NAME: _____ SECTION: _____ ROLL NO. _____

CRPF PUBLIC SCHOOL, ROHINI

PRE-BOARD I (2023-24)

CLASS XII

Time: 3 hours **MATHEMATICS SET - A** **Max Marks 80**

GENERAL INSTRUCTIONS

(i) This Question paper contains- **five sections A, B, C, D** and **E.** Each section is compulsory. However, there are internal choices in some questions.

(ii) **Section A has 18 MCQ's and 2 Assertion Reasoning based Questions** of 1 mark each.

*(iii)***Section B has 5 Very short Answer (VSA) – type questions** of 2 mark each.

(iv) **Section C has 6 Short Answer (SA) – type questions** of 3 mark each.

(v) **Section D has 4 Long Answer (LA) – type questions** of 5 mark each.

(vi) **Section E has 3 source based / case based /integrated units of assessment** (4 mark each) with sub parts.

SECTION A

Q1. Let $A = \{3,5\}$ then the number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

Q2. If the points $A(3,-2), B(k,2)$ and $C(8,8)$ are collinear, then the value of k is:

(a) 2 (b) -3 (c) 5 (d) -4

Q3. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is:

(a) 6 (b) -6 (c) 0 (d) -7

Q4. If $|A| = |KA|$, where A is a square matrix of order 2, then sum of all possible values of K is

(a) 1 (b) -1 (c) 2 (d) 0

Q5. If $A.(adjA) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |adjA|$ is equal to:

(a) 12 (b) 9 (c) 3 (d) 27

Q6. If $f(x) = \begin{cases} \frac{sinx}{x} + cosx & ,x \neq 0 \\ 2k & ,x = 0 \end{cases}$ is continuous at $x = 0$, the value of k is:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

Q7. If $x = a\cos^2\theta, y = b\sin^2\theta$, then $\frac{dy}{dx}$ is:

(a) $-\frac{a}{b}$ (b) $\frac{a}{b}\cot\theta$ (c) $-\frac{b}{a}$ (d) none of these

Q8. The feasible region for an LPP is always a _____ polygon.

(a) Convex (b) Concave (c) the feasible region depends on LPP (d) none of these

Q9. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}\, dx = ?$

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) none of these

Q10. What is the product of order and the degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 siny + \left(\frac{dy}{dx}\right)^3 cosy = \sqrt{y}$$

(a) 4 (b) 6 (c) 3 (d) not defined

Q11. The integrating factor of the differential equation $xlogx\frac{dy}{dx} + y = \frac{2logx}{x}$ is

(a) x (b) $\log(logx)$ (c) $logx$ (d) $\frac{1}{x}$

Q12. The projection of the vector $\vec{a} = 3\hat{i} - \hat{j} - 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ is:

(a) $\sqrt{14}/2$ (b) $14/\sqrt{2}$ (c) $\sqrt{14}$ (d) 7

Q13. If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\pi/6$ (b) $2\pi/3$ (c) $5\pi/3$ (d) $\pi/3$

Q14. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ is:

(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$

Q15. The value of λ for which the lines $\frac{x-5}{7} = \frac{2-y}{5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{2y-1}{\lambda} = \frac{z}{3}$ are at right angles, is

(a) 2 (b) 4 (c) -2 (d) -4

Q16. Direction cosines of a line perpendicular to both $x - axis\ and\ z - axis$ are

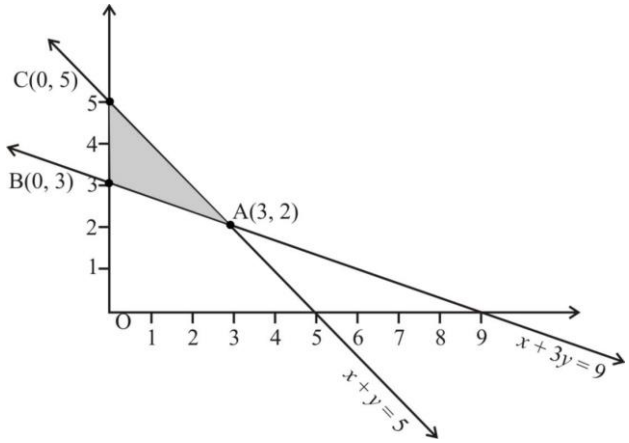
(a) 1,0,1 (b) 1,1,1 (c) 0,0,1 (d) 0,1,0

Q17. $\int e^{-x}\left(\frac{x+1}{x^2}\right)dx$ is equal to:

(a) $\frac{e^{-x}}{x}+C$ (b) $\frac{e^{x}}{x}+C$ (c) $\frac{e^{x}}{x^2}+C$ (d) $-\frac{e^{-x}}{x}+C$

Q18. The feasible region for an LPP is shown in the following figure. Then the minimum value

of Z = 11x + 7y is



(a) 21 (b) 47 (c) 20 (*d*) 31

Assertion Reasoning Based Question

Q19. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : The value of $cot\left(cos^{-1}\frac{7}{25}\right) is\ \frac{7}{24}$

Reason (R): $cot^{-1}(cot\theta) = \theta\ for\ all\ \theta \in (0, \pi)$

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c) **A** is correct but **R** is not correct
d) **A** is not correct but **R** is correct

Q20. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, then $P(A \cap B) = \frac{4}{5}$, if A ,B are independent events.

Reason (R): If A and B are independent events, then $P(A \cap B) = P(A).P(B)$

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c) **A** is correct but **R** is not correct
d) **A** is not correct but **R** is correct

SECTION B

Q21. Evaluate the following: $\sin\left(\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

OR

Write the following in the simplest form: $y = \sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$

Q22. Find the intervals in which the following function is increasing or decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Q23. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1: 4, where

$\vec{a} = 2\hat{i} + 3j + 4\hat{k}$ and $\vec{b} = -\hat{i} + j + \hat{k}$.

OR

If the sum of two unit vector is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Q24. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}+3j-6\hat{k}$ and $\vec{b}=3\hat{i}-4j-\hat{k}$.

Q25. Evaluate the following : $\int \frac{\sin 3x}{\sin x}dx$

SECTION C

Q26. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

Find $\frac{dy}{dx}$ if: $y=e^{\sin x}+(\tan x)^x$

Q27. Evaluate: $\int \frac{e^x}{(1+e^x)(2+e^x)}dx$

Q28. Find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$.

Q29. Solve the following differential equation:

$$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy=\left[2x\sin\left(\frac{y}{x}\right)-y\cos\left(\frac{y}{x}\right)\right]dx$$

OR

Find the particular solution of the differential equation

$(x^2+1)\frac{dy}{dx}-2xy=(x^4+2x^2+1)\cos x$, given that $y(0)=0$

Q30. Two persons A and B throw a coin alternately till one of them gets a 'head' and win the game. Find their respective probabilities of winning if A starts first.

Q31. Solve the following linear programming problem (L.P.P) graphically.

Minimize $Z=5x+10y$

Subject to constraints:

$$x+2y\le 120; x+y\ge 60; x-2y\ge 0; x,y\ge 0$$

SECTION D

Q32. Find the inverse of matrix $A=\begin{bmatrix}2 & 3 & 4\\ 3 & -2 & 2\\ 4 & 2 & -3\end{bmatrix}$

Hence solve the given system of equations:

$2x+3y+4z=17, 3x-2y+2z=11, 4x+2y-3z=8$.

Q33. Let $A = \{1,2,3, \ldots, 9\}$ and R is the relation in $A \times A$ defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d)$ in $A \times A$ .Prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

Q34. A perpendicular is drawn from the point $(0,2,7)$ to the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$.

Find (i) foot of perpendicular (ii) length of perpendicular (iii) image of point in the line.

OR

Find the shortest distance between the following pair of parallel lines:

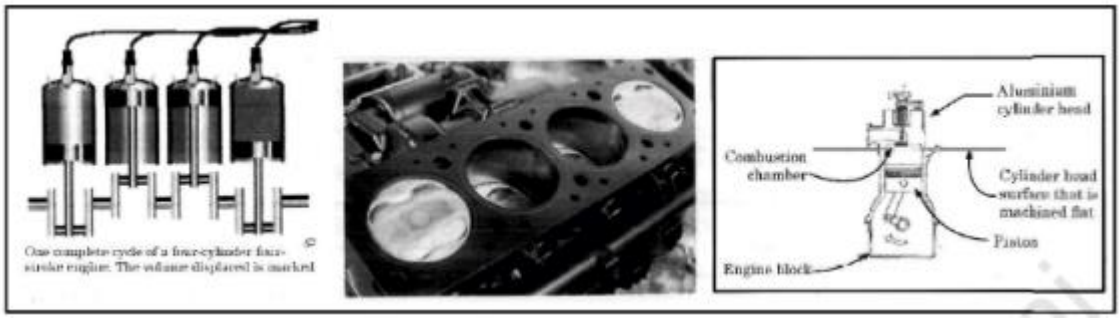
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$; $\frac{x-3}{4} = \frac{3-y}{-6} = \frac{z+5}{12}$

Q35. Evaluate : $\int_{-5}^{0} (|x| + |x+2| + |x+5|)\, dx$

SECTION E

Q36. Read the following passage and answer the questions given below:

Engine displacement is the measure of the cylinder volume swept by all the pistons of the piston engine. The piston moves inside the cylinder bore. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$.



(i) If the radius of the cylinder is r cm and height is h cm, then write the volume V of the cylinder in terms of radius r

(ii) Find $\frac{dV}{dr}$

(iii) Find the radius of the cylinder when its volume is maximum

OR

For maximum volume, $h > r$. State true or false and justify.

Q37. Read the following passage and answer the questions given below:

To reduce global warming environmentalists and scientists came up with an innovative idea of developing a spherical bulb that would absorb harmful gases and thereby reduce global warming. But during the process of absorption the bulb would get inflated and it's radius would be increasing at 1cm/sec.

(i) Find the rate at which the volume increases when radius is 6 cm.

(ii) At an instant when volume was increasing at the rate of $400\pi cm^3$/sec find the rate at which it's surface area is increasing?

Q38. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are $4:4:2$ respectively. The germination rates of three types of seeds are 45%, 60%, and 35%respectively.

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.