SAMPLE QUESTION PAPER (SET-3)

CLASS:XII Marking Scheme AppliedMathematics(241)

| Sr. No. | SECTION–A | Marks |
|---|---|---|
| 1 | 157≡ 13(modm)<br>⇒157− 13≡m<br>⇒144 ≡ m<br>m is a divisor of 144 , m = 2,3,4,6,8,9,12,18,24,36,72,144<br>50 < m <144<br>(C)option | 1 |
| 2 | (B)option | 1 |
| 3 | (C) Skew-symmetric matrix | 1 |
| 4 | (B) $\|A\|^3$ | 1 |
| 5 | Speed in swimming in still water is = 8 km/h<br>Speed of stream = 4km/h<br>Speed of swimming in downstream = 8+4 = 12km/h<br>Time = DISTANCE /SPEED = 24/12 = 2hours<br>(B)option | 1 |
| 6 | (c) $x^3 / 3 - e^x + C$ | 1 |
| 7 | at R =60 and i =0.04/2 = 0.02<br>Then present value of a perpetuity<br>P = R/I = 60/0.02 = 3000<br>(a) 3000 | 1 |
| 8 | (C) Symmetric matrix | 1 |
| 9 | 14/3 (A)option | 1 |
| 10 | CAGR = [ $(1200000/200000)^{1/4}$-1] X 100<br>CAGR = (1.56508-1) x 100<br>Hence, CAGR = 56.5 % | 1 |
| 11 | t C = 40,000; n=4; S = 8000<br>Annual depreciation = C-S/n = (40000-8000)/4 = 8000<br>(A) 8000 | 1 |
| 12 | (c) $y = x^3/3 + x + C$ | 1 |
| 13 | (a) Feasibleregion | 1 |
| 14 | c) Weighted aggregative price index | 1 |
| 15 | $$\frac{95-80}{80-50}=\frac{15}{30}=\frac{1}{2}$$<br>⇒(C) option | 1 |
| 16 | Degree =3<br>(C) option | 1 |
| 17 | (B) Consumer price index | 1 |
| 18 | (B) 0.10 | 1 |
| 19 | (A) Percentage | 1 |
| 20 | (c) 2 logx/x | 1 |

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| | SECTION–B | |
| 21 | Given sample data is<br>5,8,10,7,10,14 and n = 6<br>(i) The point estimate of population mean is sample mean<br>Mean = sum/6<br>= 54/6 = 9<br>(ii) S = square root of $\Sigma (x_i - \text{mean})^2/n-1 = 3.1$ | 1<br><br><br><br><br>1 |
| 22 | $\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$<br>= 1/2[ 3( 2 –1) – 8 (– 4 – 5 )+1 (– 4 –10)] = ½ [3+72-14] = 61/2<br>OR<br>$\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ | 1<br><br><br>1 |
| 23 | Suppose x is the number of pieces of Model A and y is the number of pieces of Model B.<br>Then Total profit (in Rs) = 8000 x + 12000 y<br>Let Z = 8000 x + 12000 y<br>We now have the following mathematical model for the given problem<br>Maximize Z = 8000 x + 12000 y ..<br>(1) subject to the constraints<br>$9x + 12y \leq 180$ (Fabricating constraint)<br>i.e. $3x + 4y \leq 60$ ...<br>(2) $x + 3y \leq 30$ (Finishing constraint) ...<br>(3) $x \geq 0, y \geq 0$ (non-negative constraint) | 1<br><br><br><br><br>1 |
| 24 | Time taken by pipe to fill the tank is 3 hours<br>Therefore in 1 hour, 1/3 portion of the tank is filled<br>Again time taken by pipe to empty the tank is 4 hours<br>Therefore in 1 hour, 1/4 portion of the tank is emptied<br>So net water filled in the tank in 1 hour when both the pipes are opened = 1/3 - 1/4 = 1/12<br>Therefore time taken to fill the tank when both the pipes are opened is 12 hours | 1<br><br><br>1 |
| 25 | $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$<br>or $f'(x) = 12x^3 + 12x^2 - 24x$<br>$= 12x(x-1)(x+2)$<br>Or $f'(x) = 0$ at $x = 0$, $x = 1$ and $x = -2$.<br>Now $f''(x) = 36x^2 + 24x - 24$<br>$= 12(3x^2 + 2x - 2)$<br>Therefore, by second derivative test, x = 0 is a point of local maxima and local maximum value of f at x = 0 is f (0) = 12 while x = 1 and x = – 2 are the points of local minima and local minimum values of f at x = – 1 and – 2 are f (1) = 7 and f (–2) = –20, respectively<br>$\begin{cases} f''(0) = -24 < 0 \\ f''(1) = 36 > 0 \\ f''(-2) = 72 > 0 \end{cases}$<br>OR<br>$y = \log(\log x)$<br>Differentiating w.r.t. x, we get,By Chain Rule<br>$\frac{dy}{dx} = \frac{1}{\log x} \times \frac{1}{x} = \frac{1}{x \log x}$ | |
| 25 | SECTION–C | |

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| 26 | **Solution**: Since the money Lender deducts ₹ 200 as interest while lending a loan of ₹ 2000 for 6 months, therefore ₹ 200 may be treated as interest on ₹ 1800 for 6 months. Consequently, interest rate per six months is<br>$i = \frac{200}{1800} = \frac{1}{9}$<br>Thus, the equivalent effective rate of interest, $r_{eff}$ is given by<br>$r_{eff} = (1 + i)^2 - 1$<br>$= (1 + \frac{1}{9})^2 - 1 = 0.23456$<br>$= 23.45\ \%$<br>OR<br>P = ₹ 2,00,000<br>$I = \frac{10}{100} \times 2,00,000 \times 5$ = ₹ 1,00,000<br>n = 5 years = 5 x 12 = 60<br>EMI is given by the formula<br>$EMI = (\frac{P+I}{n})$<br>$EMI = (\frac{2,00,000+1,00,000}{60})$<br>$= \frac{3,00,000}{60}$ = ₹ 5000 | 1<br>1<br>1<br>1<br>1<br>1 |
| 27 | IAI = 3 (2 – 3) + 2(4 + 4) + 3 (– 6 – 4) = – 17 ≠ 0<br>Hence, A is nonsingular and so its inverse exists.<br>Now<br>A11 = –1, A12 = – 8, A13 = –10<br>A21 = –5, A22 = – 6, A23 = 1<br>A31 = –1, A32 = 9, A33 = 7<br>$A^{-1} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$<br>$X = A^{-1}B = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$<br>x = 1, y = 2 and z = 3. | 1<br>1<br>1 |
| 28 | $\int x log(1+x^2)dx$ ,<br>Integration by parts<br>$= \log(1+x^2).\int xdx - \int[d/dx \log(1+x^2).\int xdx]\,dx$<br>$= x^2/2\ \log(1+x^2) - \int[2x/(1+x^2).\ x^2/2]dx$<br>$= x^2/2\ \log(1+x^2) - \int x^3/(1+x^2)dx$<br>$= x^2/2\ \log(1+x^2) - \int[x - x/(1+x^2)]\,dx$<br>$= x^2/2\ \log(1+x^2) - x^2/2 + 1/2\ \log(1+x^2) + C$<br>$= 1/2\ [(1+x^2)\log(1+x^2) - x^2] + C$ | 1<br>1<br>1 |
| 29 | Let the rate at which the stream is flowing be 5 km/hr<br>And the distance covered by the boat be x km.<br>According to the question<br>3 (5 – x) = 5+x<br>⇒ 15 – 3x = 5+x<br>⇒ 4x = 10 | 1<br>1 |

$\Rightarrow$ x = 2.5

$\therefore$ The stream is flowing at the rate of 2.5 km/hr. 1

**30** 1

| Commodity | Price 2008 ($p_0$) | 2012 ($p_1$) | Quantity 2008 ($Q_0$) | 201 ($Q_1$) | $p_0Q_0$ | $p_0Q_1$ | $p_1Q_0$ | $p_1Q_1$ |
|---|---|---|---|---|---|---|---|---|
| Rice | 10 | 13 | 4 | 6 | 40 | 60 | 52 | 78 |
| Wheat | 15 | 18 | 7 | 8 | 105 | 120 | 126 | 144 |
| Rent | 25 | 29 | 5 | 9 | 125 | 225 | 145 | 261 |
| Fuel | 11 | 14 | 8 | 10 | 88 | 110 | 112 | 140 |
| | | | **Total** | | **359** | **515** | **435** | **623** |

1

Fisher's price index number = $\sqrt{\frac{\Sigma p_1Q_0}{\Sigma p_0Q_0} \times \frac{\Sigma p_1Q_1}{\Sigma p_0Q_1}} \times 100 = \sqrt{\frac{435}{359} \times \frac{623}{515}} \times 100 = 121.2$ 1

**31** 1

a. Test statistic.

$z_* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{26.4-25}{6/\sqrt{40}} = 1.4757$ 1

b. P-value.

$P(Z > z_*) = P(Z > 1.4757) = 0.0700$

c. Conclusion.

Since P-value $= 0.0700 > \alpha = 0.01$, we CAN'T reject $H_0 : \mu = 25$.

We conclude that we don't have enough evidence to claim that 1

$\mu > 25$. (Some of us would say that we accept that $\mu = 25$).

## SECTION–D

**32** Probability of defective razor blade = 0.002 2

m =np=10 x0.002 =0.02

Let X=numberofdefectiverazor bladesinasampleof 10

(i) P (nodefectiverazor blades)= 0.9802 3

(ii) P (onedefectiverazor blades)= 0.9802x0.02

= 0.0196

OR 1

X=scoresof students,$\mu$=30,$\sigma$=10

(i) When X =33,$Z$=0.3

P (X<33)=P(Z<0.3) =0.6179 2

$\Rightarrow$1000 x 0.6179 = 617.9 studentsscoredless than 33 marks

(ii) WhenX =30,$Z$=0 andwhenX =45, $Z$=1.5 2

P (30<X<45)=P(0<Z<1.5)

= 0.9332-0.5 = 0.4332

$\Rightarrow$1000 x 0.4332 = 433 studentsscoredbetween30and45marks

**33** Cost of 1 share = Rs. [25+5+1/4)] = Rs. (121/4).

Cost of 88 shares = Rs.[(121/4)*88] = Rs. 2662. 1

Investment made = Rs. 2662. 1

Face value of 88 shares = Rs. (88*25) = Rs. 2200. 1

Dividend on Rs. 100 = (15/2). 1

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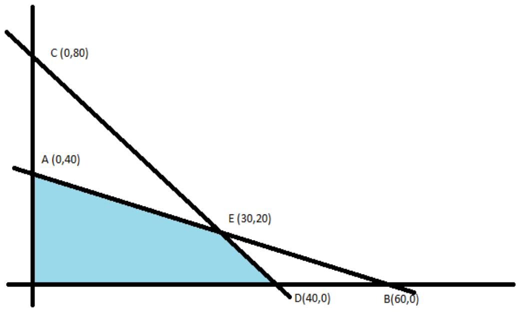
| | Dividend on Rs. 2200 = Rs. [(15/20*(1/100)*2200] = Rs. 165.<br>Income derived = Rs. 165.<br>Rate of interest on investment = [(165/2662)*100] = 6.2 %. | 1 |
| 34 | Let x be the number of packages of screws A and y be the number of packages of screws B that we can make.<br>Clearly, $x,y \geq 0$.<br>Total time on machine is 4 hr = 240 min<br>∴ for automatic machine , time = $4x+6y \leq 240 \Rightarrow 2x+3y \leq 120$<br>∴ for hand machine, time = $6x+3y \leq 240 \Rightarrow 2x+y \leq 80$<br>The shaded region is the feasible region.<br>The profits on Screw A is Rs. 0.7 and on Screw B is Rs. 1. We need to maximize the profits, i.e. maximize $z=0.7x+y$, given the above constraints.<br><br>Now, at A, z=28<br>at O, z=0<br>at E, z=41<br>at D, z=40<br>Hence, maximum profit is at point E(30,20). | 1<br>2<br>2 |
| 35 | $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$<br>put $y = vx$<br>$\frac{dy}{dx} = v + x\frac{dv}{dx}$<br>$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x^2 + x.vx}$<br>$= \frac{1+v^2}{1+v}$<br>$x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$<br>$x\frac{dv}{dx} = \frac{1+v^2-v-v^2}{1+v} = \frac{1-v}{1+v}$<br>$\int \frac{1+v}{1-v} \times dv = \int \frac{dx}{x} \Rightarrow \int\left(1+\frac{2}{v-1}\right)dv = -\int \frac{dx}{x}$<br>$\Rightarrow v + 2\log\lvert v-1\rvert = -\log\lvert x\rvert + \log C$ | 1<br>1<br>1 |

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| 35 | $\Rightarrow v + 2\log\lvert v-1\rvert = -\log\lvert x\rvert + \log C$<br>$\Rightarrow 2\log\lvert v-1\rvert + \log\lvert x\rvert = -v + \log C$<br>$\Rightarrow \log\left\lvert\frac{(v-1)^2(x)}{C}\right\rvert = -v$<br>$\Rightarrow \log\left\lvert\frac{\left(\frac{y}{x}-1\right)^2(x)}{C}\right\rvert = \frac{-y}{x}$<br>$\Rightarrow \log\left\lvert\frac{(y-x)^2}{Cx}\right\rvert = \frac{-y}{x}$<br>$\Rightarrow \frac{(y-x)^2}{Cx} = e^{\frac{-y}{x}}$<br>$\Rightarrow (y-x)^2 = Cxe^{\frac{-y}{x}}$ | 1<br><br>1 |
| | Given, $(x^2+y^2)dx - 2xydy = 0$<br>$\Rightarrow (x^2+y^2)dx = 2xydy$<br>$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ .... (i)<br>Let $y = vx$<br>Thus, $\frac{dy}{dx} = v + x\frac{dv}{dx}$<br>Thus, $v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{2x(vx)}$<br>$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$<br>$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$<br>$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$<br>$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$<br>$\Rightarrow \frac{dx}{x} = \frac{2v}{1-v^2}dv$<br>$\Rightarrow \frac{dx}{x} - \frac{2v}{1-v^2}dv = 0$ .... (ii)<br>Integrating both sides, we have<br>$\log x + \log(1-v^2) = \log C$<br>$\Rightarrow \log x(1-v^2) = \log C$<br>$\Rightarrow x(1-v^2) = C$<br>$\Rightarrow x\left(1-\frac{y^2}{x^2}\right) = C$<br>$\Rightarrow x\left(\frac{x^2-y^2}{x^2}\right) = C$<br>$\Rightarrow x^2 - y^2 = Cx$ | 1<br><br>1<br><br>1<br>1<br>1 |
| | SECTION–E | |
| 36 | CASESTUDY –I<br>(i)2800276.80<br>(ii)24167.82<br>(iii)410293.41 | <br>1<br>1<br>2 |

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| 37 | <u>CASESTUDY –II</u><br>Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.<br>It is given that, $p = 0.05$<br>$\therefore q = 1 - p = 1 - 0.05 = 0.95$<br>X has a binomial distribution with $n = 5$ and $p = 0.05$<br>$\therefore P(X = x) = {}^nC_x q^{n-x} p^x$, where $x = 1, 2, \ldots n$<br>$= {}^5C_x (0.95)^{5-x} \cdot (0.05)^x$<br><br>(i) $P(\text{none}) = P(X = 0)$<br>$= {}^5C_0 (0.95)^5 \cdot (0.05)^0$<br>$= 1 \times (0.95)^5$<br>$= (0.95)^5$ | 1 |
| 37 | (ii) $P(\text{not more than one}) = P(X \le 1)$<br>$= P(X = 0) + P(X = 1)$<br>$= {}^5C_0 (0.95)^5 \times (0.05)^0 + {}^5C_1 (0.95)^4 \times (0.05)^1$<br>$= 1 \times (0.95)^5 + 5 \times (0.95)^4 \times (0.05)$<br>$= (0.95)^5 + (0.25)(0.95)^4$<br>$= (0.95)^4 [0.95 + 0.25]$<br>$= (0.95)^4 \times 1.2$<br><br>(iii) $P(\text{more than 1}) = P(X > 1)$<br>$= 1 - P(X \le 1)$<br>$= 1 - P(\text{notmorethan1})$<br>$= 1 - (0.95)^4 \times 1.2$ | 2<br><br>1 |
| 38 | <u>CASESTUDY –III</u><br>**<u>(i)</u>** Pipe Cempties1tankin20h⇒ 2/5 x20 = 8 hours<br>**<u>(ii)</u>** Partoftankfilled in 1hour = 1/15+1/12-1/20=1/10<br>⇒timetakentofilltankcompletely=10hours<br>**<u>(iii)</u>** Let the tank be completely filled in ‘t’ hours<br>⇒pipe A is opened for ‘t’ hours | 1<br>1<br><br>2 |

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| | pipe B is opened for ‘t−3’ hours<br>And, pipe C is opened for ‘t−4’ hours<br>⇒ In one hour, part of tank filled by pipe A = $t$ /15 th<br>part of tank filled by pipe B = $t$−3 /15 th<br>and, part of tank emptied by pipe C = $t$−4/ 15 th<br>Therefore$t$/ 15 + $t$−3/ 12 − $t$−4/ 20 = 1<br>⇒$t$ = 10.5 , Total time to fill the tank = 10 hours 30 minute | | |