28	Find : $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$ **OR** Find : $\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$

Q29 Find the general solution of the differential equation :

$$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

Q30 Find the area of the following region using integration :

$$\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$$

Q31 Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

SECTION – D (Long Answer (LA)-type questions) 5 Marks Each

Q32 If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

Q33 Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines.

OR

A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

Q34 Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$$

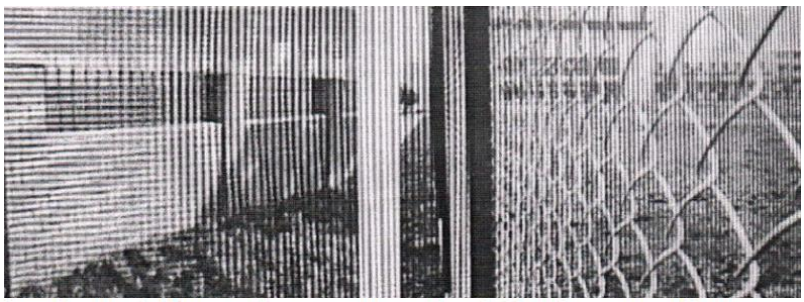
OR

Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$

Q35 A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

SECTION – E (Case Study Based Questions) 4 Marks Each

Q36 Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*

Q37 A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

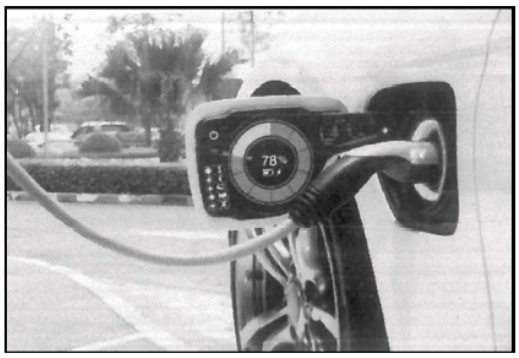
(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Q38

The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**