CRPF PUBLIC SCHOOL, ROHINI

SAMPLE PAPER-02 (2023-24)

CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E.** Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of 1 **mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

Q1. If A is a square matrix of order 3 such that $|adj\ A| = 144$, the value of $\left|A^T\right|$ is:

(a) 0 (b) 144 (c) $\pm$ 12 (d) 12

Q2. If $0 < x < \pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, the value(s) of x is:

(a) $\pi/3$ (b) $\pi/6$ (c) $5\pi/6$ (d) $2\pi/3,\ \pi/3$

Q3 If the points $A(3,-2), B(k,2)$ and $C(8,8)$ are collinear, then the value of k is:

(a) 2 (b) -3 (c) 4 (d) -4

Q4. If $A = \begin{bmatrix} 0 & x+2 \\ 2x-3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then x is equal to:

(a) $\frac{1}{3}$ (b) 5 (c) 3 (d) 1

Q5. If $y = \tan^{-1}(\sec x + \tan x)$, then $\frac{dy}{dx}$ is:

(a) $1/2$ (b) $-1/2$ (c) 1 (d) none of these

Q6. If the function $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2} & ,x\neq 0\\ k & ,x=0\end{cases}$ is continuous at $x=0$, then value of k is:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Q7. $\int \frac{x+3}{(x+4)^2}e^x dx = ?$

(a) $\frac{e^x}{x+4}+c$ (b) $\frac{e^x}{x+3}+c$ (c) $\frac{1}{(x+4)^2}+c$ (d) $\frac{e^x}{(x+4)^2}+c$

Q8. $\int \frac{dx}{e^x+e^{-x}}$ is equal to

(a) $\tan^{-1}(e^x)+C$ (b) $\tan^{-1}(e^{-x})+C$

(c) $\log(e^x-e^{-x})+C$ (d) $\log(e^x+e^{-x})+C$

Q9. The sum of the cofactors of element 13 and 1 in $\begin{vmatrix}10 & 19 & 2\\ 0 & 13 & 1\\ 9 & 24 & 2\end{vmatrix}$ is:

(a) 71 (b) -69 (c) -67 (d) -71

Q10. The integrating factor of the differential equation $(x\log x)\frac{dy}{dx}+y=2\log x$ is:

(a) $\log(\log x)$ (b) e^x (c) $\log x$ (d) x

Q11. Sum of order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^5+4x=0$ is

(a) 6 (b) 3 (c) 4 (d) 5

Q12. The p.v.'s of the points A,B,C are $\hat{i}+x\hat{j}+3\hat{k}$, $3\hat{i}+4\hat{j}+7\hat{k}$ and $y\hat{i}-2\hat{j}-5\hat{k}$ respectively, if A,B,C are collinear, then $(x,y)=?$

(a) $(2,-3)$ (b) $(-2,3)$ (c) $(0,3)$ (d) $(2,3)$

Q13. If $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=|\vec{b}|=\sqrt{2}$ and $\vec{a}.\vec{b}=-1$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\pi/3$ (b) $\pi/4$ (c) $2\pi/3$ (d) $\frac{\pi}{2}$

Q14. The diagonals of a parallelogram are represented by the vectors $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$. The area of the parallelogram is:

(a) $7\sqrt{3}$ sq.units (b) $5\sqrt{3}$ sq.units (c) $3\sqrt{5}$ sq.units (d) $\frac{3\sqrt{2}}{2}$ sq. units

Q15. If a line makes angle $\pi/3$ and $\pi/4$ with x-axis and y-axis respectively, then the acute angle made by the line with z-axis is:

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $5\pi/12$

Q16. A bag contains 5 red, 7 green and 4 white balls, three balls are drawn one after the other without replacement. Then the probability that the balls drawn are white, green and green respectively, is:

(a) $\frac{1}{20}$ (b) $\frac{3}{20}$ (c) $\frac{7}{20}$ (d) $\frac{3}{7}$

Q17.

The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

Q18.

The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2 (b) 3

(c) 4 (d) 5

Assertion Reasoning Based Questions

Q19. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A)

The equation of the line passing through $(1,1,2)$ *and* $(2,3,-1)$ is $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{-3}$

Reason (R)

Equation of the line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

(a) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(c) **A** is correct but **R** is not correct

(d) **A** is not correct but **R** is correct

Q20. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

(a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(c) **A** is correct but **R** is not correct
(d) **A** is not correct but **R** is correct

SECTION B

Q21. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represents two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

If $\vec{a}$, $\vec{b}$ are unit vectors such that the vector $\vec{a} + 3\vec{b}$ is perpendicular to the vector $7\vec{a} - 5\vec{b}$ and $\vec{a} - 4\vec{b}$ is perpendicular to $7\vec{a} - 2\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

Q22. If A, B, C and D are the points with position vectors

$\hat{i} - \hat{j} + \hat{k}, 2\hat{i} - \hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ respectively, then find the projection of $\vec{AB}$ along $\vec{CD}$.

Q23. Write the following in the simplest form: $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

OR

A relation R is defined on a set of real numbers $\mathbb{R}$ as

R = {(x, y) : x . y is an irrational number}.

Check whether R is reflexive, symmetric and transitive or not.

Q24. If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

Q25. Sand is pouring from a pipe at the rate of 12 cm^3/sec The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

SECTION C

Q26.

Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1+5^x}\,dx$$

OR

Evaluate: $\int_1^4 |x-1| + |x-2| + |x-3| dx$

Q27. Evaluate the following integral x: $\int \frac{3x+1}{(x-2)^2(x+2)} dx$

Q28.

Find : $\int \frac{x^3+x}{x^4-9}\,dx.$

Q29. Solve the following differential equation:

$$(tan^{-1}y - x)dy = (1+y^2)dx$$

OR

Solve the following differential equation:

$$x\frac{dy}{dx} - y + xsin\left(\frac{y}{x}\right) = 0, \text{given that when } x = 2, y = \pi$$

Q30.

Let X denote the number of colleges where you will apply after your results and $P(X=x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x) = \begin{cases} kx & , \text{if } x = 0 \text{ or } 1 \\ 2kx & , \text{if } x = 2 \\ k(5-x) & , \text{if } x = 3 \text{ or } 4 \\ 0 & , \text{if } x > 4 \end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in

(i) exactly one college
(ii) atmost 2 colleges
(iii) atleast 2 colleges.

OR

There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Find the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution.

Q31. Solve the following Linear Programming Problem graphically:

Maximize $z = 2x + 5y$ subject to the following constraints:

$2x + 4y \le 8$

$3x + y \le 6$

$x + y \le 4$

$x \ge 0, y \ge 0$

SECTION D

Q32. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$.

OR

Find the product of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.$

Q33.

Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

Q34. Consider $f: R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Check it is bijective or not. Justify your answer.

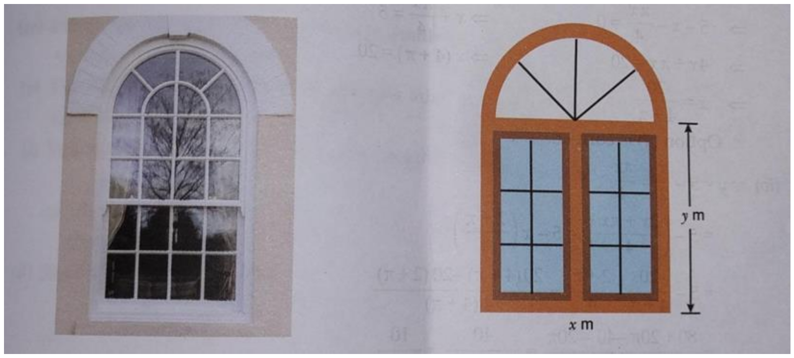
OR

Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

Q35. Dr. Dewan residing in Delhi went to see an apartment of 3BHK in Noida. The window of the house was in the form of rectangle surmounted by a semicircular opening having a perimeter of the window 10m as shown in the figure.



If x and y represents the length and breadth of the rectangular region, what is the relation between the variables?

(i) What is the area of the window in terms of x?

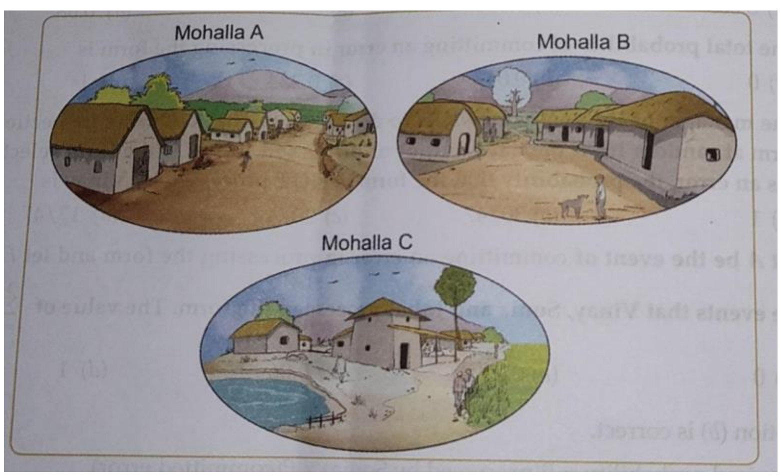
(ii) What should be the value of x for the area to be maximum?

SECTION E

(Question numbers 36 to 38 carry 4 mark each.)

This section contains three Case-study/Passage based questions.
First two questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, and 2 respectively.
Third question has two sub-parts of 2 marks each.

Q36. In a village there are three mohallas A, Band C. In A,60% farmers believe in new technology of agriculture, while in B, 70% and in C, 80%. A farmer is selected at random from village.



(i) What is the conditional probability that a farmer believe in new technology if he belongs to mohalla A?

(ii) What is the total probability that a farmer believe in new technology of agriculture?

(iii) District agriculture officer selects a farmer at random in a village and he found that selected farmer believe in new technology of agriculture, what is the probability that the farmer belongs to mohalla B ?

Q37.

Ravindra started to run a small factory of manufacturing LED bulbs. He can sell x bulbs at a price of ₹ (300 – x) each. The cost price of x bulbs is ₹ $(2x^2 - 60x + 18)$.

Based on the above information, answer the following questions :

(i) Find the profit function P(x) for selling x bulbs.

(ii) What is $\frac{d}{dx}[P(x)]$?

(iii) (a) How many bulbs should he sell to earn maximum profit ?

OR

(iii) (b) How many bulbs is he selling if he is incurring a loss of ₹ 18 ?

Q38. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ respectively.

Based on the above information, answer the following questions:

(i) Find the shortest distance between the given lines.

(ii) Find the point at which the motorcycles may collide.