CRPF PUBLIC SCHOOL, ROHINI
SAMPLE PAPER-01 (2023-24)
CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has **18 MCQ's and 02 Assertion Reasoning based questions** of 1 mark each.

3. Section B has **5 Very Short Answer (VSA)-type questions** of 2 marks each.

4. Section C has **6 Short Answer (SA)-type questions** of 3 marks each.

5. Section D has **4 Long Answer (LA)-type questions** of 5 marks each.

6. Section E has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A (MCQ) 1 Mark Questions	
Q1	If $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to (a) 0 (b) 1 (c) 2 (d) -1
Q2	If $A=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -3 & 4 & 1 \end{bmatrix}$, the value of $\lvert adjA \rvert$ is: (a) 2^0 (b) 2^1 (c) 2^2 (d) 2^3
Q3	If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$, then the magnitude of $\vec{x}$ is: $(a)\ \sqrt{12}$ (b) 12 (c) 13 $(d)\ \sqrt{13}$
Q4	If $f(x)=\begin{cases} \dfrac{3\sin \pi x}{5x} & ,x\neq 0 \\ 2k & ,x=0 \end{cases}$ is continuous at $x=0$, then the value of k is: (a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$ (c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$
Q5	The value of $\int \frac{1}{x\cos^2(1+\log x)}dx$ is: (a) $\tan(1+\log x)+c$ (b) $\cot(1+\log x)+c$ (c) $\sec(1+\log x)+c$ (d) $\cos(1+\log x)+c$

Q6

The solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is:

(a) $y+\sin^{-1}y=\sin^{-1}x+c$ (b) $\sin^{-1}y-\sin^{-1}x=c$

(c) $\sin^{-1}y+\sin^{-1}x=c$ (d) $\sin^{-1}y-\sin^{-1}x=cxy$

Q7

Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).

Let F = 4x + 6y be the objective function. The minimum value of F occurs at:

(a) Only (0, 2)

(b) Only (3, 0)

(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)

(d) any point on the line segment joining the points (0, 2) and (3, 0)

Q8

If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}.\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|$ is: equal to:

(a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) 7 (d) 3

Q9

$\int_0^{\frac{2}{3}}\frac{dx}{4+9x^2}$ is equal to:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{9}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{36}$

Q10

If A and B are 2 × 2 square matrices and A + B = $\begin{bmatrix}4 & -3\\1 & 6\end{bmatrix}$ and A – B = $\begin{bmatrix}-2 & -1\\5 & 2\end{bmatrix}$,

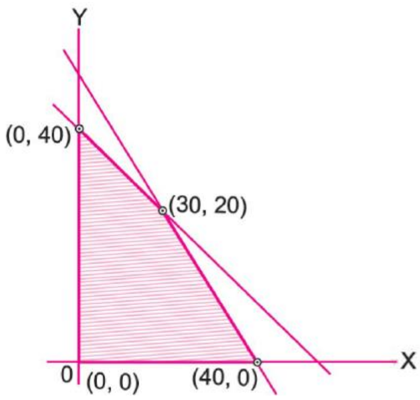
then the value of AB is:

(a) $\begin{bmatrix}-7 & 5\\1 & -5\end{bmatrix}$ (b) $\begin{bmatrix}7 & -5\\1 & 5\end{bmatrix}$ c) $\begin{bmatrix}7 & -1\\5 & -5\end{bmatrix}$ (d) $\begin{bmatrix}7 & -1\\-5 & 5\end{bmatrix}$

Q11

Feasible region (shaded) for a LPP is shown in the given figure.

The maximum value of the Z = 0.4x + y is:



(a) 45 (b) 40 (c) 50 (d) 41

Q12

If $\begin{vmatrix}2x+5 & 3\\5x+2 & 9\end{vmatrix}$ = 0, then the value of x is:

(a) 13 (b) 3 (c) -13 (d) √3

Q13	If $A=\begin{bmatrix}4 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} exist if (a) $\lambda = 2$ (b) $\lambda \neq 2$ (c) $\lambda \neq -2$ (d) $\lambda = -2$
Q14	If A and B are two events such that P(A) = $\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P\left(\frac{A}{B}\right)=\frac{1}{4}$, then $P(A' \cap B')$ equals (*a*) $\frac{1}{12}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{16}$
Q15	The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is : (a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$
Q16	If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx}$ is : (*a*) $\cot\theta$ (*b*) $\tan\theta$ (*c*) $a\cot\theta$ (*d*) $a\tan\theta$
Q17	If three points A, B and C have position vectors $\hat{i} + x\hat{j} + 3\hat{k}$, $3\hat{i} + 4\hat{j} + 7\hat{k}$ and $y\hat{i} - 2\hat{j} - 5\hat{k}$ respectively are collinear, then (*x*, *y*) is: (*a*) (2, –3) (*b*) (–2, 3) (*c*) (–2, –3) (*d*) (2, 3)
Q18	A line is such that it is inclined with *y*-axis and *z*-axis at 60°, then the angle this line is inclined with *x*-axis, is: (*a*) 45° (*b*) 30 ° (*c*) 75 ° (*d*) 60 °
	Assertion Reasoning Based Questions
Q19	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R** **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **Reason (R) :** Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
Q20	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

	Assertion A: If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is $\vec{r}=5\hat{i}-4j+6k+\lambda(3\hat{i}+7j+2k)$ **Reason R:** The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x-3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{-2}=\frac{y-4}{4}=\frac{z+8}{-5}$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
	SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each
Q21	Let $f:R-\{-1\}\to R$ be defined by, $f(x)=\frac{1+x^2}{1+x}$. Show that f is not 1-1. **OR** If y = $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$
Q22	A sphere increases its volume at the rate of π cm^3/s. Find the rate at which its surface area increases, when the radius is 1 cm.
Q23	Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that angle between them is 60^0 and their scalar product is $\frac{9}{2}$. **OR** Find the vector equation of the line passing through the point $A(1,2,-1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.
Q24	Differentiate $y=\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.
Q25	Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).
	SECTION – C (Short Answer (SA)-type questions) 3 Marks Each
Q26	Evaluate : $\int\frac{8}{(x+2)(x^2+4)}dx$

Q27	One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. **OR** Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Let X denote the number of red cards drawn. Find the probability distribution of X. Also, find the mean of this distribution.
Q28	Evaluate : $\int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4} dx$ **OR** Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x . \operatorname{cosec} x} dx$
Q29	Solve the following differential equation: $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x = y\sin\left(\frac{y}{x}\right)$; $y(1) = \frac{\pi}{2}$ **OR** Solve the following differential equation: $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$
Q30	Solve the following Linear Programming Problem graphically: Maximise Z = x + 2y Subject to the constraints: $x + 2y \geq 100$; $2x - y \leq 0$; $2x + y \leq 200$; $x, y \geq 0$
Q31	Evaluate : $\int \frac{x^3 + x + 1}{x^2 - 1} dx$
SECTION – D (Long Answer (LA)-type questions) 5 Marks Each	
Q32	Consider $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is one-one and onto. **OR** Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 10\}$ given by $S = \{(a, b) : a, b \in Z, \lvert a - b \rvert$ is divisible by 4$\}$. Show that S an equivalence relation. Find the set of all elements related to 1.
Q33	Find the sub-intervals in which $f(x) = \log(2 + x) - \frac{x}{2 + x}$, $x > -2$ is increasing or decreasing.
Q34	Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B. **OR**

	Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.
Q35	Using Matrix Method, solve the following system of linear equations. $x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$
SECTION – E (Case Study Based Questions) 4 Marks Each	
Q36	Read the following passage and answer the questions given below. In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$  (i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. (ii) Find the critical point of the function. (iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area. **OR** (iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

Q37 Rohit cuts a cake with a knife on his eighteenth birthday. The circular cake is represented by $x^2 + y^2 = 4$. The sharp edge of the knife represents a straight line given by $x = \sqrt{3}y$.

Based on the above information, answer the following questions.

(a) Draw the suitable geometrical figure representing the above situation. Also shade the smaller region formed by the line and the circle in the first quadrant and above x-axis.

(b) Using integration, find the area of the smaller region shaded above.

Q38

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time. B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random.

(a) What is the probability of getting a defective item?

(b) If the item so chosen is found to be defective. What is the probability that it was produced by B?