DELHI PUBLIC SCHOOL, GBN

SESSION 2023-24

PRE BOARD I EXAMINATION

CLASS XII- MATHEMATICS

SET A

DURATION: 3 Hour **MM: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. I to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

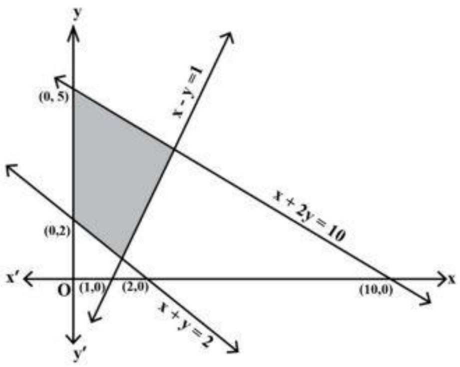
(vii) In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.

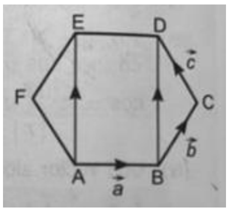
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E

(ix) Use of calculators is not allowed.

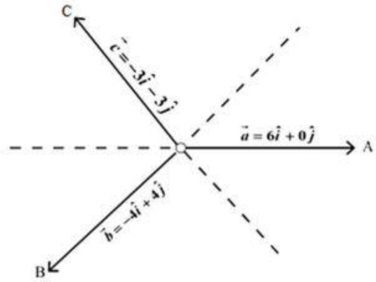
Q. No	Part No.	Question	Marks
		SECTION A (Multiple Choice Questions) Each question carries 1 mark	
1.		Let $A=\{1,2,3,\ldots,100\}$. Let a relation R be defined on A, given by $R=\{(x,y): xy \text{ is a perfect square}\}$. Then the equivalence class [2] is (a) {2, 8, 18, 32, 50} (b) {2, 8, 18, 32} (c) $\{2,8,18,32,50,72,98\}$ (d) None of these	1
2.		If $\left\|\frac{A^{-1}}{2}\right\|=\frac{1}{k\|A\|}$, where A is a 3×3 matrix, then the value of k is (a) $\frac{1}{8}$ (b) 8 (c) 2 (d) $\frac{1}{2}$	1
3.		If $\begin{vmatrix}\alpha & 3 & 4\\ 1 & 2 & 1\\ 1 & 4 & 1\end{vmatrix}=0$, then the value of α is (a)1 (b) 2 (c)3 (d) 4	1

4.		If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then $A^5 - A^4 - A^3 + A^2$ is equal to (a) 2A (b) 4A (c)3A (d) O	1
5.		The order of the matrix A such that $\begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 3 & 4 \end{bmatrix} A = \begin{bmatrix} 1 & -8 \\ 1 & -2 \\ 9 & 2 \end{bmatrix}$, is (a)$2 \times 3$ (b) 3×2 (c) 2×2 (d) 2×1	1
6.		If $(a, b), (c, d)$ and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to (a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ	1
7.		The function $f(x) = x\|x\|$ is (a) continuous and differentiable at $x = 0$ (b) continuous but not differentiable at $x = 0$ (c) differentiable but not continuous at $x = 0$ (d) neither differentiable nor continuous at $x = 0$	1
8.		If $f(x) = \log x$, then $f'(x) + f'\left(\frac{1}{x}\right) =$ (a) $\frac{x^2 - 1}{x}$ (b) $\frac{1 - x^2}{x}$ (c) $\frac{x^2 + 1}{x}$ (d) $\frac{1 + x}{x}$	1
9.		If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to (a) $\frac{2e^{2x}}{1 + e^{4x}}$ (b) $\frac{1}{1 + e^{4x}}$ (c) $\frac{2}{e^{2x} + e^{-2x}}$ (d) $\frac{1}{e^{2x} - e^{-2x}}$	1
10.		$\int_{e}^{e^2} \frac{\log x}{x} dx$ equals (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$	1
11.		Integration factor for differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$, is (a) $2\sqrt{x}$ (b) $e^{2\sqrt{x}}$ (c) $e^{\sqrt{x}}$ (d) $e^{-2\sqrt{x}}$	1
12.		The number of arbitrary constants in the particular solution of a differential equation of second order is (are) (a) 0 (b) 1 (c)2 (d) 3	1
13.		$\int e^x \left(\log\sqrt{x} + \frac{1}{2x}\right) dx =$ (a) $e^x \times \log x + C$ (b) $e^x \times \log\sqrt{x} + C$	1

		(c) $e^x \times \frac{1}{2x} + C$ (d) $\frac{e^x}{\log\sqrt{x}} + C$	
14.		The set of all points where the function $f(x) = x + \lvert x \rvert$ is differentiable, is (a)$(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$	1
15.		The lines $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) and\ \vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu(6\hat{\imath} + 9\hat{\jmath} - 18\hat{k})$ are (a) coincident (b) skew (c) intersecting (d) parallel	1
16.		The corner points of the bounded feasible region determined by a system of linear constraints are $(0,3), (1,1)\ and\ (3,0)$. Let $z = px + qy$ where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at $(3,0)$and $(1,1)$ is (a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$	1
17.		A point P lies on the line segment joining the points $(-1, 3, 2)$ and $(5, 0, 6)$. If x-coordinate of P is 2, then its z-coordinate is (a)-1 (b) 4 (c) $\frac{3}{2}$ (d) 8	1
18.		The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below Which of the following is not a constraint to the given Linear Programming Problem? (a)$x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$  	1
		ASSERTION-REASON BASED QUESTIONS Followings are Assertion-Reason based questions. In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true and R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19.		**Assertion (A) :** Acute angle between the vectors $\hat{i} + \hat{j} - k$ and $2\hat{i} + 3k$ is, $\cos^{-1}\left(\frac{1}{\sqrt{39}}\right)$. **Reason (R) :** For vectors $\vec{a}$ and $\vec{b}$, the acute angle between them is, $\cos^{-1}\left(\frac{\lvert\vec{a}.\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$.	1
20.		**Assertion (A):** if A and B are independent events then $P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B})$ **Reason (R):** for independent events $P(A \cap B) = P(A).P(B)\ and\ P(\bar{A}) = 1 - P(A)$	1

		SECTION B This section comprises of very short answer type-questions (VSA) of 2 marks each	
21.		ABCDEF is a regular hexagon, $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{CD} = \vec{c}$, find $\overrightarrow{AE}$ 	2
22.		The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?	2
23.		Find the interval/s in which the function $f: R \to R$ defined by $f(x) = xe^x$, is increasing.	2
24.		Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$. **OR** Find range of the function $f(x) = \tan^{-1} x + \frac{1}{2}\sin^{-1} x$.	2
25.		Evaluate $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$ **OR** Evaluate: $\int \sqrt{\frac{x}{1-x^3}}\, dx; x\epsilon(0,1)$	2
		SECTION C (This section comprises of short answer type questions (SA) of 3 marks each)	
26.		Show that: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx = \frac{\pi}{6}$. **OR** Find: $\int_{-1}^{2} \lvert x^3 - x\rvert dx$.	3
27.		Find $\int \frac{2x^2+3}{x^2(x^2+9)} dx; x \neq 0$	3
28.		Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$, where c is a parameter.	3
29.		If $y = (\sin^{-1} x)^2$, $x > 0$, show that $(1-x^2)\left(\frac{d^2y}{dx^2}\right) - x\left(\frac{dy}{dx}\right) - 2 = 0$. **OR** If $y = \left(x + \frac{1}{x}\right)^x + (\log x)^x$, then find $\frac{dy}{dx}$.	3
30.		Solve the following Linear Programming Problem graphically: Maximize $Z = 30x + 20y$ subject to $1.5x + 3y \leq 42$, $3x + y \leq 24$, $x \geq 0$, $y \geq 0$.	3

31.		A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to be both kings. Find the probability of the lost card being a king. **OR** An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.	3
		SECTION D (This section comprises of long answer-type questions (LA) of 5 marks each)	
32.		Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ and find the area of the region, using the method of integration.	5
33.		Let $f: R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.	5
34.		If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations: $x + 2y - 3z = 6,\ 3x + 2y - 2z = 3,\ 2x - y + z = 2$. **OR** Evaluate the product AB, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Hence solve the system of linear equations: $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.	5
35.		An aeroplane is flying along the line $\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$ where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them. **OR** Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect. Also, find the coordinates of the point of intersection.	5
		SECTION E (Question numbers 36 to 38 carry 4 marks each.) This section contains three Case-study/Passage based questions. First two questions have three sub-parts (i), (ii) and (iii) of marks 1, 1 and 2 respectively. Third question has two sub-parts of 2 marks each.	
36.		Lucifer was doing a survey on a school. Theme of the survey was 'the average number of hours spent on study' by students selected at random. At the end of survey, he prepared the following report related to the data. Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some	4

		constant. $P(X=x)=\begin{cases} 0.2, \text{ if } x=0 \\ kx, \text{ if } x=1 \text{ or } 2 \\ k(6-x), \text{ if } x=3 \text{ or } 4 \\ 0, \text{ otherwise} \end{cases}.$ **(i)** What is the value of k? **(ii)** What is the probability that the average study time of students is not more than 1 hour? **(iii)** What is the probability that the average study time to students is at least 3 hours? **OR** What is the probability that the average study time of students is exactly 2 hours?	
37.		Read the following passage and answer the questions given below: Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area. Team A pulls with force $F_1 = 6\hat{\imath} + 0\hat{\jmath}\ kN$, Team B pulls with force $F_2 = -4\hat{\imath} + 4\hat{\jmath}\ kN$, Team C pulls with force $F_3 = -3\hat{\imath} - 3\hat{\jmath}\ kN$  (i) What is the magnitude of the force of Team A ? (ii) Which team will win the game? (iii) Find the magnitude of the resultant force exerted by the teams. **OR** (iii) In what direction is the ring getting pulled?	4
38.		Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.   $\int$ Based on the above information, answer the following questions: (i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. (ii) Determine the maximum value of A(x).	4