CRPF PUBLIC SCHOOL, ROHINI

SAMPLE PAPER-01 (2023-24)- SOLUTIONS

SECTION-A

(1) $A^2 = I \Rightarrow \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow x = 0$ ∴ option (a)

(2) $|\text{adj}A| = |A|^{n-1} = |A|^2$; $|A| = 2$ ∴ Ans $= 2^2$ ∴ option (c)

(3) $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 12 \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \Rightarrow |\vec{x}|^2 = 13 \Rightarrow |\vec{x}| = \sqrt{13}$ ∴ option (d)

(4) $\lim_{x\to 0} f(x) = f(0) \Rightarrow 2k = \lim_{x\to 0} \frac{3\sin\pi x}{5x} \Rightarrow k = \frac{3}{10}\cdot\left(\lim_{x\to 0}\frac{\sin\pi x}{\pi x}\right).\pi = \frac{3\pi}{10}$ ∴ option (b)

(5) $I = \int \frac{\sec^2(1+\log x)}{x}dx$; Put $1+\log x = t \Rightarrow \frac{1}{x}dx = dt$ ∴ $I = \int \sec^2 t\, dt = \tan t + C$

∴ $I = \tan(1+\log x) + C$ ∴ option (a)

(6) $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = C$ ∴ option (c)

(7) clearly option (d)

(8) $|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$; $\vec{a}.\vec{b} = |\vec{b}|^2$

$|\vec{a}-\vec{b}|^2 = 7 \Rightarrow |\vec{a}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2 = 7 \Rightarrow 14 - 2|\vec{b}|^2 + |\vec{b}|^2 = 7$

$\Rightarrow |\vec{b}|^2 = 7 \Rightarrow |\vec{b}| = \sqrt{7}$ ∴ option (a)

(9) $I = \int\frac{dx}{4+9x^2} = \int\frac{dx}{(2)^2+(3x)^2} = \frac{1}{2}\times\frac{1}{3}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right] + C$

∴ $\int_0^{2/3}\frac{dx}{4+9x^2} = \frac{1}{6}\cdot\left[\tan^{-1}1 - \tan^{-1}0\right] = \frac{\pi}{24}$ ∴ option (c)

(10) adding, $2A = \begin{bmatrix} 2 & -4 \\ 6 & 8 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$; subtracting, $2B = \begin{bmatrix} 6 & -2 \\ -4 & 4 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

∴ $AB = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 3+4 & -1-4 \\ 9-8 & -3+8 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ ∴ option (b)

(11) $Z_{(0,0)} = 0$; $Z_{(0,40)} = \underline{40}$; $Z_{(40,0)} = 40(0.4) = 16$; $Z_{(30,20)} = 30(0.4) + 20 = 32$

clearly, $Z_{max} = 40$ ∴ option (b)

(12) $9(2x+5) - 3(5x+2) = 0 \Rightarrow 18x - 15x + 45 - 6 = 0 \Rightarrow 3x = -39 \Rightarrow x = -13$ ∴ option (c)

(13) $|A| \neq 0 \Rightarrow 4(6-5) - \lambda(-5) - 3(-2) \neq 0 \Rightarrow 4 + 5\lambda + 6 \neq 0 \Rightarrow \lambda \neq -2$ ∴ option (c)

(14) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$; $P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(A\cap B) = P(B)\times\frac{1}{4} = \frac{1}{12}$

Now, $P(A'\cap B') = P[(A\cup B)'] = 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right] = 1 - \left(\frac{6+4-1}{12}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

∴ option (c)

(15) $m+n = 2+2 = 4 \quad \therefore$ option (d)

(16) $x = a(\cos\theta + \theta\sin\theta) \Rightarrow \frac{dx}{d\theta} = a(-\sin\theta + \theta.\cos\theta + \sin\theta.1) = a\theta\cos\theta$

$y = a(\sin\theta - \theta\cos\theta) \Rightarrow \frac{dy}{d\theta} = a\left[\cos\theta - (\theta.(-\sin\theta)) + \cos\theta.1)\right] = a\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta \quad \therefore$ option (b)

(17) $\overrightarrow{AB} = 2\hat{i} + (4-x)\hat{j} + 4\hat{k} \quad ; \quad \overrightarrow{BC} = (y-3)\hat{i} + 6\hat{j} + 12\hat{k}$

A, B, C are collinear , $\frac{2}{y-3} = \frac{4-x}{6} = \frac{4}{12} \Rightarrow x = 2, y = 3 \quad \therefore$ option (a)

(18) $\cos^2\alpha + \frac{1}{4} + \frac{1}{4} = 1 \Rightarrow \cos^2\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ \quad \therefore$ option (a)

(19) option (a)

(20) option (c)

Section - B

(21) here, $f(0) = \frac{1+0^2}{1+0} = 1 \quad ; \quad f(1) = \frac{1+1^2}{1+1} = 1 \quad$ ie. $f(0) = f(1)$ but $0 \neq 1$

$\therefore f$ is not one-one.

OR

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$\therefore y = \sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2}\right) = \sin^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)$

ie. $y = \sin^{-1}\left(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$

$\therefore \frac{dy}{dx} = \frac{1}{2} \times \frac{-1}{\sqrt{1-x^2}}$ Hence shown

(22)  $\frac{dV}{dt} = \pi \text{ cm}^3/s \quad ;$ To find $\left.\frac{dS}{dt}\right]_{r=1cm}$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt} \Rightarrow \pi = \frac{4}{3}\pi.3r^2.\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4r^2}$

Now, $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r.\frac{dr}{dt} \Rightarrow \frac{dS}{dt} = 8\pi r \times \frac{1}{4r^2} = \frac{2\pi}{r}$

$\therefore \left.\frac{dS}{dt}\right]_{r=1} = \frac{2\pi}{1} = 2\pi \text{ cm}^2/s$

(23) $|\vec{a}| = |\vec{b}| = k$ (say) $\quad ; \quad \theta = 60^\circ \quad , \quad \vec{a}.\vec{b} = \frac{9}{2} \Rightarrow |\vec{a}|.|\vec{b}|\cos\theta = \frac{9}{2}$

$\Rightarrow k.k.\cos 60^\circ = \frac{9}{2} \Rightarrow k^2 \times \frac{1}{2} = \frac{9}{2} \Rightarrow k = 3$

OR

Passing point is $(1,2,-1)$; line : $5x-25 = 14-7y = 35z$

$\Leftrightarrow 5(x-5) = -7(y-2) = 35z \Leftrightarrow \frac{x-5}{\left(\frac{1}{5}\right)} = \frac{y-2}{\left(-\frac{1}{7}\right)} = \frac{z}{\left(\frac{1}{35}\right)}$

$\Leftrightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$

$\therefore$ Req. vector eqn :

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

(24) $y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right) = \tan^{-1}\left(\cot\frac{x}{2}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right)$

$\therefore y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

(25) $A(1,1,2),\ B(2,3,5),\ C(1,5,5)$

$\vec{BA} = -\hat{i} - 2\hat{j} - 3\hat{k}, \quad \vec{BC} = -\hat{i} + 2\hat{j} + 0\hat{k}$

$ar(\Delta ABC) = \frac{1}{2}|\vec{BA} \times \vec{BC}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ -1 & 2 & 0 \end{vmatrix} = \frac{1}{2}|\hat{i}(6) - \hat{j}(-3) + \hat{k}(-4)|$

$= \frac{1}{2}\sqrt{36+9+16} = \frac{\sqrt{61}}{2}$ sq. units

Section-C

(26) $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \Rightarrow 8 = A(x^2+4) + (Bx+C)(x+2)$

$A+B=0, \quad 2B+C=0, \quad 4A+2C=8 \quad ; \quad A=1, B=-1, C=2$

$I = \int\frac{dx}{x+2} + \int\frac{-x+2}{x^2+4}dx = \int\frac{dx}{x+2} - \frac{1}{2}\int\frac{2x}{x^2+4}dx + 2\int\frac{dx}{x^2+4}$

$= \log|x+2| - \frac{1}{2}\log|x^2+4| + \cancel{2}\times\frac{1}{\cancel{2}}\tan^{-1}\left(\frac{x}{2}\right) + C$

(27) $P(A) = \frac{4}{5}, \quad P(B) = \frac{3}{4}, \quad P(C) = \frac{2}{3}$

(i) $P(B \cap C \cap \bar{A}) = P(B).P(C).P(\bar{A}) = \frac{3}{4}\times\frac{2}{3}\times\frac{1}{5} = \frac{1}{10}$

(ii) $P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC) = \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3} + \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} + \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{13}{30}$

OR

X = no. of red cards drawn ; $X = 0,1,2$

$P(X=0) = \frac{26}{52}\times\frac{25}{51} = \frac{650}{2652}$; $P(X=1) = \frac{26}{52}\times\frac{26}{51}\times 2 = \frac{1352}{2652}$; $P(X=2) = \frac{26}{52}\times\frac{25}{51} = \frac{650}{2652}$

Probability Dist. is

X	0	1	2
P(X)	$\frac{650}{2652}$	$\frac{1352}{2652}$	$\frac{650}{2652}$

ie.

X	0	1	2
P(X)	$\frac{25}{102}$	$\frac{52}{102}$	$\frac{25}{102}$

mean = 1

(28) $x^2 - 2x = 0 \Rightarrow x = 0, 2 \quad \therefore I = -\int_1^2(x^2-2x)dx + \int_2^3(x^2-2x)dx$

$= -\left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_1^2 + \left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_2^3 = 2$

OR

$I = \int_0^\pi \frac{x\tan x}{\sec x.\operatorname{cosec} x}dx = \int_0^\pi x.\sin^2 x\, dx = \int_0^\pi x.\left(\frac{1-\cos 2x}{2}\right)dx$

$\Rightarrow I = \frac{1}{2}\int_0^\pi x\,dx - \frac{1}{2}\int_0^\pi \underset{I}{x}.\underset{II}{\cos 2x}\,dx$

$= \frac{1}{4}.[x^2]_0^\pi - \frac{1}{2}\left[x.\frac{\sin 2x}{2} - \int 1.\frac{\sin 2x}{2}dx\right]_0^\pi$

$= \frac{\pi^2}{4}$

(29) $\frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)}$

Put $\frac{y}{x} = v$ ie. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{\sin v} \Rightarrow x\frac{dv}{dx} = -\operatorname{cosec} v \Rightarrow \frac{dv}{\operatorname{cosec} v} = -\frac{dx}{x} \Rightarrow \int \sin v\, dv = -\int\frac{dx}{x}$

$\Rightarrow -\cos v = -\log|x| + C \Rightarrow \cos v = \log|x| - C \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + c'$

when $x = 1$, $y = \frac{\pi}{2} \Rightarrow C = 0$ $\therefore$ solution is $\cos\left(\frac{y}{x}\right) = \log|x|$

OR

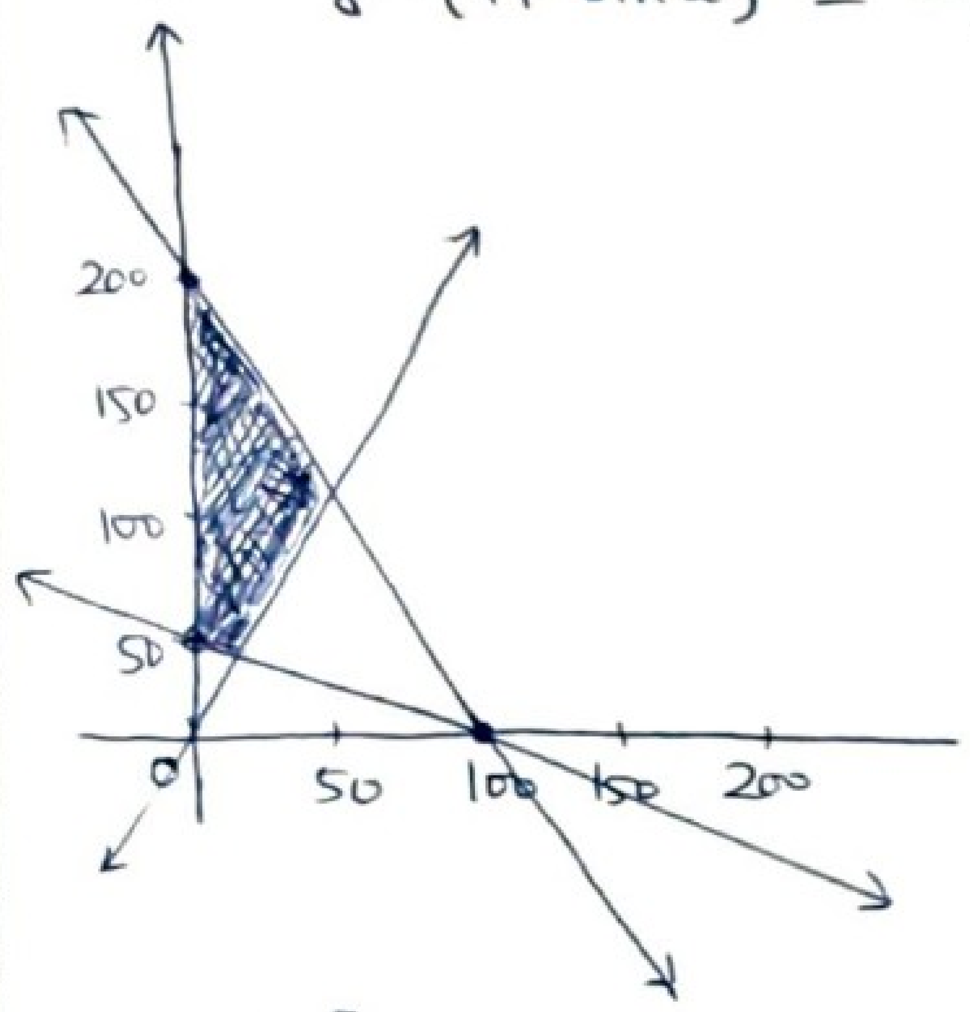
$\frac{dy}{dx} + \underbrace{\left(\frac{\cos x}{1+\sin x}\right)}_{P} \cdot y = \underbrace{-\frac{x}{1+\sin x}}_{Q}$

I.F. $= e^{\int P dx} = 1 + \sin x$

Sol'n : $y\cdot(1+\sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\, dx = \int -x\, dx = -\frac{x^2}{2} + C$

ie. $y\cdot(1+\sin x) = -\frac{x^2}{2} + C$

(30)



Corner Pt	$Z = x + 2y$
(0,50)	100
(50,100)	250
(0,200)	400
(20,40)	100

$\rightarrow$ max. at $x = 0$, $y = 200$

(31) $I = \int \frac{x^3+x+1}{x^2-1}\, dx = \int x\, dx + \int \frac{2x}{x^2-1}\, dx + \int\frac{dx}{x^2-1}$

$= \frac{x^2}{2} + \log|x^2-1| + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

Section-D

(32) for one-one : let $x_1, x_2 \in R - \{-\frac{4}{3}\}$

let $f(x_1) = f(x_2)$

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one

for onto let $y = f(x) \Rightarrow y = \frac{4x+3}{3x+4} \Rightarrow x = \frac{3-4y}{3y-4} \in$ co-domain

$\Rightarrow f$ is onto

OR

$|a-a| = 0$, which is divisible by 4 $\therefore$ S is reflexive.

$(a,b) \in S \Rightarrow |a-b|$ is div. by 4 $\Rightarrow |b-a|$ is div. by 4 $\Rightarrow (b,a) \in S \Rightarrow S$ is symm.

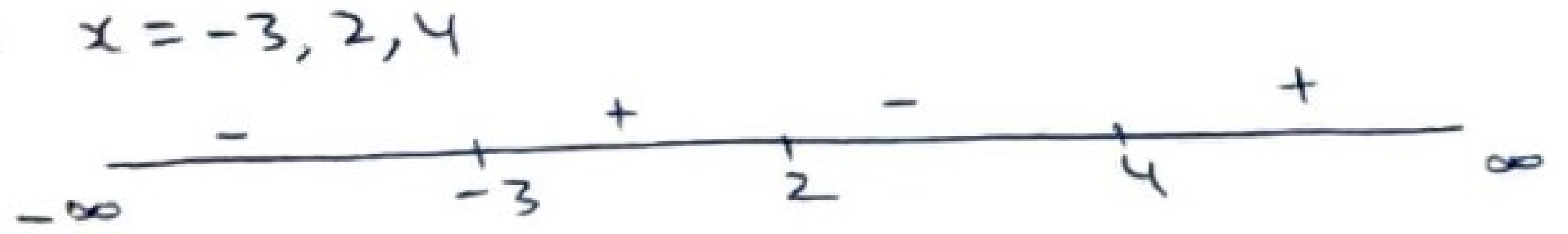
$(a,b) \in S$ and $(b,c) \in S \Rightarrow |a-b| = 4\lambda$, $|b-c| = 4\mu \Rightarrow |a-c| = 4\lambda' \Rightarrow S$ is tran.

$\therefore S$ is equivalence.

$[1] = \{1, 5, 9\}$

(33) $f'(x) = x^3 - 3x^2 - 10x + 24 = (x-2)(x-4)(x+3)$

$f'(x) = 0 \Rightarrow x = -3, 2, 4$



f is inc. when $(-3, 2) \cup (4, \infty)$

f is dec. when $(-\infty, -3) \cup (2, 4)$

(34) Let eqn of req. line is $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$

here, $3a - 16b + 7c = 0$ and $3a + 8b - 5c = 0$

solving: $\frac{a}{24} = \frac{b}{36} = \frac{c}{72} = \lambda'$ or $\frac{a}{2} = \frac{b}{3} = \frac{z}{6}$

Req. Ans: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

OR

here, $\vec{a_1} = \hat{i} + \hat{j} - \hat{k}$, $\vec{a_2} = \hat{i} + \hat{j} + 2\hat{k}$; $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

given lines are parallel.

here, $\vec{a_2} - \vec{a_1} = 3\hat{k}$; $|\vec{b}| = \sqrt{6}$

Req. Dist. $= \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right| = \frac{|3\hat{i} - 6\hat{j}|}{\sqrt{6}} = \frac{\sqrt{45}}{\sqrt{6}} = \sqrt{\frac{15}{2}}$

(35) $|A| = 6 \neq 0 \Rightarrow A^{-1}$ exists.

Cofactors: $A_{11} = -6$; $A_{12} = 14$; $A_{13} = -15$

$A_{21} = 17$; $A_{22} = 5$; $A_{22} = 9$

$A_{31} = 13$; $A_{32} = -8$; $A_{33} = -1$

$$\text{adj } A = \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj} A = \frac{1}{6} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$$

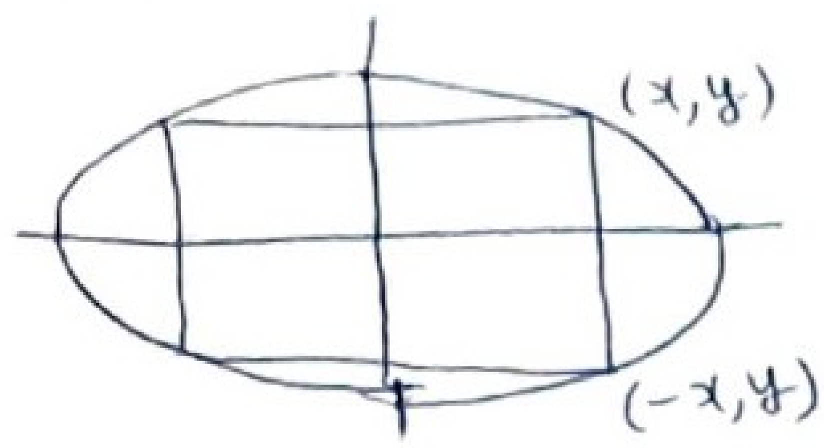
$$X = A^{-1} \cdot B = \frac{1}{6} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 201 \\ -134 \\ 67 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

$\Rightarrow x = 3$, $y = 2$, $z = -1$ Ans

Section D

(36)



$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

(i) $A = (2x)(2y) = 4x \cdot \frac{b}{a}\sqrt{a^2-x^2} \Rightarrow A = \frac{4b}{a} x \sqrt{a^2-x^2}$

(ii) $A^2 = A' = \frac{16b^2}{a^2} \cdot x^2(a^2 - x^2)$

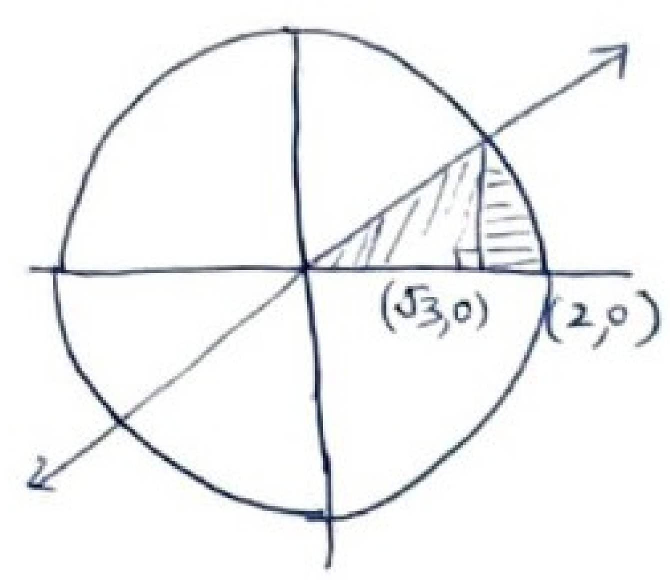
$\frac{dA'}{dx} = x(2a^2 - 4x^3) = 0 \Rightarrow x = \pm\frac{a}{\sqrt{2}}, 0 \Rightarrow x = \frac{a}{\sqrt{2}}$; when $x = \frac{a}{\sqrt{2}}$,

(iii) $\left.\frac{d^2A'}{dx^2}\right]_{x<\frac{a}{\sqrt{2}}} = +ve$; $\left.\frac{d^2A'}{dx^2}\right]_{x>\frac{a}{\sqrt{2}}} = -ve$ $\therefore$ A' or A is max.

also, $\frac{d^2A'}{dx^2} = \frac{16b^2}{a^2}\cdot\left(2x^2 - \frac{12a^2}{2}\right) < 0$ $\therefore$ A' or A is max. at $x = \frac{a}{\sqrt{2}}$

Now, $l = 2x = \sqrt{2}a$ and $b = 2y = \sqrt{2}b$

(37)



$$A = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

$$= \frac{1}{2\sqrt{3}}\left[x^2\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2} = \frac{\pi}{3} \text{ sq. units}$$

(38) E_1: item manufactured by A

E_2: " " " B

E_3: " " " C

A : defective item is found.

(a) $P(A) = \frac{50}{100}\times\frac{1}{100} + \frac{30}{100}\times\frac{5}{100} + \frac{20}{100}\times\frac{7}{100} = \frac{340}{10000} = \frac{17}{500}$

(b) $P(E_2|A) = \dfrac{\frac{30}{100}\times\frac{5}{100}}{P(A)} = \frac{15}{34}$